Modeling Term Structure of Default Correlation

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Motivation

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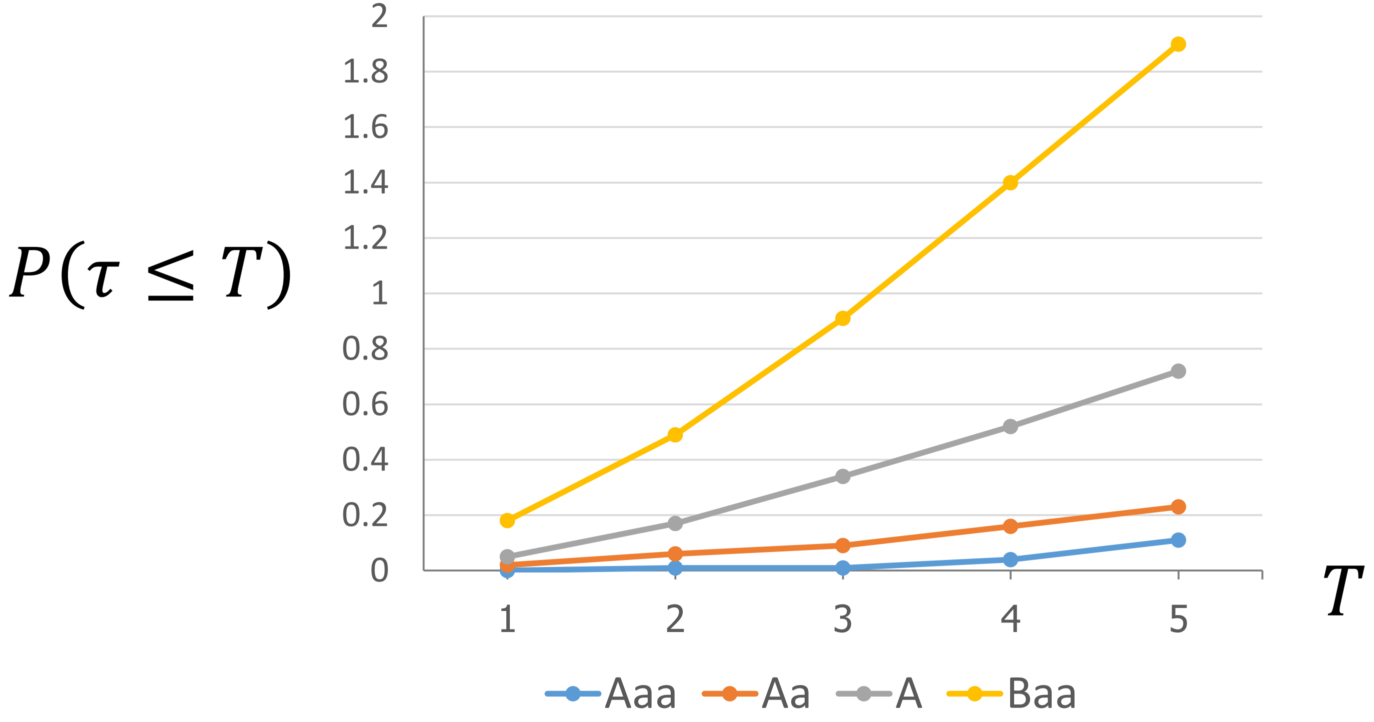
Motivation

- Multiname credit modeling:
  - ➢ Marginal Probability of Default (PD)
  - ➢ Default Correlation
- PD has a “term structure”

	1yr	2yr	3yr	4yr	5yr
Aaa	.00	.01	.01	.04	.11
Aa	.02	.06	.09	.16	.23
A	.05	.17	.34	.52	.72
Baa	.18	.49	.91	1.4	1.9

Motivation



Motivation

- Extracting term structure of PD…
  - Single-name products (i.e., CDSs) of different maturities
- What about "term structure of correlation"?...
- I.e., how correlation of $\{\tau^{(i)} < t\}$ and $\{\tau^{(j)} < t\}$ varies with t
- Extracted using different information:
  - "Correlation products" such as index tranches
  - Comovement between CDSs of different maturities

