Collateral Posting and Choice of Collateral Currency
-Implications for derivative pricing and risk management- *

Masaaki Fujii†, Yasufumi Shimada‡, Akihiko Takahashi§

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†Graduate School of Economics, The University of Tokyo

‡General Manager, Capital Markets Division, Shinsei Bank, Limited

§Graduate School of Economics, The University of Tokyo

Outlines

Textbook-style IR Model has been obsolete...

Textbook-style Interest Rate Model (such as LMM) has been out of use for many years, at least, in major investment banks.

- **Unable to explain non-negligible basis spread in cross currency swap (CCS) market.**

$\Downarrow$

- **Unable to construct consistent multi-currency framework.**
  - **FX forward $\leftrightarrow$ (CCS + IRS)**

Extended IR models (with multi curves) have been used for long, especially by U.S. banks, to reflect (and to exploit) funding cost asymmetry (such as Japan premium) in derivative prices.

Textbook-style IR Model has been obsolete...

New market realties after the Financial Crisis

- **Much more volatile CCS basis spread.**
- **Non-negligible basis spreads even in the single currency market.**

  Tenor swap spread, Libor-OIS spread, etc.
- **Widespread use of Collateralization.**

OTC Market and Collateralization

- Collateralization
  - The most important credit risk mitigation tool.
    - margin call, settlement and associated procedures.
    - legal specifications are provided by CSA (Credit Support Annex).
  - Dramatic increase in recent years (ISDA [4])
    - 30%(2003) $\rightarrow$ 70%(2009) in terms of trade volume for all OTC.
    - Coverage is up to 78% (for all OTC) and 84% (for fixed income) among major financial institutions.
    - More than 80% of collateral is Cash.
    - About half of the cash collateral is USD.
    - Almost all the credit derivatives are collateralized.

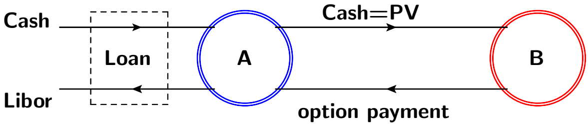
Impact of Collateralization

Impact of collateralization :

- **Reduction of Counter-party Exposure.**
  - **Associated change in CVA has been actively studied.**
- **Change of Funding Cost (topic of this talk)**
  - **Require new methodology for term structure construction.**
  - **Add "cheapest-to-deliver" optionality when there are multiple eligible collateral.**
  - **Significant impact on derivative pricing and risk management.**

Source of Funding Cost Difference

- **Unsecured Funding and Contract (old picture)**

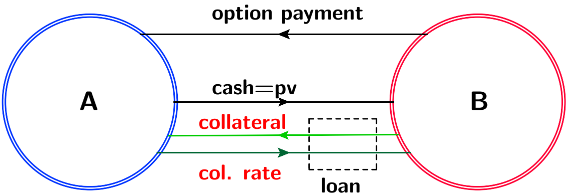


  - **Libor is unsecured offer rate in the interbank market.**
  - **Libor discounting makes the present value of Loan zero.**
  - **Libor discounting is appropriate for unsecured trades between financial firms with Libor credit quality.**

$\Rightarrow$ **Libor discounting (+ CVA) has been the standard market practice.**

Source of Funding Cost Difference

- Collateralized (Secured) Contract (current picture)

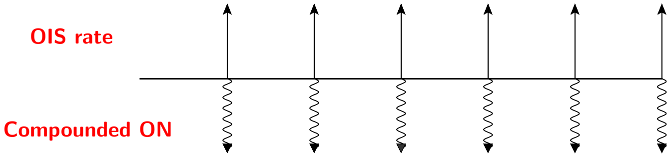


- No outright cash flow (collateral=PV)
- No external funding is needed.
- Funding is determined by over-night (ON) rate.

$\Rightarrow$ Libor discounting is inappropriate.

