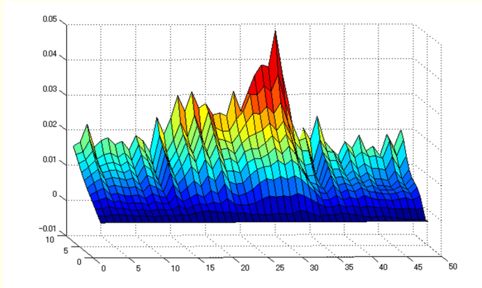
Measuring Credit Risk of Individual CBs and Deriving Term Structure of Default Probabilities

-Market approach



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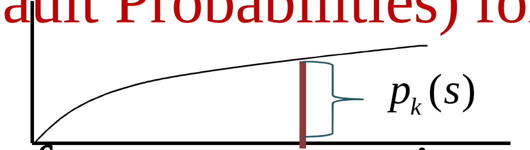
Abstract Via Kariya's cross-sectional CB model (2012) with GB model in Kariya, et al (2012), for individual CBs we define a measure of **CRPS** (**Credit Risk Price Spread**) for the k-th CB by

$$y_k^{(i)} = V_k - \hat{P}_k^{(i)}, \quad \hat{P}_k^{(i)} = \sum_{j=1}^{M(k)} C_k(s_{kj}) \overline{D}_k^{(i)}(s_{kj})$$

and derive the **TSDP**s (Term Structure of Default Probabilities) for the k-th firm; $p_k(s) = P(\tau_k \le s)$



- Market Prices are formed by investors with a future perspective on credit risks and take into account **bond attributes of GBs and CBs**
- **Credit-homogeneous groups** via R&I rating, cluster analysis and **FIR (fixed interval rating)** via S-CRPS are analyzed and compared where industry category is combined.
- **TSDPs** of these groups and some individual firms are derived. For TEPCO and Mitsubishi Corp, time series changes of TSDPs are described.

Basic information on credit risk

Forward-Looking vs Backward-Looking

Backward-Looking：model using past time series data over a period, *generated under different regimes*

ー interest rates：statistical or econometric model

ー credit data: past defaults and non-defaults, intensity model, classification, transition, logit-probit model

Forward-Looking :model using current (cross-sectional) market price data that reflects investors' future views, projection and perspectives on economic and financial movements of firms, given the past time series information

—interest rate：current GB prices, swap rates

ー **credit:current CB prices, CDS, stock prices**,

Information content of prices at *n* of CBs

CB prices reflect

- Investors' evaluations, views and perspectives on the term structure of credit risk in the CBs over maturities, given past and current information
- The evaluation includes their considerations on the business portfolio structure of each firm.
- Hence prices at n of many CBs implicitly carry the investor's view on the **TSDP** for each specific features of firms, provided the CB market is efficient.
- **In short, the information contains investors' forward-looking evaluation on the TSDP of CBs for existing firms that have different portfolio of business lines.**

GB prices and Attribute Effects

- **GB prices are formed on investors' different motives**
- Does GB prices reflects bond-attribute effects？: investors' coupon preference and maturity preference
- **Motives** :Investment (Buy & Hold) or Trading or both

  on their future perspectives,
- **Institutional Investors (life insurance, pension etc)**: ALM, Duration⇔coupon & maturity preferences
- **The existence of these attribute effects denies the empirical validity of no-arbitrage theory in math finance（logic of trading motives）**
- **Testing Hypothesis of No Attribute Effect**

Yield Approach vs Price Approach

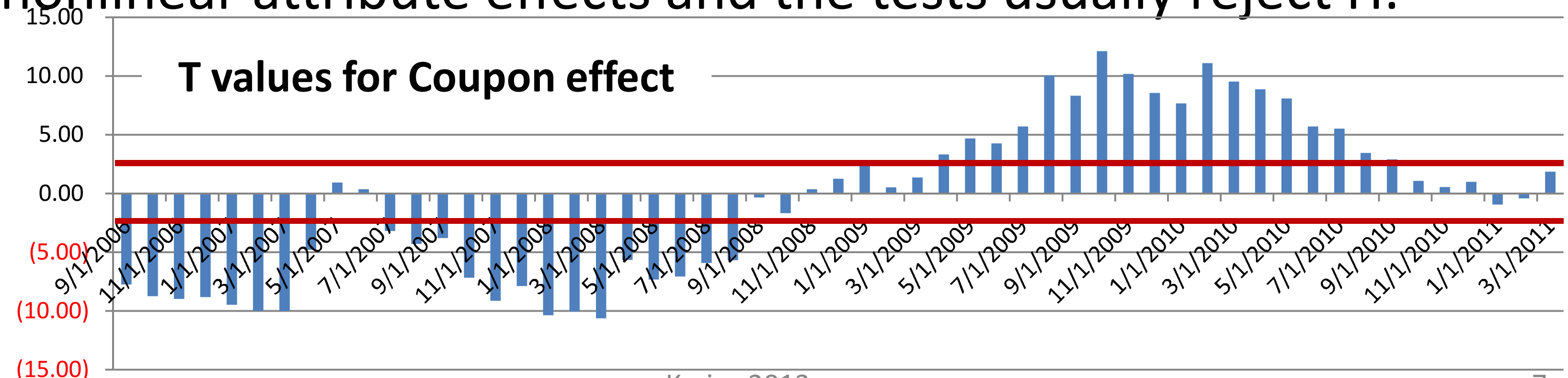
- Attribute effects are directly included in each prices
- Hypothesis of No attribute effect is tested either by price model or by yield model
- Traditionally it is often tested and rejected in a yield model (YTM, par rate) by assuming the linearity between yield and attributes
- But, this approach fails to specify a model that takes them into account in order to derive yields that get rid of the attribute effects
- Our bond model by price approach aims to price individual GBs .

Yield Approach

- GB price is a convex function of YTM ⇒YTM is a function of coupon, maturity and price, and it is convex in price
- A yield curve derived from all the GB prices is often regarded as risk-free or zero mean curve and the spreads between the mean yields and yields of individual bonds are used to test the hypothesis of no attribute effect by such a model

$$\text{Spread(M)} = \alpha + \sum_{i=1}^{9} \beta_i MT_i + \gamma Coupon$$

- However, the mean curves themselves are distorted by the nonlinear attribute effects and the tests usually reject H.



Testing Hypothesis of
Ho : No Attribute Effect

H：No　Attribute　Effect

１）what is GB-equivalent CB price?

２）Investors’ behaviors in the GB market

３）Maturity preference and coupon preference

Attribute-dependent GB pricing model

Currently at t=0, future CFs are generated at

$$s_{g1} < s_{g2} < \cdots < s_{gM(g)} \qquad (g = 1, \cdots, G)$$

$C_g(s)$; CF function＝0 unless s=s_{gj} $\qquad s_{aM(a)} = \max_g s_{gM(g)}$

$D_g(s)$; At-dependent stochastic DF $\qquad 0 < s \le s_{aM(a)}$

$$P_g = \sum_{j=1}^{M(g)} C_g(s_{gj}) D_g(s_{gj}) \qquad (g = 1, \cdots, G)$$

P_g realization ⟺ $\{D_g(s) : 0 \le s \le s_{gM(g)}\}$

$$D_g(s) = \overline{D}_g(s) + \Delta_g(s)$$

$$P_g = \sum_{m=1}^{M(g)} C_g(s_{gm}) \overline{D}_g(s_{gm}) + \eta_g \qquad \eta_g = \vec{C}_g' \vec{\Delta}_g$$

$$\underset{\sim}{y} = X \underset{\sim}{\beta} + \underset{\sim}{\eta} \qquad Cov(\underset{\sim}{\eta}) = (Cov(P_g, P_h)) \equiv \sigma^2 \Phi(\rho, \xi)$$