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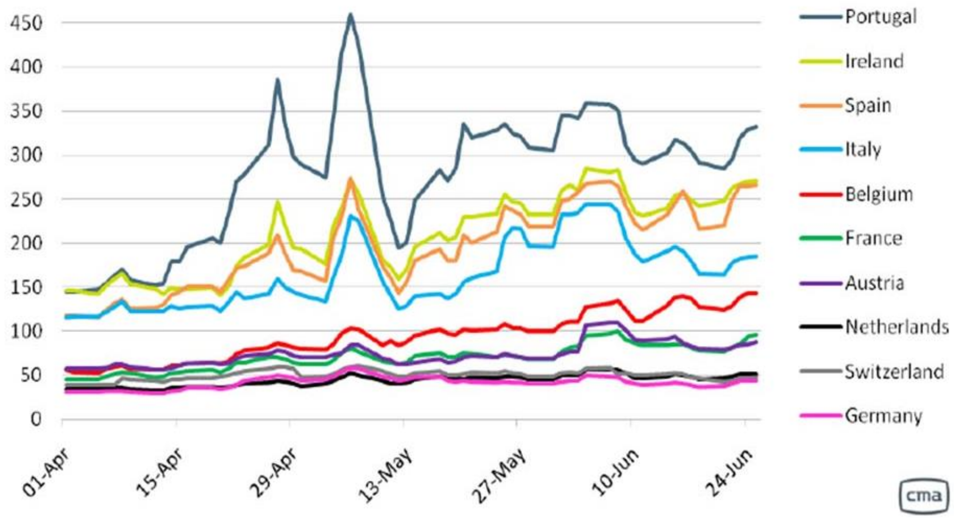
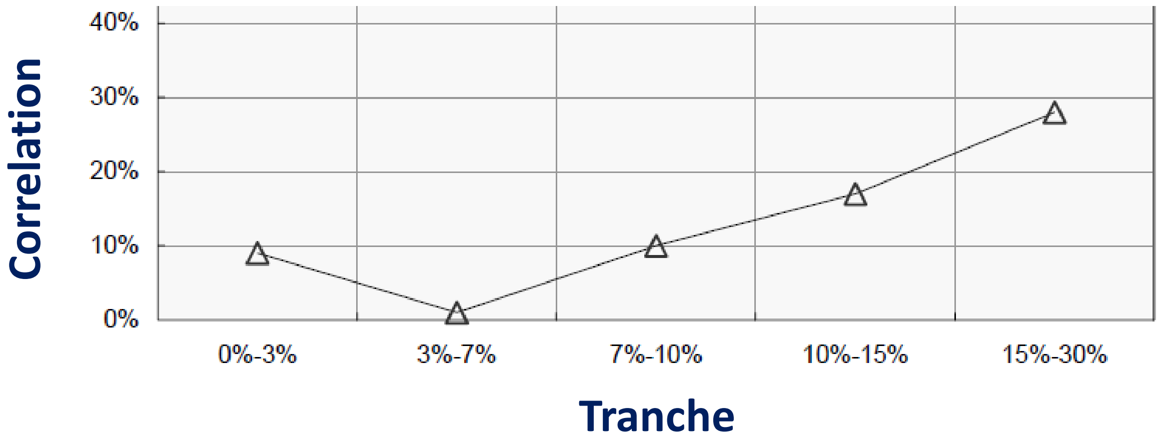
Motivation from CDS Market

5-year sovereign CDSs



- 3-yr or 10-yr CDSs also imply different levels of correlation
- Q: How to reconcile?

Motivation from Basket Credit Derivatives



- Q: Term structure of correlation might be key to correlation skew?

Objective

Develop a model that:

- Allows correlation structure to **vary with maturity**
- Imposes correlation structure ***on top of*** term structure of PD
- Consistent with **single-name & correlation products**
- **Tractable**

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Agenda:

- Existing models and challenges
- Model description
- Correlation Term Structure
- Calibration Example

Challenges in Existing Models

- Copula
- Merton's
- First-Passage
- Intensity-based Conditionally Independent Default (CID)

Challenges in Existing Models

- Copula

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad X^{(i)} > \Phi^{-1}\big(F(t)\big)$$

- Where $X^{(i)}$ is standard normal and F is cdf of $\tau^{(i)}$
- Cannot specify correlation structure that varies with t
- Attempt to turn $X^{(i)}$ into a process

Challenges in Existing Models

- Merton's (1974) model:

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad X_t^{(i)} > B^{(i)}$$

- $X_t^{(i)}$ usually interpreted as firm's net liability
- Correlation among $X_t^{(i)}$ can be made vary with t
- Schlosser and Zagst (2009), Brunlid (2006)
- But can result in "multiple defaults"