Important Instruments and Market Realities

- **Overnight Index Swap (OIS)**



  - **Floating side: Daily compounded ON rate**
  - **Usually, there is only one payment for $< 1yr$.**
  - **Market Quote : fixed rate, called OIS rate.**

Important Instruments and Market Realities

Historical behavior of IRS (1Y)-OIS (1Y) spreads (bps)

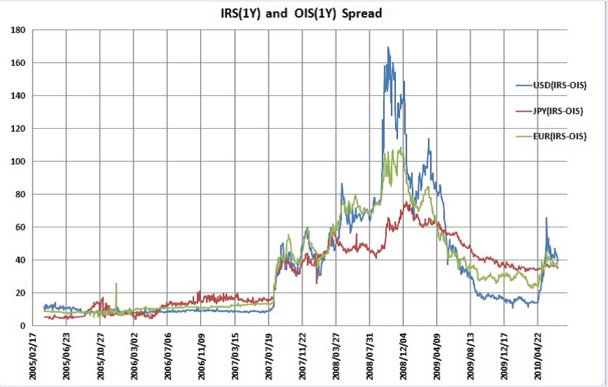
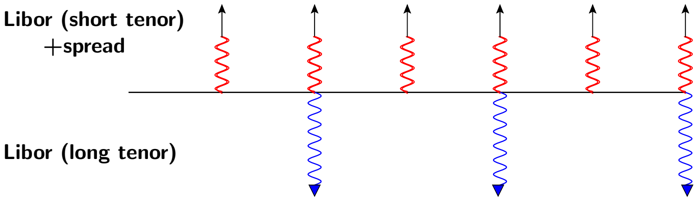


Figure: Source:Bloomberg

Important Instruments and Market Realities

- **Tenor Swap (TS)**[1]

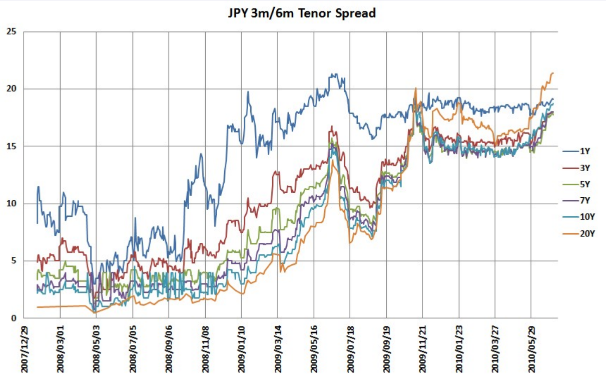


- **Textbook-style Implementation ⇒ Zero spread.**
- **Market: Spread is quite significant and volatile since late 2007.**

[1] It is also common that payment of short-tenor Leg is compounded and paid at the same time with the other Leg.

Important Instruments and Market Realities

Historical behavior of JPY TS spreads (bps)



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Important Instruments and Market Realities

Historical behavior of USD TS spreads (bps)

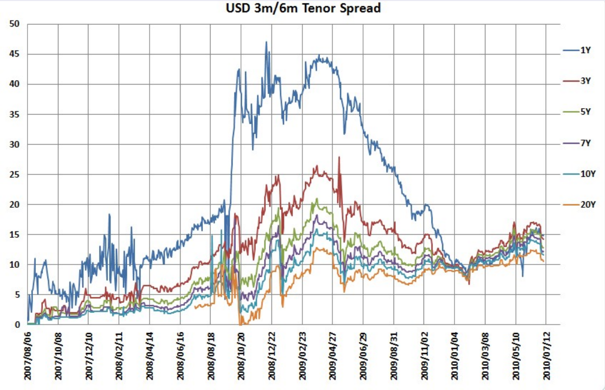


Figure: Source:Bloomberg

Important Instruments and Market Realities

Historical behavior of EUR TS spreads (bps)

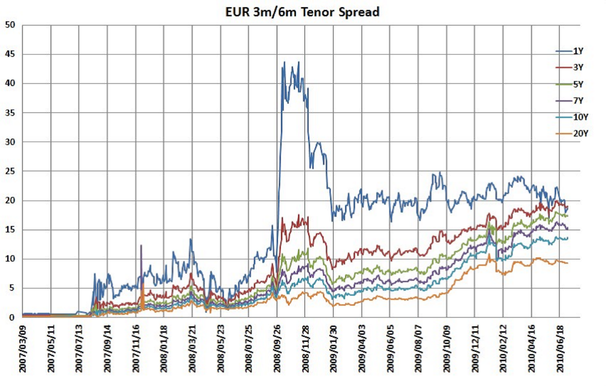
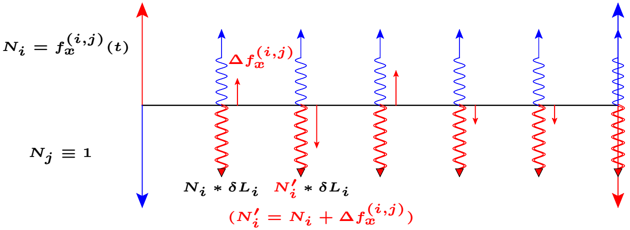