Attribute-Dependent Mean Discount Function

(w_1, w_2, w_3) $z_{g1}=1$, z_{g2} : maturity, z_{g3} : coupon

M0 : (1,0,0); basic model with no attributes
M1 : (1,1,0): M0 + maturity effect,
M2 : (1,0,1); M0 + coupon effect
M3 :(1,1,1); M0 +maturity effect+coupon effect

$$\bar{D}_g(s) = 1 + (\delta_{11} w_1 z_{g1} + \delta_{12} w_2 z_{g2} + \delta_{13} w_3 z_{g3}) s + \cdots + (\delta_{p1} w_1 z_{g1} + \delta_{p2} w_2 z_{g2} + \delta_{p3} w_3 z_{g3}) s^p$$

$$\bar{D}_g(s_{gj}) = E[D_g(s_{gj})] = E[\exp(-\int_0^{s_{gj}} f_{gs} ds)]$$

$$R_s = -\frac{1}{s} \log \bar{D}(s)$$ Term structure of interest rates

Covariance Structure and GLS Estimation

$$[Cov(P_g, P_h)] = [Cov(\eta_g, \eta_h)] = \sigma^2[\lambda_{gh}\varphi_{gh}] = \sigma^2\Phi(\rho, \xi)$$

Attribute-dependent cov naturally induces duration effect

$$\varphi_{gh} = \sum_{j=1}^{M(g)} \sum_{m=1}^{M(m)} C_g(s_{gj}) C_h(s_{hm})$$

$$\lambda_{gh} = \begin{cases} \sigma^2 & (g = h) \\ \sigma^2 \rho \exp(-\xi |s_{gM(g)} - s_{hM(h)}|) & (g \neq h) \end{cases}$$

$$\underset{\sim}{y} = X \underset{\sim}{\beta} + \underset{\sim}{\eta}$$

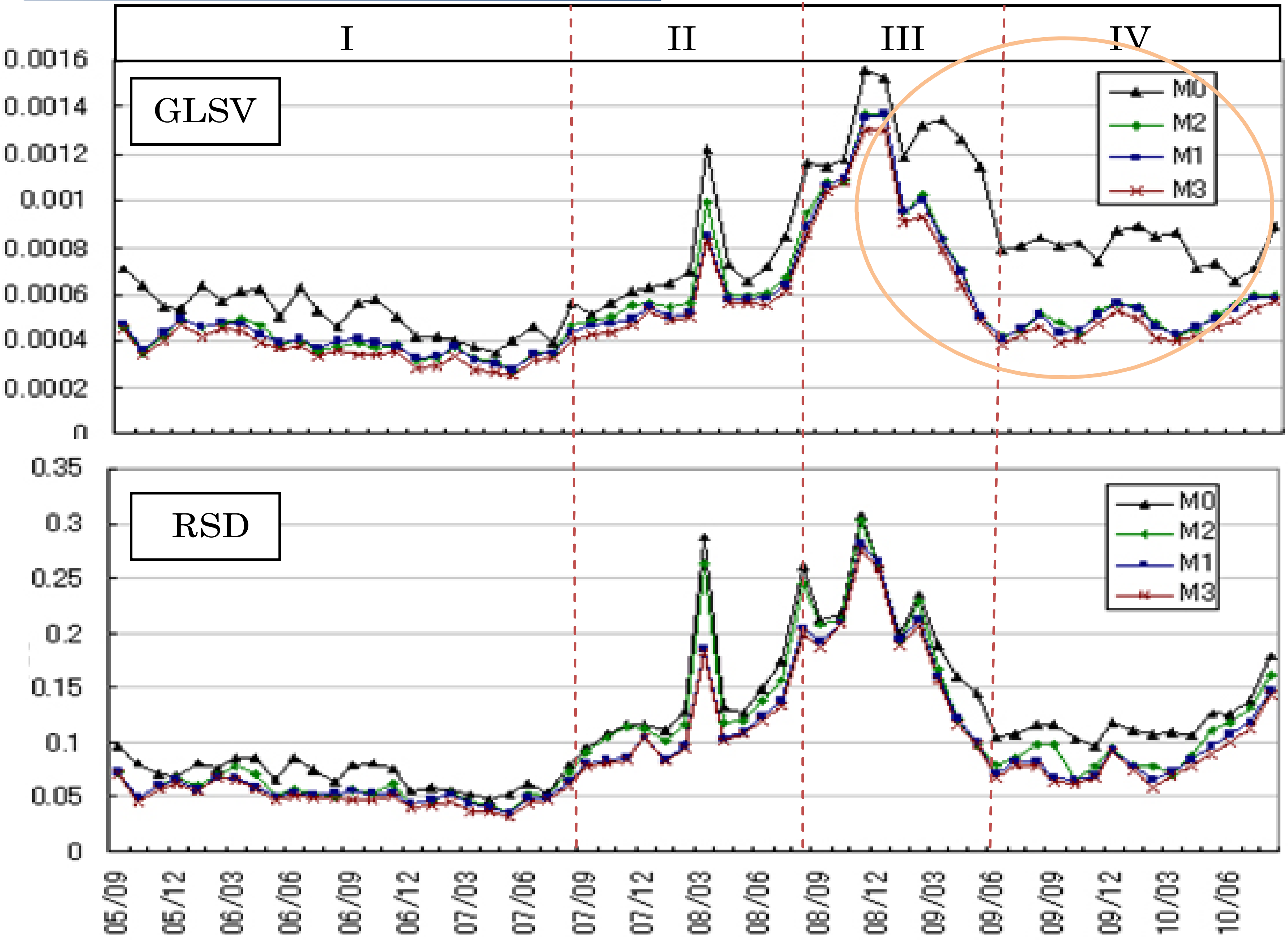
GLS Estimation to minimize

$$\psi(\underset{\sim}{\beta}, \rho) = [\underset{\sim}{y} - X\underset{\sim}{\beta}]'[\Phi(\rho, \xi)]^{-1}[\underset{\sim}{y} - X\underset{\sim}{\beta}]$$