Challenges in Existing Models

- First-passage model (Hull and White (2001), Zhou (2001))

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad \sup_{s \le t} X_t^{(i)} > B^{(i)}$$

- First time process $X_t^{(i)}$ crosses barrier $B^{(i)}$
- Time-varying correlation: Metzler (2008), Hull et al. (2010), …
- $\sup_{s \le t} X_t^{(i)}$ loosely interpreted as how close we've come to default
- This "maximum-to-date" process makes the model intractable

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Challenges in Existing Models

- Intensity-based CID models:

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad \int_0^t \lambda_s^{(i)}\, ds \; > E^{(i)}$$

- Where $\lambda_t^{(i)}$ is the default intensity, $E^{(i)} \sim$ exponential
- Correlation introduced through factor structure among $\lambda_t^{(i)}$'s
- Tractable, and allow time-varying correlation
- But factor structure usually affects marginal distribution of τ

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Model Description

- Default by time t

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad \int_0^t \delta_s^{(i)} \, dN_s^{(i)} \; > \; b_t^{(i)}$$

- $\tau^{(i)} =$ first passage of a pure jump process across $b_t^{(i)}$
- $N_t^{(i)}$ is a Cox process with intensity $\lambda_t^{(i)}$
- $\delta_t^{(i)}$ is the jump size

Model Description

- Default by time t

$$\tau^{(i)} < t \quad \Longleftrightarrow \quad \int_0^t \delta_s^{(i)}\, dN_s^{(i)} > b_t^{(i)}$$

- Approximate traditional first-passage time model
- Pure-jump process approximates maximum-to-date process
- Able to calibrate to any marginal distribution of $\tau^{(i)}$

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Model Description

- Default by time t

$$\tau^{(i)} < t \qquad \Longleftrightarrow \qquad \int_0^t \delta_s^{(i)}\, dN_s^{(i)} \; > \; b_t^{(i)}$$

- How to introduce correlation?
- Let's first assume homogeneity and constant jump size…

$$\tau^{(i)} < t \qquad \Longleftrightarrow \qquad \delta N_t^{(i)} \; > b_t$$

- Correlation introduced by letting $N_t^{(i)} \; = \; M_t^{(i)} \; + \; M_t^{*}$

$$(1 - a_t)\lambda_t \qquad\qquad a_t\lambda_t$$

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Model Description

- Assuming fixed jump size δ

$$\tau^{(i)} < t \qquad \Leftrightarrow \qquad \underbrace{\delta M_t^{(i)}}_{(1-a_t)\lambda_t} + \underbrace{\delta M_t^*}_{a_t\lambda_t} > b_t^{(i)}$$

- <u>Note:</u> a_t used to allocate *intensity*, not *magnitude*
- Instead of *one* Cox process with factor structure in intensity, we've two Cox processes whose intensities are fraction of λ_t
- a_t does not affect marginal distribution of $\tau^{(i)}$

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Model Description

- Assuming fixed jump size δ

$$\tau^{(i)} < t \qquad \Longleftrightarrow \qquad \delta M_t^{(i)} + \delta M_t^{*} > b_t^{(i)}$$

$$(1 - a_t)\lambda_t \qquad a_t\lambda_t$$

- Conditional independence
- Condition on value of M_t^{*}, not on its intensity

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Model Description

- Joint default probability

$$P\left(\tau^{(i)} < t,\ \tau^{(j)} < t\right)$$

$$= P_{m_t^*}(b_t/\delta) + \sum_{k=0}^{\lfloor b_t/\delta \rfloor} p_{m_t^*}(k)\left[P_{m_t}(b_t/\delta - k)\right]^2$$

- where $p_\nu(x) = e^{-\nu}\nu^x/x!$ and $P_\nu(x) = 1 - \sum_{k=0}^{\lfloor x \rfloor} p_\nu(k)$
- $m_t^* = \int_0^t a_s \lambda_s ds$ and $m_t = \int_0^t (1 - a_s)\lambda_s ds$

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Model Description

- For portfolio of credit instruments
  - Laplace transform of its aggregate loss is available
  - Use inverse transform to compute loss distribution