Figure: Source:Bloomberg

Important Instruments and Market Realities

- **Mark-to-Market Cross Currency Swap**



- USD Libor is exchanged by Libor +spread of the other currency.
- USD leg notional is reset every start of accrual period.
- Textbook-style Implementation ⇒ Zero spread.
- Market:
  - Spread is quite significant and volatile for long time.
  - Drastic/Rapid change in recent years.

Important Instruments and Market Realities

Historical behavior of USDJPY CCS spreads (bps)

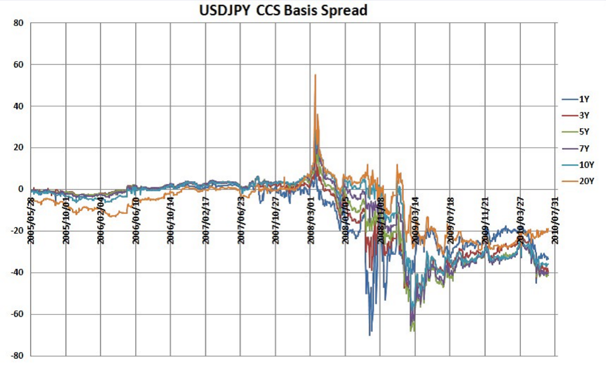


Figure: Source:Bloomberg

Important Instruments and Market Realities

Historical behavior of EURUSD CCS spreads (bps)

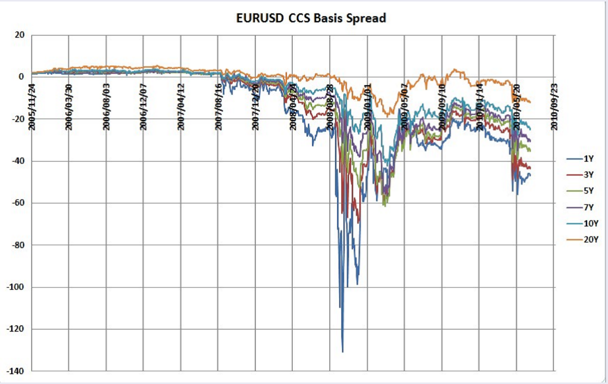


Figure: Source:Bloomberg

Important Instruments and Market Realities

It is very dangerous to use textbook-style implementation of IR models, because...

- **mispricing of various fundamental instruments:**
  - **Tenor Swap (TS)**
  - **Cross Currency Swap (CCS) $\Rightarrow$ FX**
  - **Overnight Index Swap (OIS)**
- **Hence, all the products are mispriced...**
  - **Potential loss can be $\geq$ a few % of outstanding notional.**
- **Unable to recognize the important delta exposure, such as to Libor-OIS spread.**
  - **Proper control of risk exposure is impossible.**

Criteria for Models Workable in Real Business

Criteria

- **Consistent discounting/forward curve construction**
  - Price all types of IR swaps correctly:
    - OIS, IRS and TS
  - Take collateralization into account.
  - Maintain consistency in multi-currency environment
    - CCS basis spreads need to be recovered.
- **Stochastic Modeling of Basis spreads**
  - Allow systematic calibration procedures
  - Flexible enough to allow non-trivial term structure of spreads.

Pricing under the Collateralization

- **Assumption**
  - Continuous adjustment of collateral amount
  - Perfect collateralization by Cash
  - Zero minimum transfer amount

- **Comments**
  - Daily margin call/settlement is becoming popular.
  - By making use of Repo / Reverse-Repo, other collateral assets can be converted into the equivalent amount of cash collateral.
  - General Collateral (GC) repo rate closely tracks overnight rate.