Kariya&Kurata(2004) Generalized Least Squares Wiley

Performances of 4 models:M0-M3

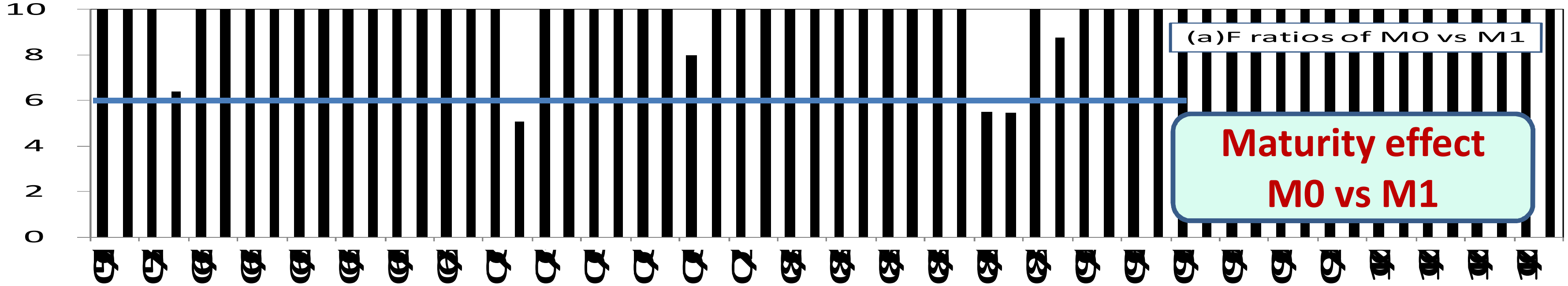
Monthly analysis 2005.09-2010.08

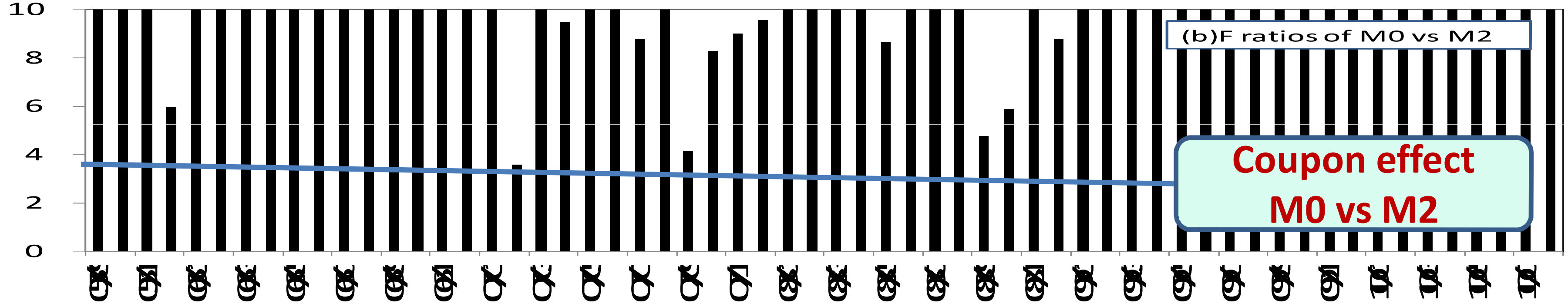


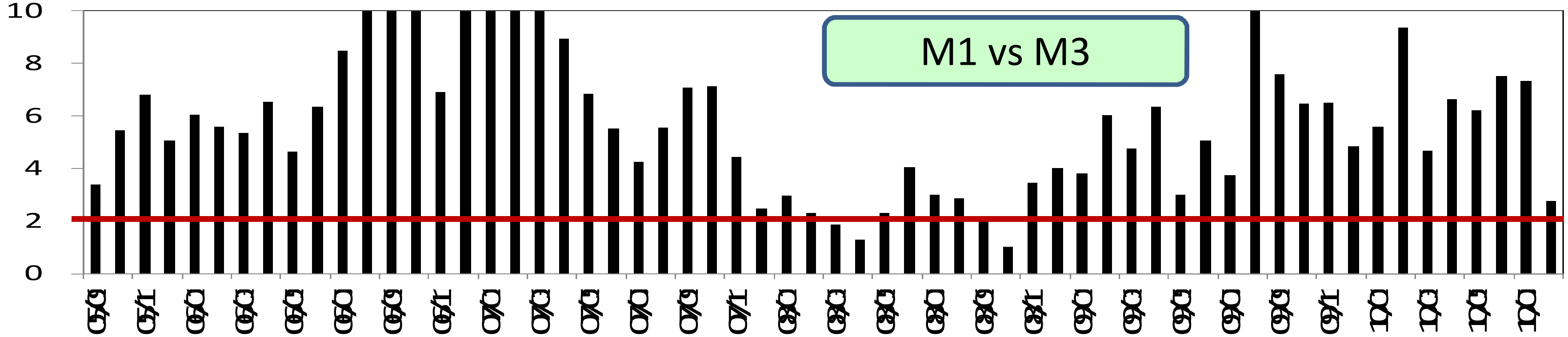
F ratios of no attribute effects JGB $\hat{\psi}$

F ratio $=[\{QSR(0) - QSR(1)\}/\#]/[QSR(1)/df]$,

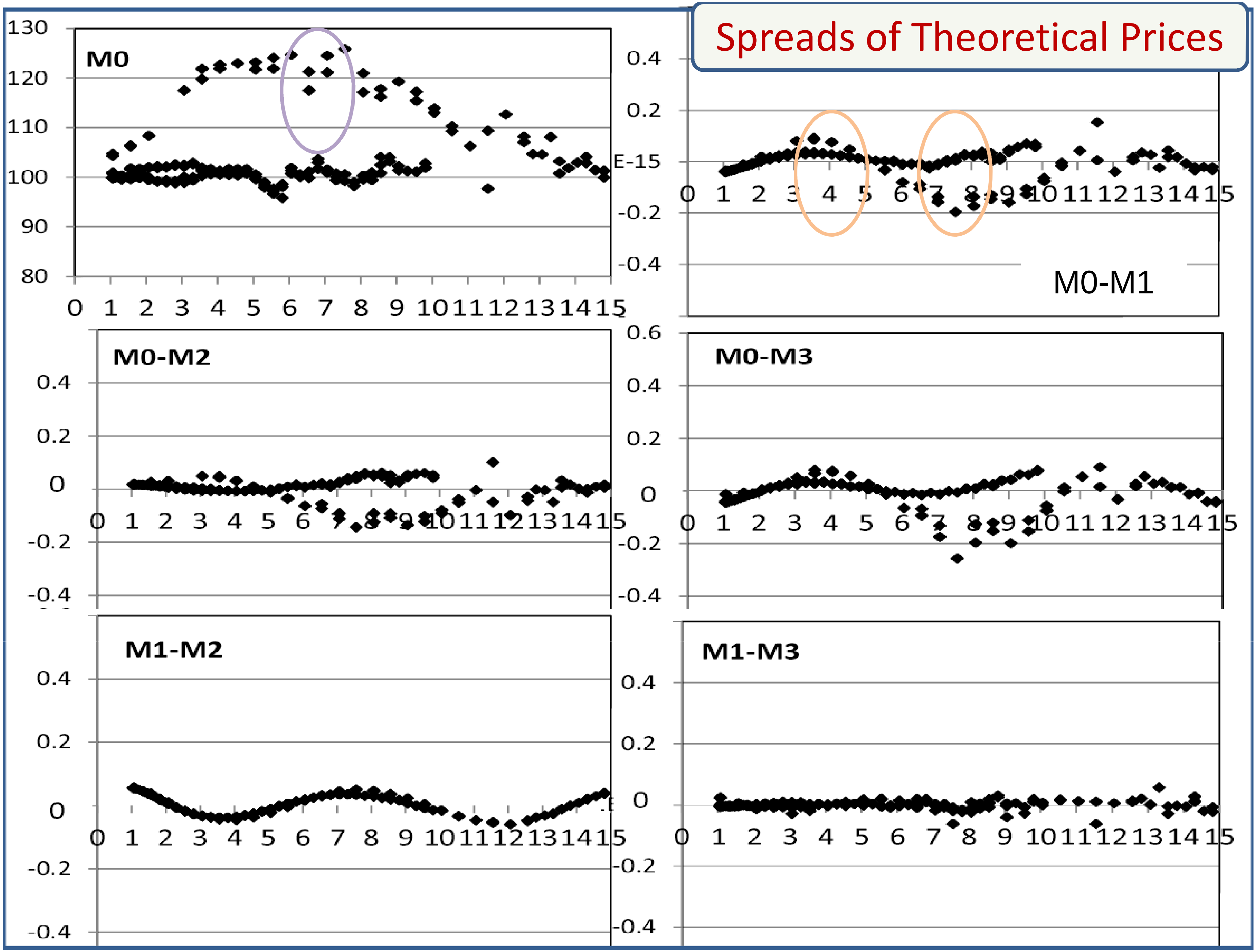
$[\{QSR(0) - QSR(1)\}/\#] > 2[QSR(1)/df]$.