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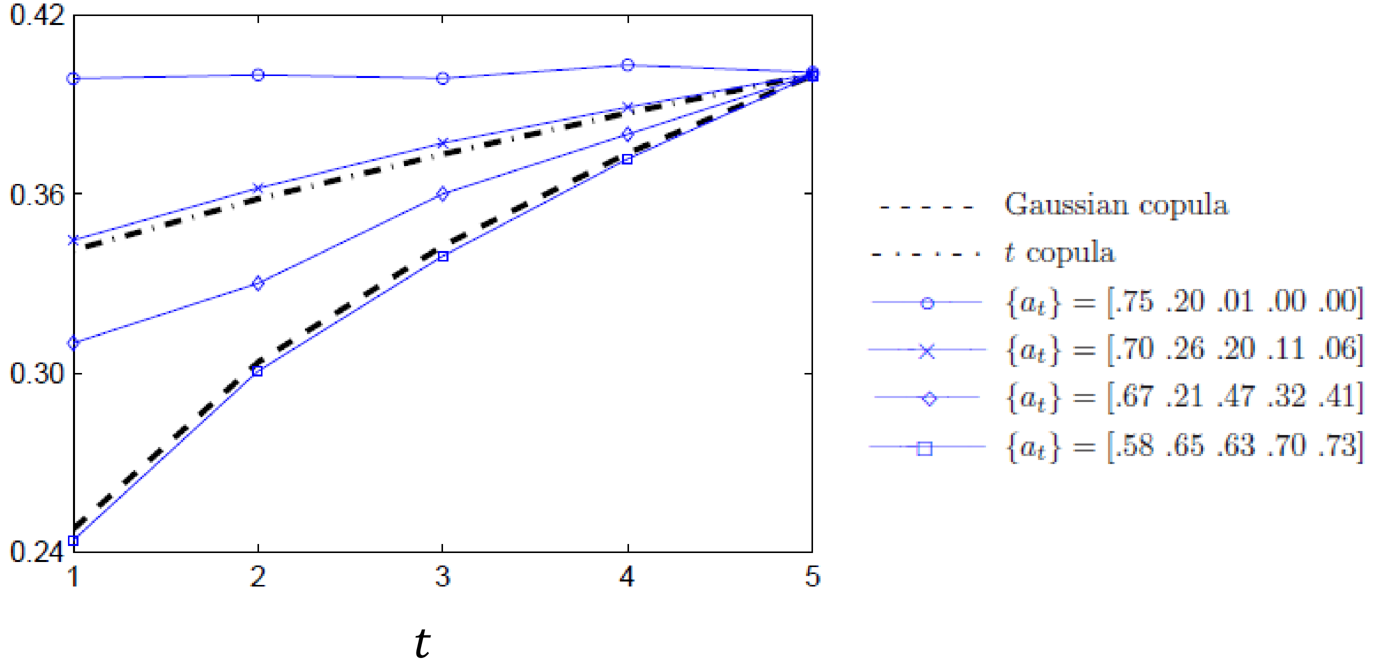
Modeling Term Structure of Correlation

- How correlation between $\{\tau^{(i)} \leq t\}$ and $\{\tau^{(j)} \leq t\}$ depends on t
- This term structure of correlation is controlled by a_t

Tail Dependence
$P\left(\tau^{(i)} \leq t \mid \tau^{(j)} \leq t\right)$



Base Correlation Curve

- Tranche pricing, detached at 3%, 6%, 9%, 12%, 22%
- Use correlation term structure to control base correlation curve