Pricing under the Collateralization

Proposition

T-maturing European option under the collateralization is given by

$$\begin{aligned} h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}\left(e^{\int_t^T y^{(j)}(s)ds}\right)h^{(i)}(T)\right] \\ &= D^{(i)}(t,T)E_t^{\mathcal{T}^c_{(i)}}\left[\left(e^{-\int_t^T y^{(i,j)}(s)ds}\right)h^{(i)}(T)\right] \end{aligned}$$

where,

$$\begin{aligned} y^{(j)}(s) &= r^{(j)}(s) - c^{(j)}(s) \quad , \quad y^{(i,j)}(s) = y^{(i)}(s) - y^{(j)}(s) \\ D^{(i)}(t,T) &= E_t^{Q_i}\left[e^{-\int_t^T c^{(i)}(s)ds}\right] \end{aligned}$$

- $h^{(i)}(T)$: option payoff at time T in currency i
- collateral is posted in currency j
- $c^{(j)}(s)$: instantaneous collateral rate of currency j at time s
- $r^{(j)}(s)$: instantaneous risk-free rate of currency j at time s
- $E^{\mathcal{T}^c_{(i)}}[\cdot]$: expectation under the fwd measure associated with $D^{(i)}(\cdot,T)$

Pricing under the Collateralization

- Collateral amount in currency j at time s is given by $\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}$, which is invested at the rate of $y^{(j)}(s)$:

$$
\begin{aligned}
h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds} h^{(i)}(T)\right] \\
&\quad + f_x^{(i,j)}(t) E_t^{Q_j}\left[\int_t^T e^{-\int_t^s r^{(j)}(u)du} y^{(j)}(s) \left(\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}\right) ds\right] \\
&= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds} h^{(i)}(T) + \int_t^T e^{-\int_t^s r^{(i)}(u)du} y^{(j)}(s) h^{(i)}(s) ds\right].
\end{aligned}
$$

Note that $X(t) = e^{-\int_0^t r^{(i)}(s)ds} h^{(i)}(t) + \int_0^t e^{-\int_0^s r^{(i)}(u)du} y^{(j)}(s) h^{(i)}(s) ds$ is a Q_i-martingale. Then, the process of the option value is written by

$$
dh^{(i)}(t) = \left(r^{(i)}(t) - y^{(j)}(t)\right) h^{(i)}(t)dt + dM(t)
$$

with some Q_i-martingale M. This establishes the proposition.

$f_x^{(i,j)}(t)$: Foreign exchange rate at time t representing the price of the unit amount of currency "j" in terms of currency "i".

Pricing under the Collateralization

Corollary

- **If payment and collateral currencies are the same, the option value is given by**

$$\begin{aligned} h(t) &= E_t^Q\left[e^{-\int_t^T c(s)ds} h(T)\right] \\ &= D(t,T) E_t^{\mathcal{T}^c}\left[h(T)\right] \ . \end{aligned}$$

- **The discounting is determined by "collateral rate", which is consistent with the schematic picture seen before.**

Building Blocks for IR Term Structure Model

Building Blocks

$$
\begin{aligned}
c^{(i)}(t,T) &= -\frac{\partial}{\partial T}\ln D^{(i)}(t,T) \\
B^{(i)}(t,T_k;\tau) &= E_t^{\mathcal{T}^c_{k,(i)}}\left[L^{(i)}(T_{k-1},T_k;\tau)\right] - \frac{1}{\delta_k^{(i)}}\left(\frac{D^{(i)}(t,T_{k-1})}{D^{(i)}(t,T_k)} - 1\right) \\
y^{(i,k)}(t,T) &= -\frac{\partial}{\partial T}\ln\left(E_t^{Q_i}\left[e^{-\int_t^T y^{(i,k)}(s)ds}\right]\right)
\end{aligned}
$$

- **These building blocks are enough to calibrate all the relevant OIS, IRS, TS and CCS.**

Construction of Term Structure

Term structure construction procedures

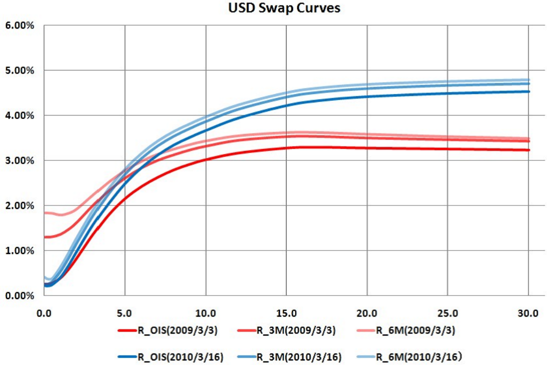
See, (Fujii, Shimada, Takahashi 2009) [1] for details.

