ON REAL CENTER SINGULARITIES OF COMPLEX VECTOR FIELDS ON SURFACES

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Abstract. One of the various versions of the classical Lyapunov-Poincaré center theorem states that a nondegenerate real analytic center type planar vector field singularity admits an analytic first integral. In a more proof of this result, R. Moussu establishes important connection between this result and the theory of singularities of holomorphic foliations ([2]). In this paper we consider generalizations for two main frameworks: (i) planar real analytic vector fields with "many" periodic orbits near the singularity and (ii) germs of holomorphic foliations having a suitable singularity in dimension two.

In this talk we discuss some versions of Poincaré-Lyapunov center theorem, including for the case of holomorphic vector fields. We also give some applications, hinting that there is much more to be explored in this framework.

References

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