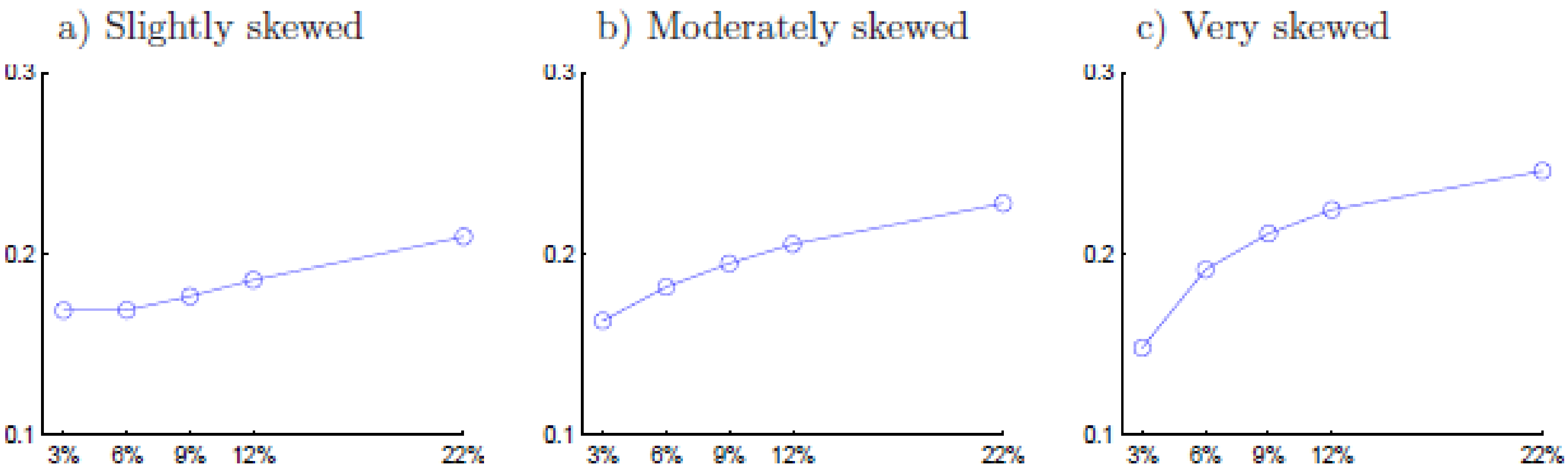


Figure 2: Base correlation curves using three different term structures of correlation. The leftmost curve uses $\{\varrho(t)\}_{t=1,\ldots,5} = [.00\ .04\ .10\ .17\ .26]$, the middle $[.00\ .02\ .08\ .17\ .28]$, and the rightmost $[.00\ .01\ .06\ .16\ .29]$

Extension

- Correlation between recovery rate and default
- Random correlation
- Random jump size to take care of clustering
- Randomizing the jump size equivalent to multifactor model

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Calibration Example

- 5-year iTraxx Europe monthly fixings, Mar ’08 – Jan ’09

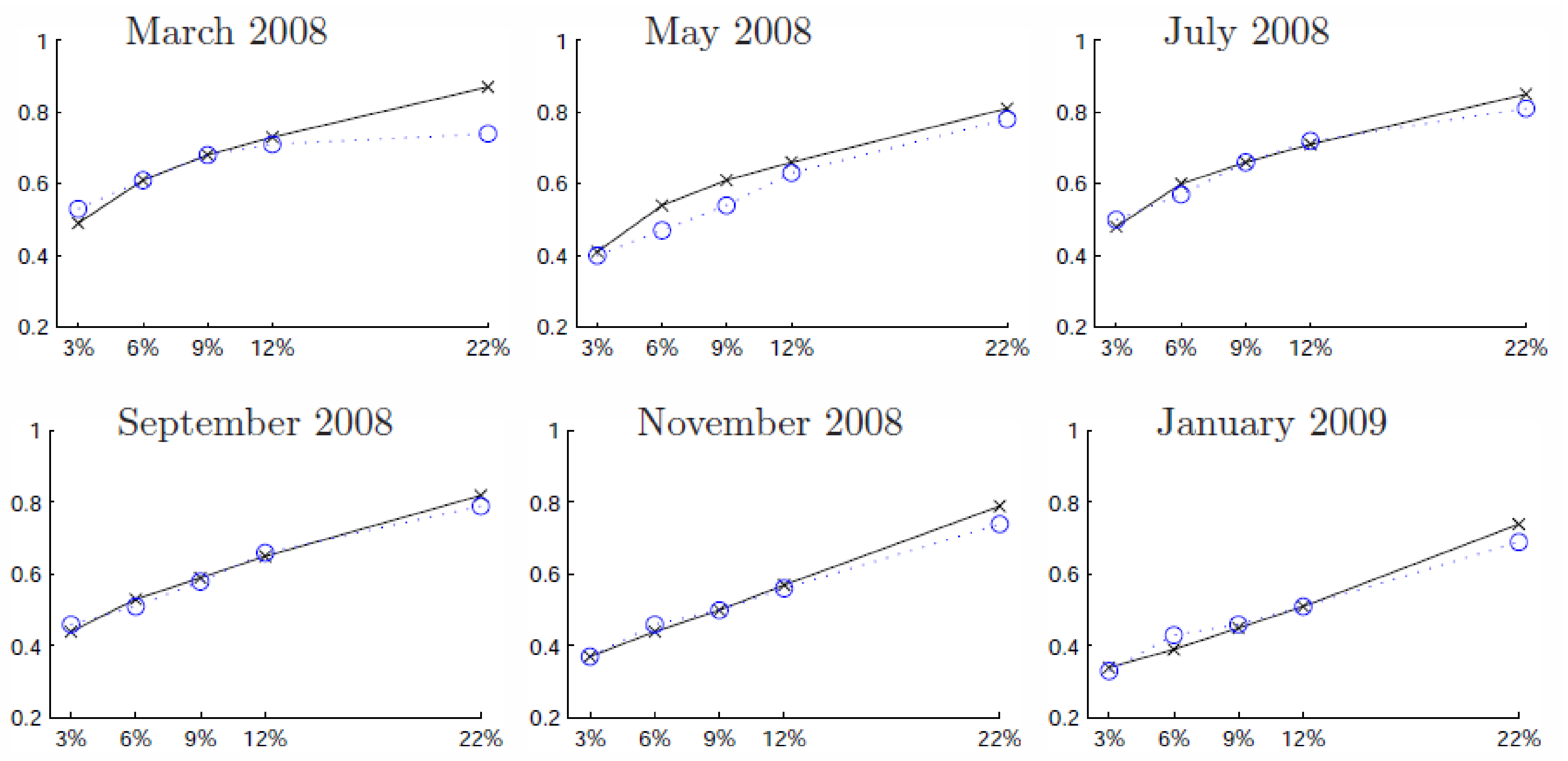
	0–3%	3–6%	6–9%	9–12%	12–22%	Index Level
Mar ’08	40%	484	310	216	110	123
May ’08	34%	301	189	127	62	80
Jul ’08	31%	356	220	141	70	90
Sep ’08	47%	672	388	208	97	130
Nov ’08	64%	1176	601	325	127	180
Jan ’09	64%	1186	607	316	97	165