- **(1), OIS** $\Rightarrow c^{(i)}(t,s)$
- **(2), IRS+TS+(1)** $\Rightarrow B^{(i)}(t,s;\tau)$
- **(3), CCS+(1)+(2)** $\Rightarrow y^{(i,j)}(t,s)$

- **Assume collateralization in domestic currency for OIS, IRS and TS [2].**
- **Assume collateralization in USD for CCS (USD crosses).**
- **No-arbitrage dynamics of these underlyings in HJM framework is given in (Fujii, Shimada, Takahashi 2009)[2].**

[2] Assumption on collateral currency has only minor impact on the market par quotes.

Construction of Term Structure



$$
\begin{aligned}
R_{\mathrm{OIS}}(T) &= -\ln(D(0,T))/T \\
E^{\mathcal{T}^c_m}[L(T_{m-1},T_m;\tau)] &= \frac{1}{\delta_m}\left(\frac{e^{-R_\tau(T_{m-1})T_{m-1}}}{e^{-R_\tau(T_m)T_m}}-1\right)
\end{aligned}
$$

Choice of Collateral Currency

Role of $y^{(i,j)}$

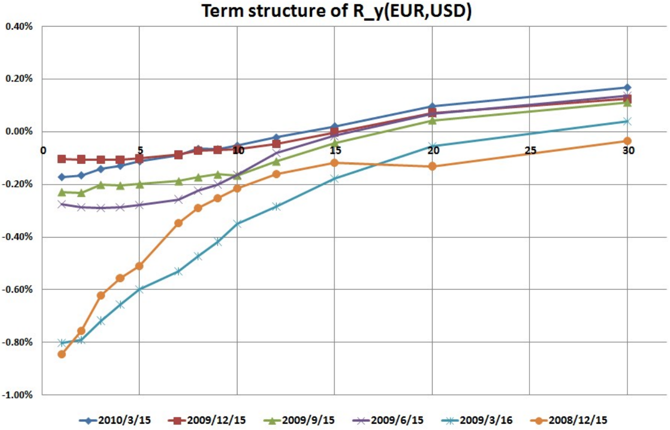
- Payment currency i with Collateral currency j

$$D^{(i)}(t,T) \Rightarrow E_t^{Q_i}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right] D^{(i)}(t,T)$$

after neglecting small corrections from possible non-zero correlations.

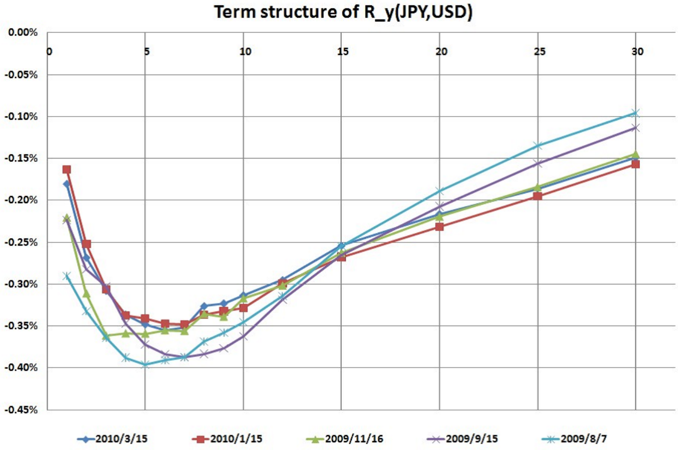
- To choose "strong" currency, such as USD, is expensive (for the collateral payer).

Choice of Collateral Currency



$$R_{y(i,j)}(T) = -\frac{1}{T}\ln\left(E^{Q_i}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right]\right) = \frac{1}{T}\int_0^T y^{(i,j)}(0,s)ds$$

Choice of Collateral Currency



Choice of Collateral Currency

Role of $y^{(i,j)}$

Optimal behavior of collateral payer can significantly change the derivative value.