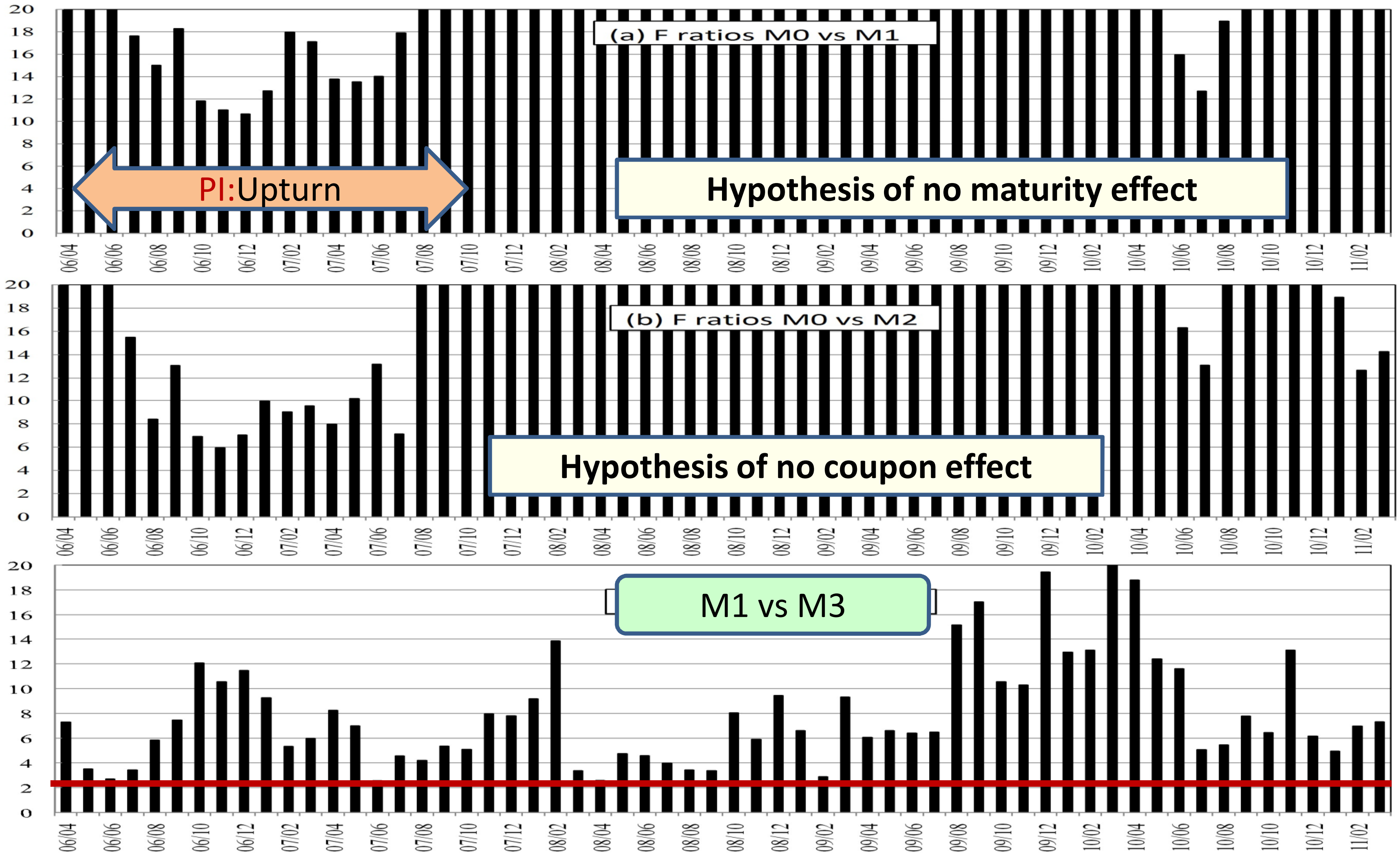
Spreads of Theoretical Prices



F-ratios of no attribute effects based on $\hat{\psi}$ USGB

$$F \text{ ratio} = [\{QSR(0) - QSR(1)\}/\#]/[QSR(1)/df],$$

$$[\{QSR(0) - QSR(1)\}/\#] > 2[QSR(1)/df].$$



CRPS for individual CBs and R&I ratings

2010.8 Cross-section analysis

１）Current CRPS Information

２）Do such categorical information as agency's rating and industry provide credit-homogeneous groups ?

CRPS & S-CRPS for CBs

Mean DFs derived From GB prices

- Individual credit risk price spread (CRPS):

CRPS＝CB price－its GB-equivalent CB price with the same coupon and maturity

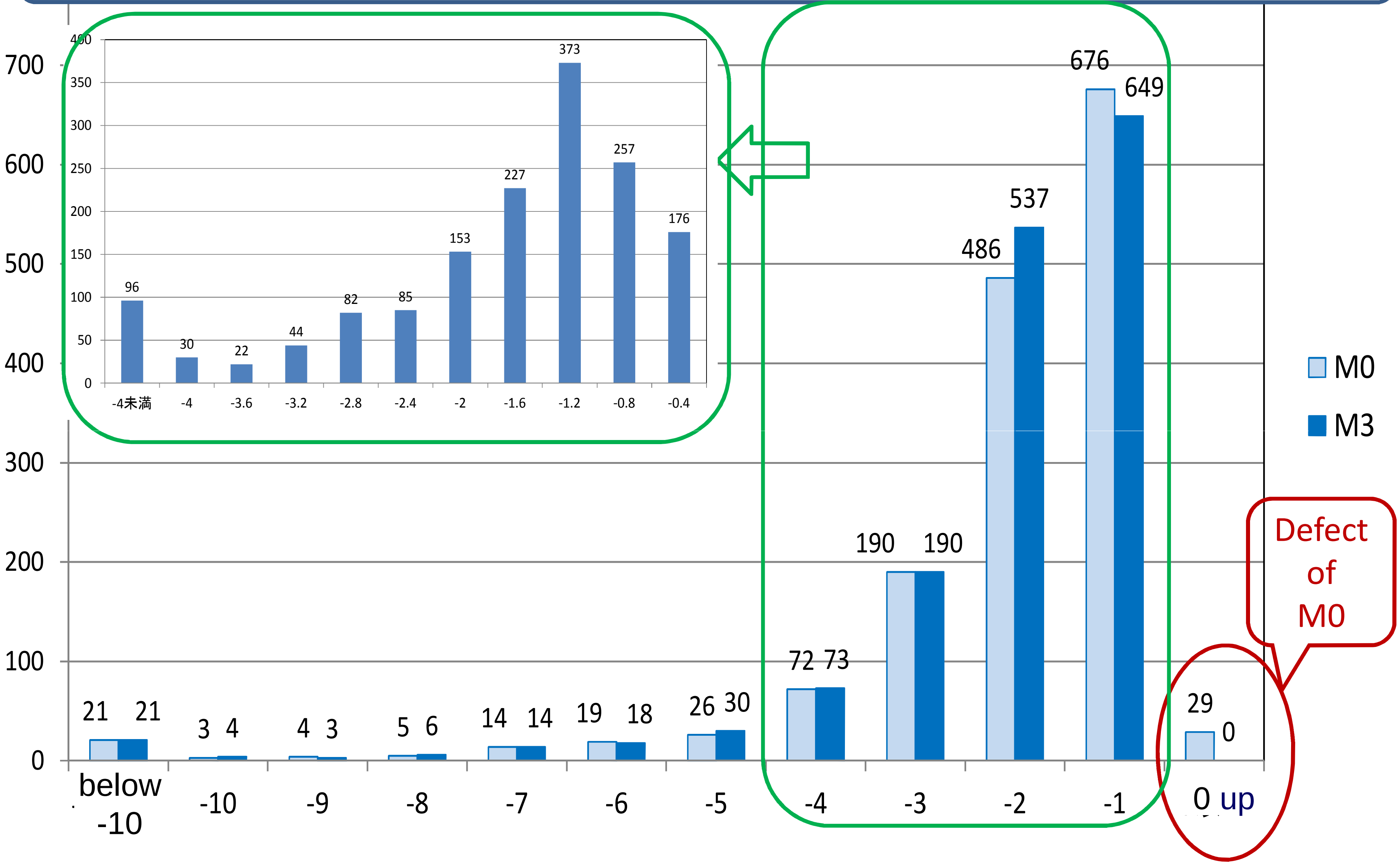
$$y_k^{(i)} = V_k - \hat{P}_k^{(i)}, \quad \hat{P}_k^{(i)} = \sum_{j=1}^{M(k)} C_k(s_{kj}) \overline{D}_k^{(i)}(s_{kj})$$