- Calibrate the correlation, assuming

$$P(\tau < t) = 1 - e^{-st/(1-R)}$$

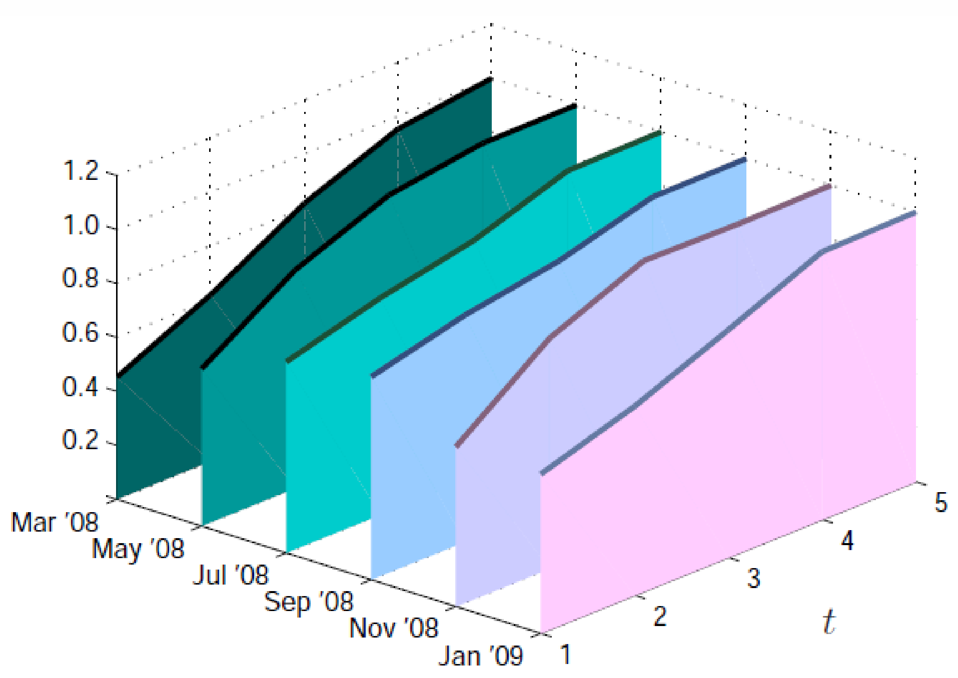
Calibration Example

- Base correlation:



Calibration Example

- Can produce the skew by calibrating only the correlation…



- Note: compared with “implied copula” (Hull 2006)

Conclusion

- **Motivate the concept of correlation term structure**
  - Implied from market data
  - Important role in explaining correlation skew
- **Develop a model of correlated default that:**
  - Imposes term structure of correlation on top of PD
  - Easy to calibration to single-name & correlation products
  - Tractable, even under generalizations

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