- **Payment currency i with multiple currencies as eligible collateral choice $\mathcal{C}$**

$$D^{(i)}(t,T) \Rightarrow E_t^{Q_i}\left[e^{-\int_t^T \max_{j\in\mathcal{C}}\{y^{(i,j)}(s)\}ds}\right] D^{(i)}(t,T)$$

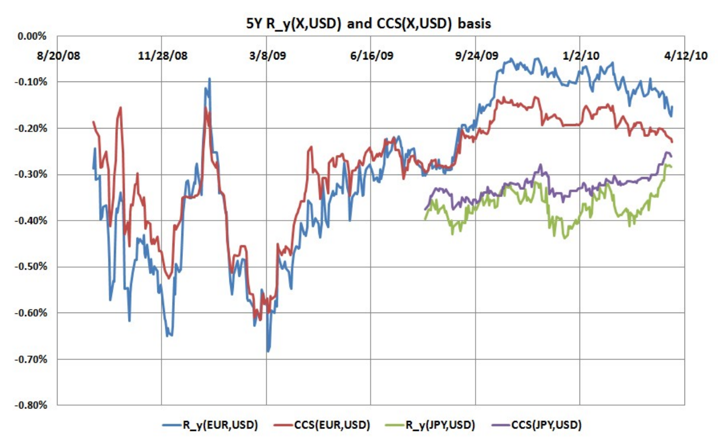
- **Payment currency and USD as eligible collateral is relatively common.**

$$D^{(i)}(t,T) \Rightarrow E_t^{Q_i}\left[e^{-\int_t^T \max\{y^{(i,USD)}(s),0\}ds}\right] D^{(i)}(t,T)$$

- **Volatility of $y^{(i,j)}$ is an important determinant.**

Choice of Collateral Currency

Close relationship to CCS basis spread



Choice of Collateral Currency

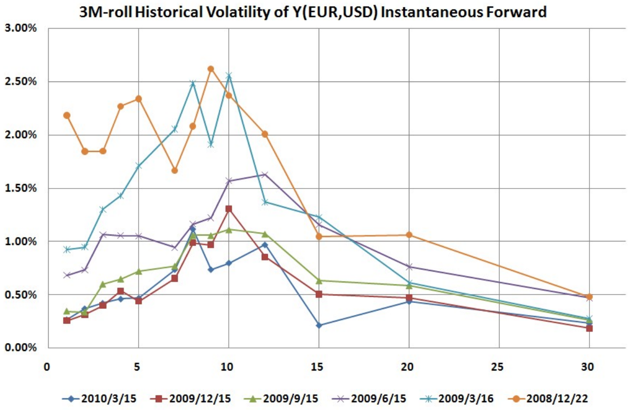


Figure: 3M-Roll historical volatility of $y^{(EUR,USD)}$ instantaneous forward. Annualized in absolute terms.

Choice of Collateral Currency

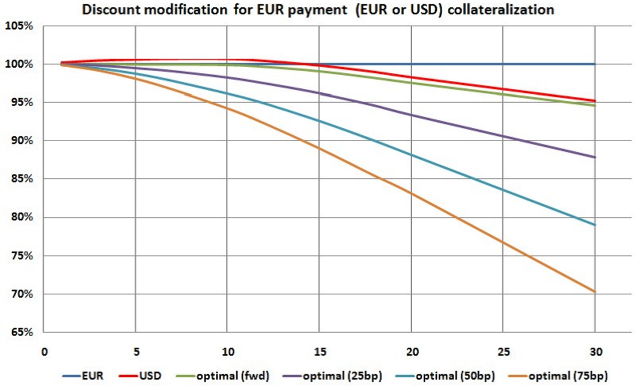


Figure: Modification of EUR discounting factors based on HW model for $y^{(EUR,USD)}$ as of 2010/3/16. The mean-reversion parameter is 1.5%, and the volatility is given at each label.