$(i = 0, 3)$ M0 Spread vs M3 Spread

- Standardized CRPS =**S-CRPS**（Standardization of uncertainty）

$$\varsigma_k^{(3)} = y_k^{(3)} / s_{kM(k)}$$

Distributions of CRPS of M0 & M3 with 1 yen interval



Credit-homogeneous grouping

- R&I rating as of 2010.8

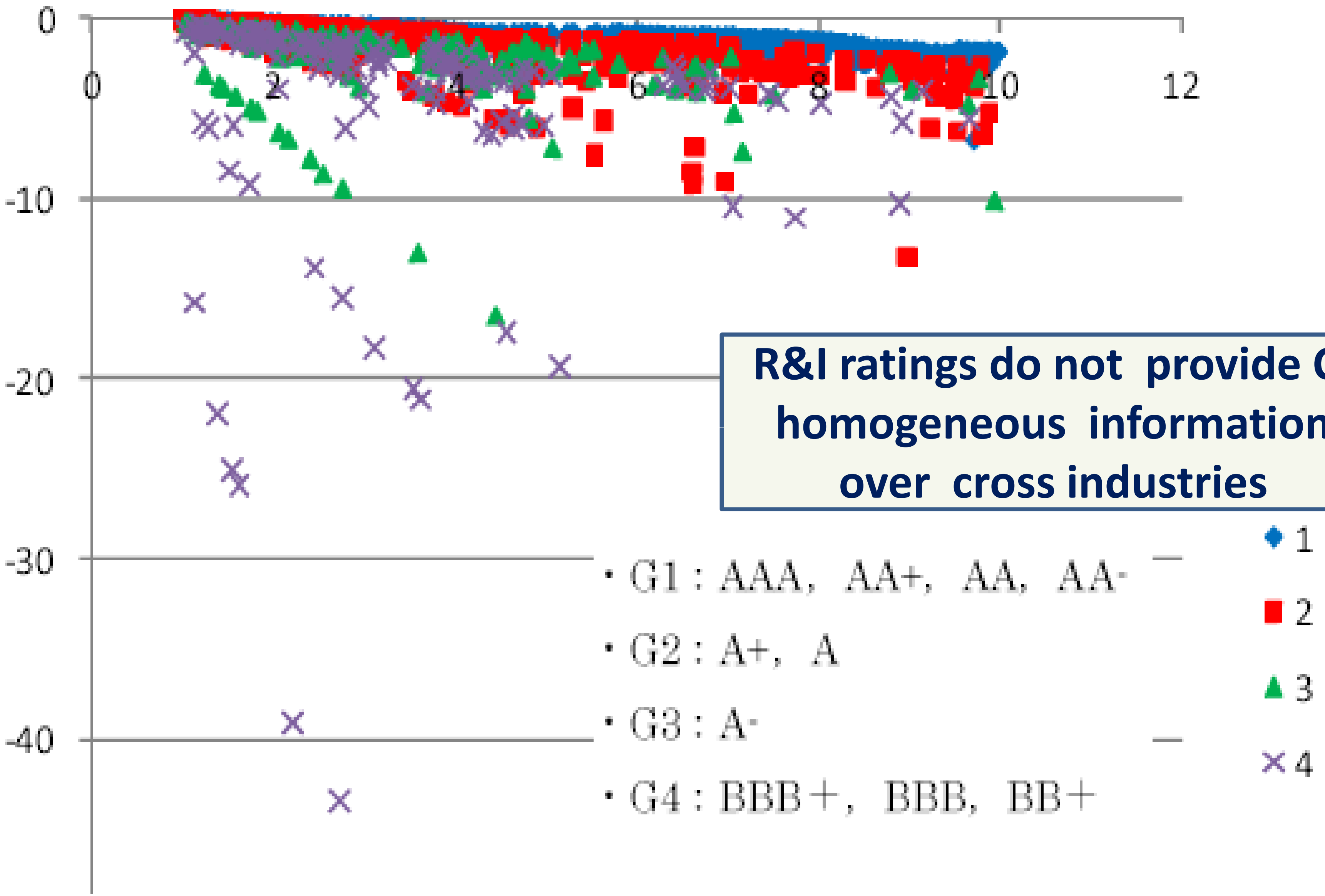
	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	C+	NA	Total
#	9	497	119	172	179	152	162	95	52	1	7	100	1545
%	0.6	54.0	8.2	11.9	12.4	10.5	11.2	6.6	3.6	0.0	0.5	NA	100

- Industry category

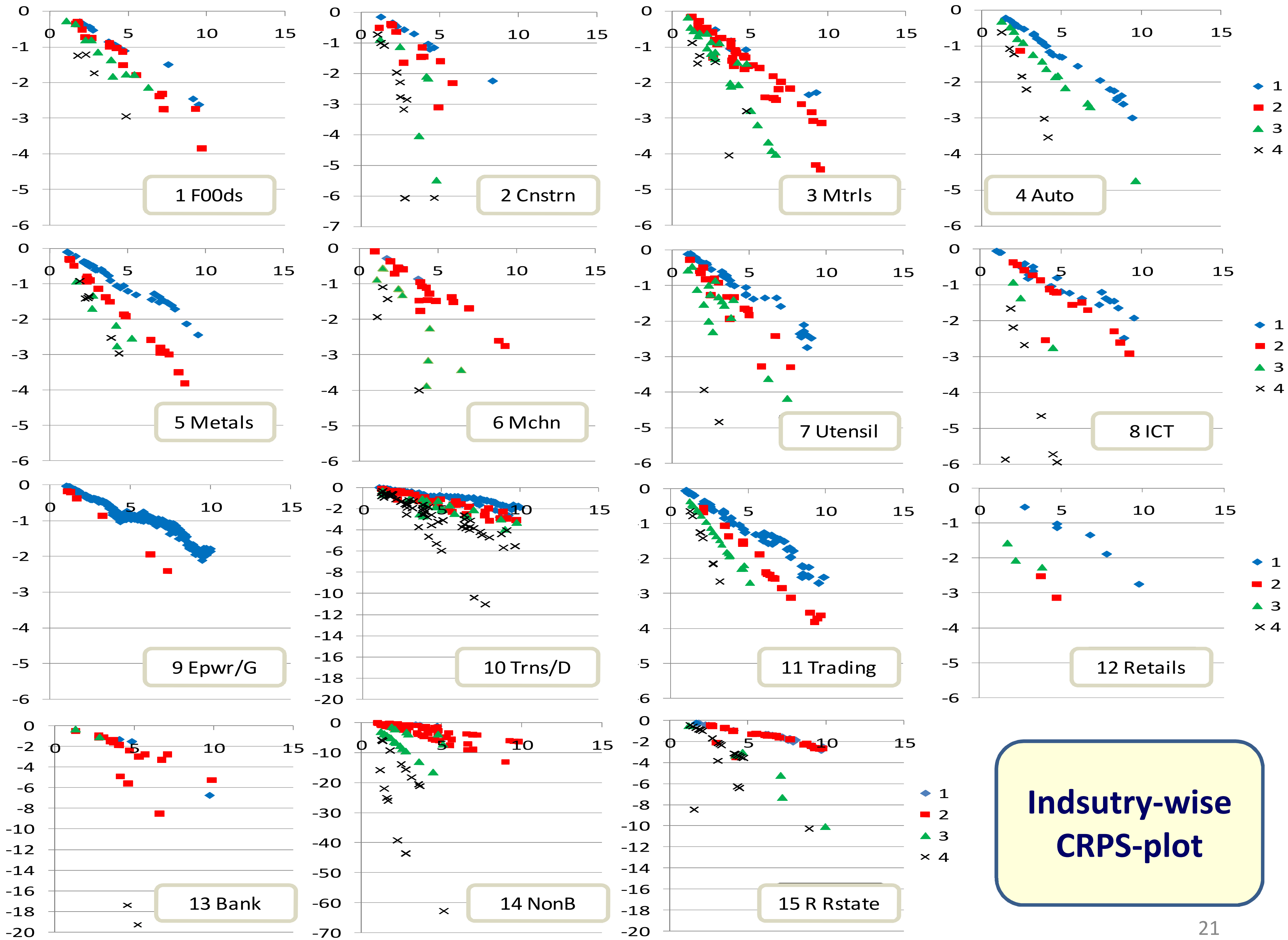
1 Foods, 2 Construction & its Materials, 3 Materials/Chemicals, 4 Transportation Equipments , 5 Steel /Non-steel/Mining, 6 Machinery, 7 Electric Appliances/Precision Instruments, 8 ICT /Services, 9 Electric Power/Gas, 10 Transportation/Distribution, 11 International Distribution (Trading), 12 Retails, 13 Banking, 14 Nonbank Financial Business, 15 Real Estate.

- Cluster Analysis
- **FIR (Fixed Interval Rating) via S-CRPS**

Effectiveness of Grouping via R&I Rating

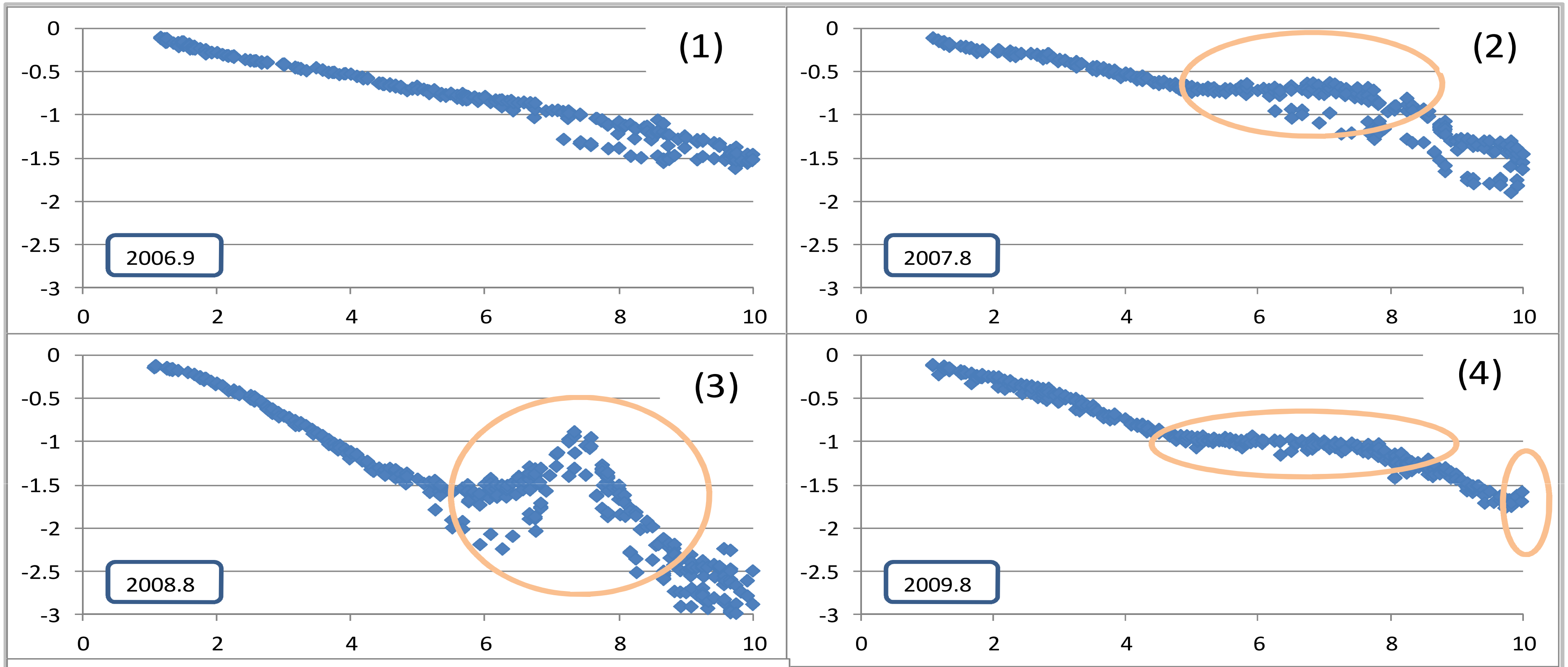


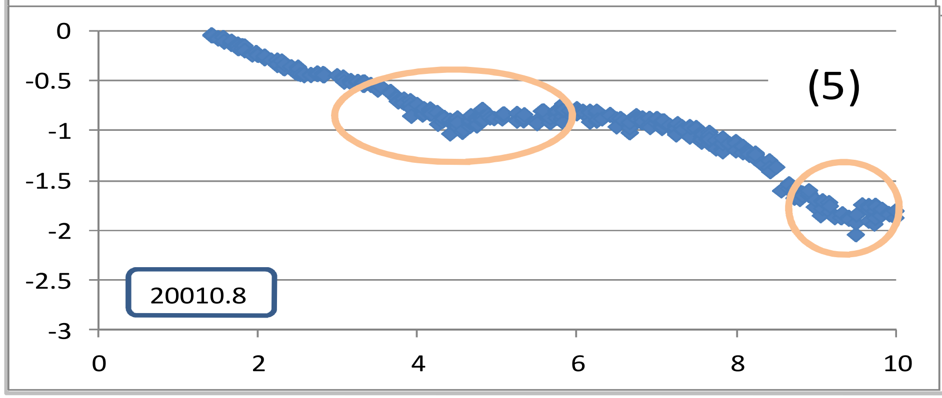
R&I ratings do not provide C-homogeneous information over cross industries



Indsutry-wise CRPS-plot

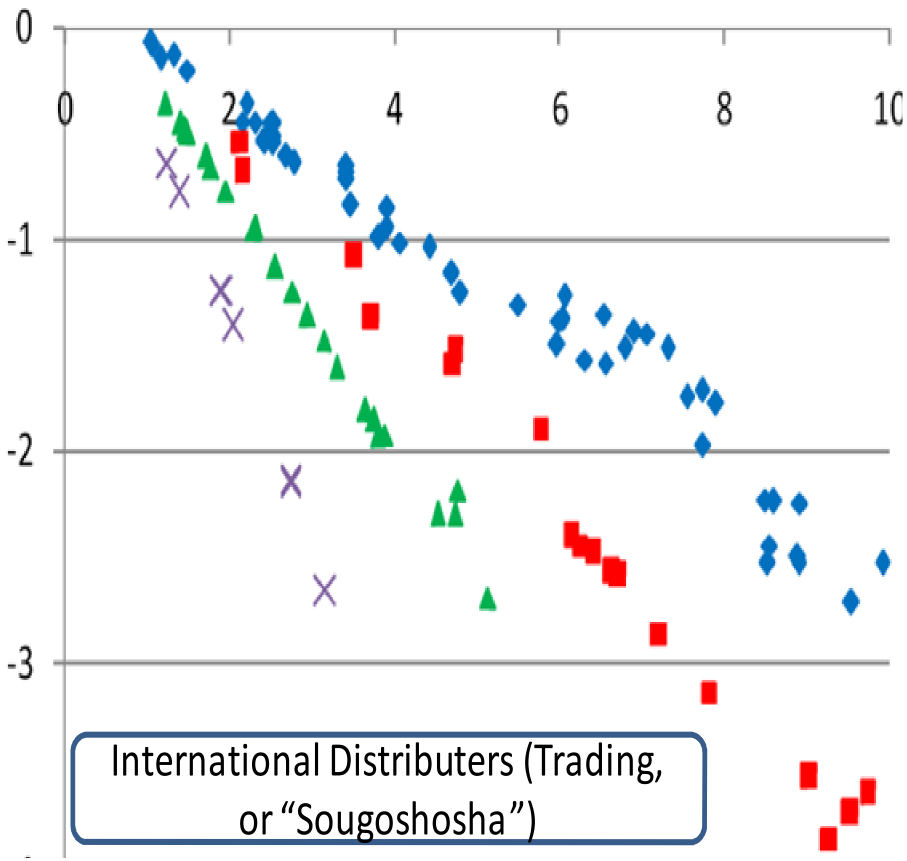
Changes of CRPSs of E-Power Indsutry over business cycles