Choice of Collateral Currency

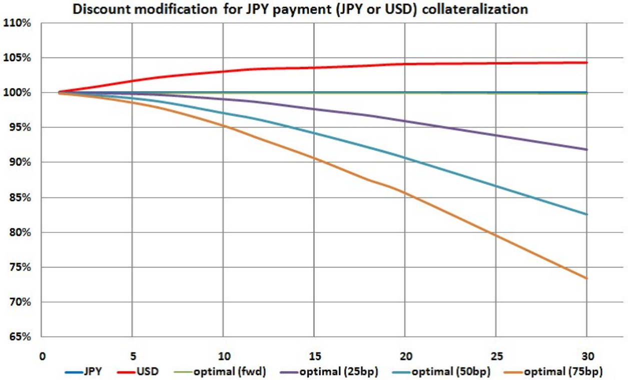


Figure: Modification of JPY discounting factors based on HW model for $\boldsymbol{y}^{(JPY,USD)}$ as of 2010/3/16. The mean-reversion parameter is $\mathbf{1.5\%}$, and the volatility is given at each label.

Conclusions

- **We proposed term structure model under the collateralization consistently with cross currency market.**
- **Choice of collateral currency is quite important.**
- **Embedded cheapest-to-deliver option can significantly change the effective discounting factor.**
- **Use of textbook-style IR model leads to significant mispricing, and unable to provide important risk exposure.**

Appendix: HJM-framework under the collateralization

$$dc^{(i)}(t,s) = \sigma_c^{(i)}(t,s)\cdot\left(\int_t^s \sigma_c^{(i)}(t,u)du\right)dt + \sigma_c^{(i)}(t,s)\cdot dW_t^{Q_i}$$

$$dy^{(i,k)}(t,s) = \sigma_y^{(i,k)}(t,s)\cdot\left(\int_t^s \sigma_y^{(i,k)}(t,u)du\right)dt + \sigma_y^{(i,k)}(t,s)\cdot dW_t^{Q_i}$$

$$\frac{dB^{(i)}(t,T;\tau)}{B^{(i)}(t,T;\tau)} = \sigma_B^{(i)}(t,T;\tau)\cdot\left(\int_t^T \sigma_c^{(i)}(t,s)ds\right)dt + \sigma_B^{(i)}(t,T;\tau)\cdot dW_t^{Q_i}$$

$$\frac{df_x^{(i,j)}(t)}{f_x^{(i,j)}(t)} = \left(c^{(i)}(t) - c^{(j)}(t) + y^{(i,j)}(t)\right)dt + \sigma_X^{(i,j)}(t)\cdot dW_t^{Q_i}$$

$$dc^{(j)}(t,s) = \sigma_c^{(j)}(t,s)\cdot\left[\left(\int_t^s \sigma_c^{(j)}(t,u)du\right) - \sigma_X^{(i,j)}(t)\right]dt + \sigma_c^{(j)}(t,s)\cdot dW_t^{Q_i}$$

$$dy^{(j,k)}(t,s) = \sigma_y^{(j,k)}(t,s)\cdot\left[\left(\int_t^s \sigma_y^{(j,k)}(t,u)du\right) - \sigma_X^{(i,j)}(t)\right]dt + \sigma_y^{(j,k)}(t,s)\cdot dW_t^{Q_i}$$

$$\frac{dB^{(j)}(t,T;\tau)}{B^{(j)}(t,T;\tau)} = \sigma_B^{(j)}(t,T;\tau)\cdot\left[\left(\int_t^T \sigma_c^{(j)}(t,s)ds\right) - \sigma_X^{(i,j)}(t)\right]dt + \sigma_B^{(j)}(t,T;\tau)\cdot dW_t^{Q_i}$$

Main References

[1] Fujii, M., Shimada, Y., Takahashi, A., 2009, "A note on construction of multiple swap curves with and without collateral," CARF Working Paper Series F-154, available at http://ssrn.com/abstract=1440633.

[2] Fujii, M., Shimada, Y., Takahashi, A., 2009, "A Market Model of Interest Rates with Dynamic Basis Spreads in the presence of Collateral and Multiple Currencies", CARF Working Paper Series F-196, available at http://ssrn.com/abstract=1520618.

[3] Fujii, M., Shimada, Y., Takahashi, A., 2010, "Collateral Posting and Choice of Collateral Currency -Implications for Derivative Pricing and Risk Management-", CARF Working Paper Series F-216, available at http://ssrn.com/abstract=1601866.

[4] ISDA Margin Survey 2010, Preliminary Results
Market Review of OTC Derivative Bilateral Collateralization Practices