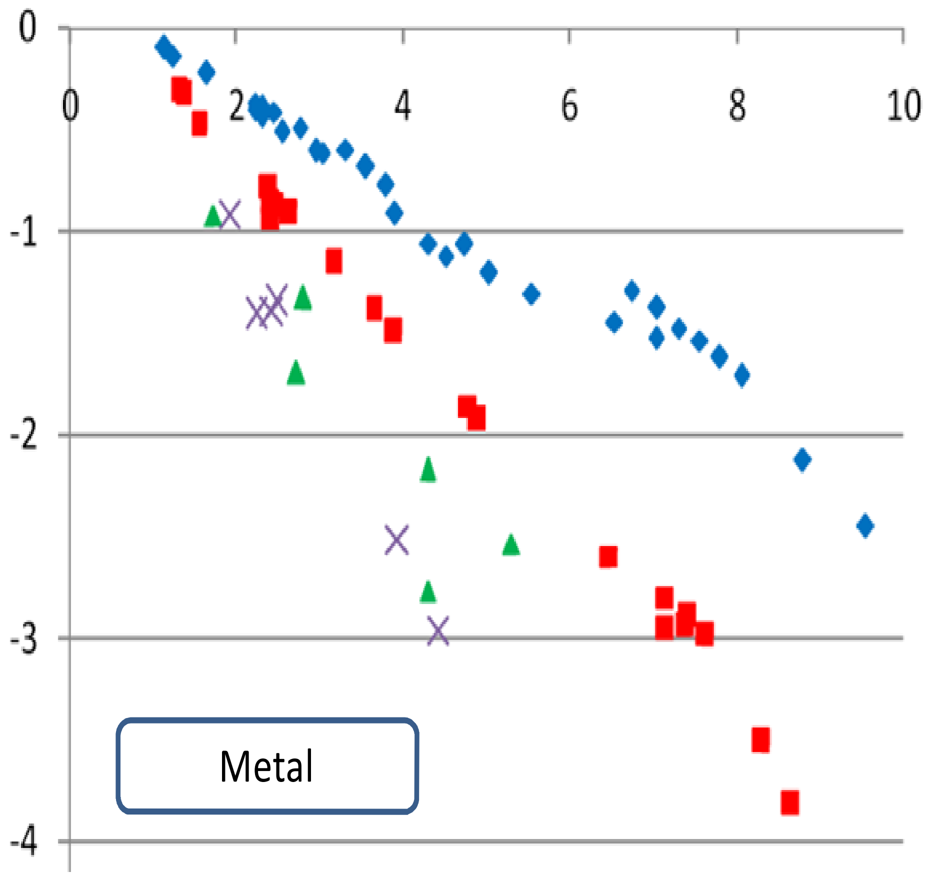


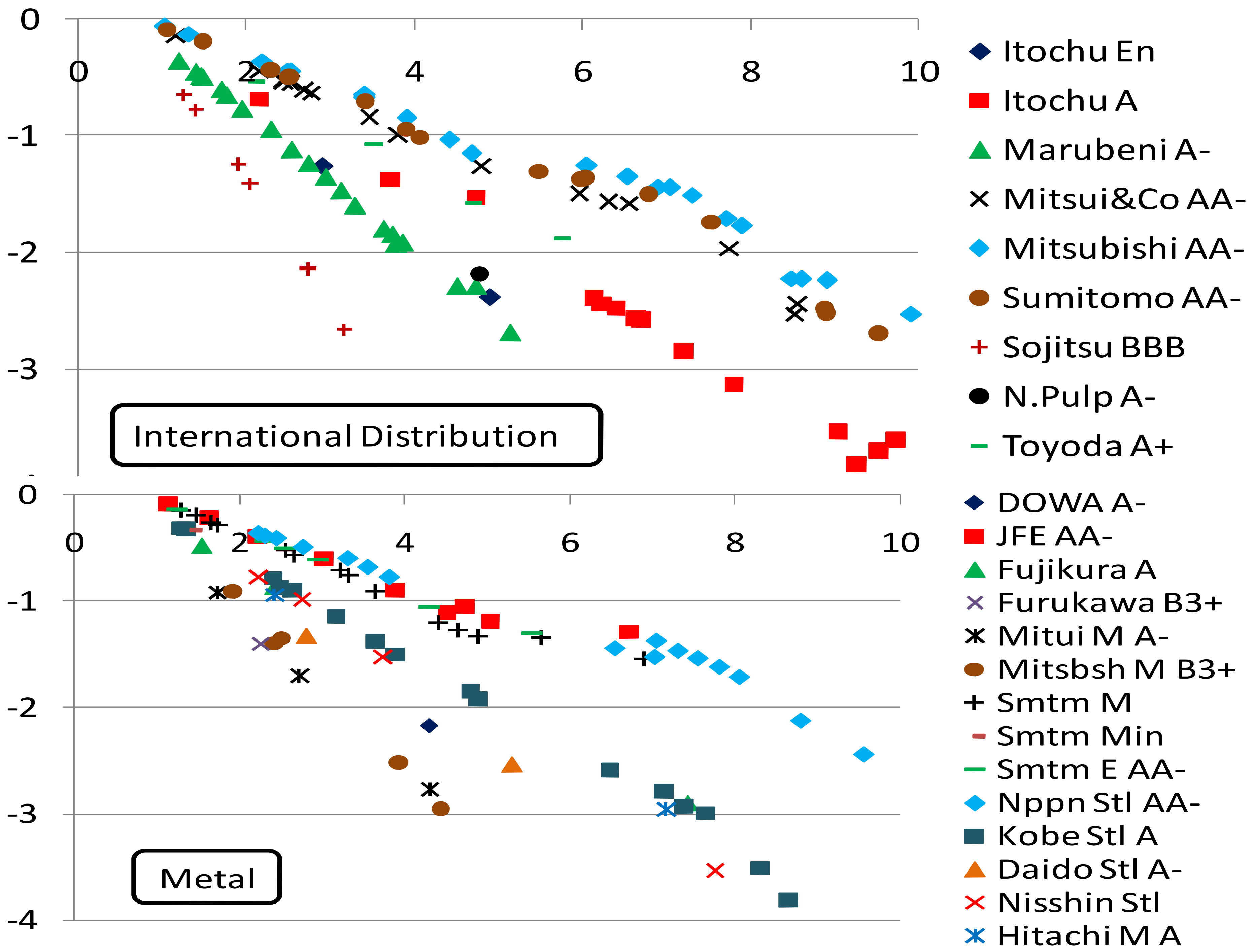


CRPS-plot for CBs in the Electric Power Industry over a business cycle

CRPS-Plots for International Distribution and Metal Industries

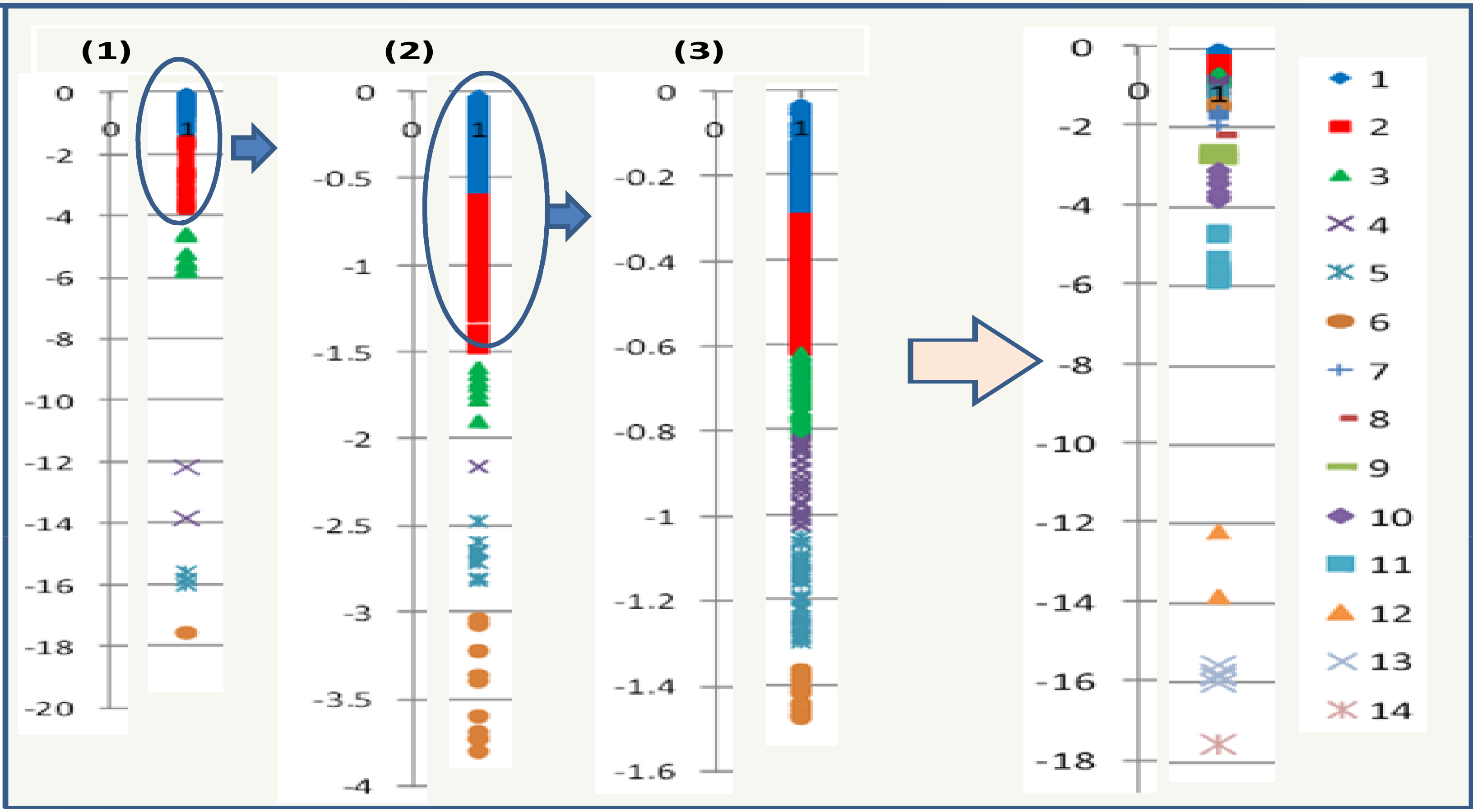






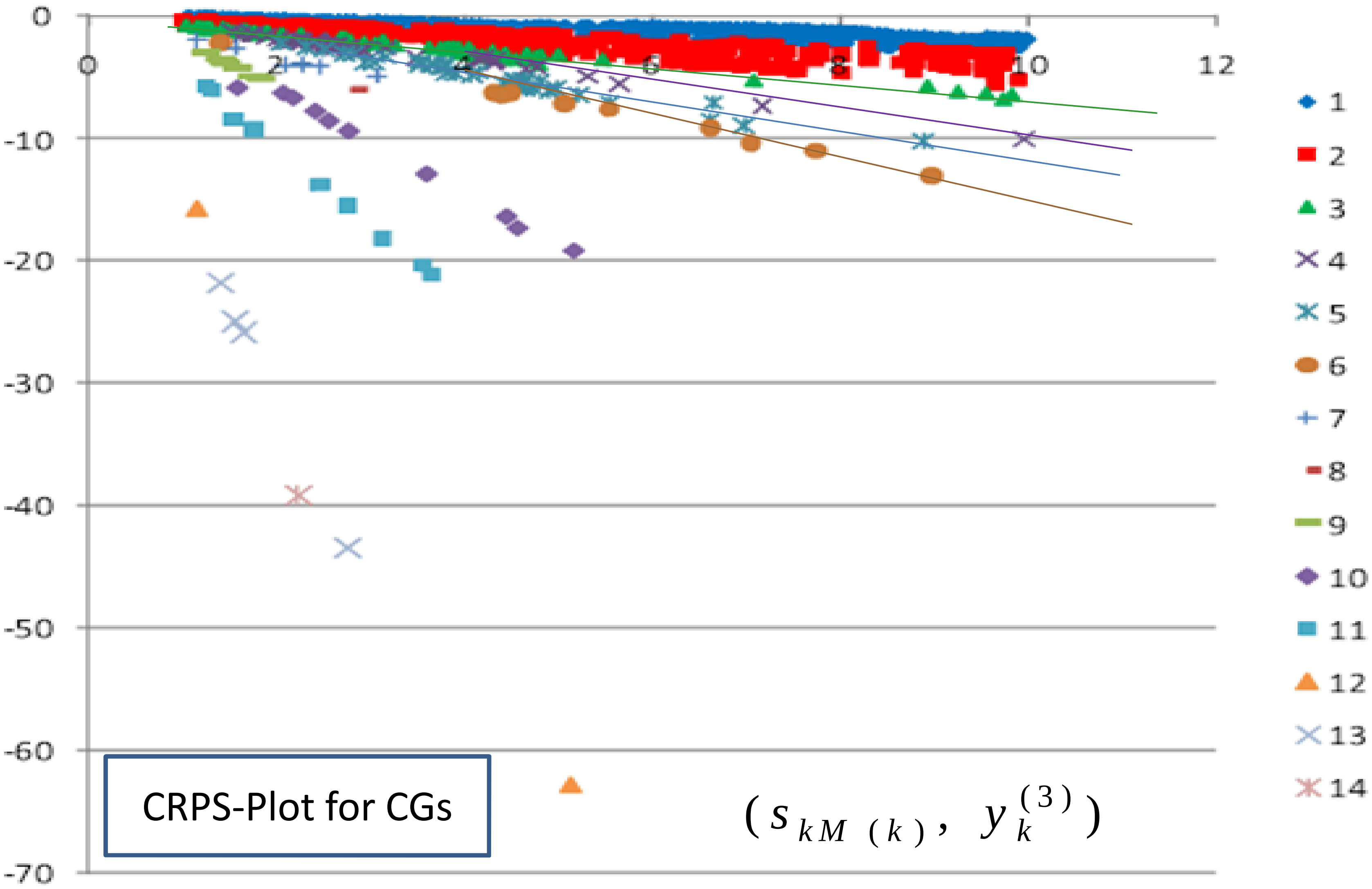
0
-1
-2
-3
0
2
4
6
8
10
International Distribution
Itochu En
Itochu A
Marubeni A-
Mitsui&Co AA-
Mitsubishi AA-
Sumitomo AA-
Sojitsu BBB
N.Pulp A-
Toyoda A+
0
-1
-2
-3
-4
0
2
4
6
8
10
Metal
DOWA A-
JFE AA-
Fujikura A
Furukawa B3+
Mitui M A-
Mitsbsh M B3+
Smtm M
Smtm Min
Smtm E AA-
Nppn Stl AA-
Kobe Stl A
Daido Stl A-
Nisshin Stl
Hitachi M A

Credit-homogeneous Groups via 3 Stage Cluster Analysis



1) # of clusters is fixed as 6 for each stage
2) Stage (1) separates 4 groups, called CG11, CG12, CG 13, CG14
3) Stage (2) separates 4 groups, called CG7, CG8, CG9, CG10
4) Stage (3) gives 6 groups;CG1-CG6, within the distance 1.5

Credit-homogeneous groups via cluster analysis



FIR (Fixed Interval Rating) via S-CRPS

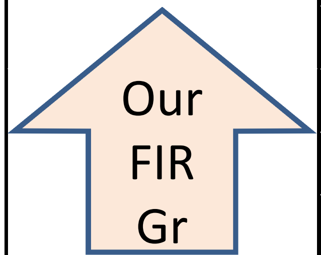
- Grouping by Cluster Analysis is a posterior grouping given CB and GB prices, which depends on the economic environment at the point of analysis
- Alternative we propose an absolute and prior criterion for grouping based on the intervals formed by S-CRPS

$$\varsigma_k^{(3)} = y_k^{(3)} / s_{kM(k)}$$

- If ς_k belongs to $I_m = (x_{m-1}, x_m]$, CB k is assigned to group FIR k. This is an market approach to grouping.
- To choose an interval scheme, we consider the distribution of S-CRPS measures.

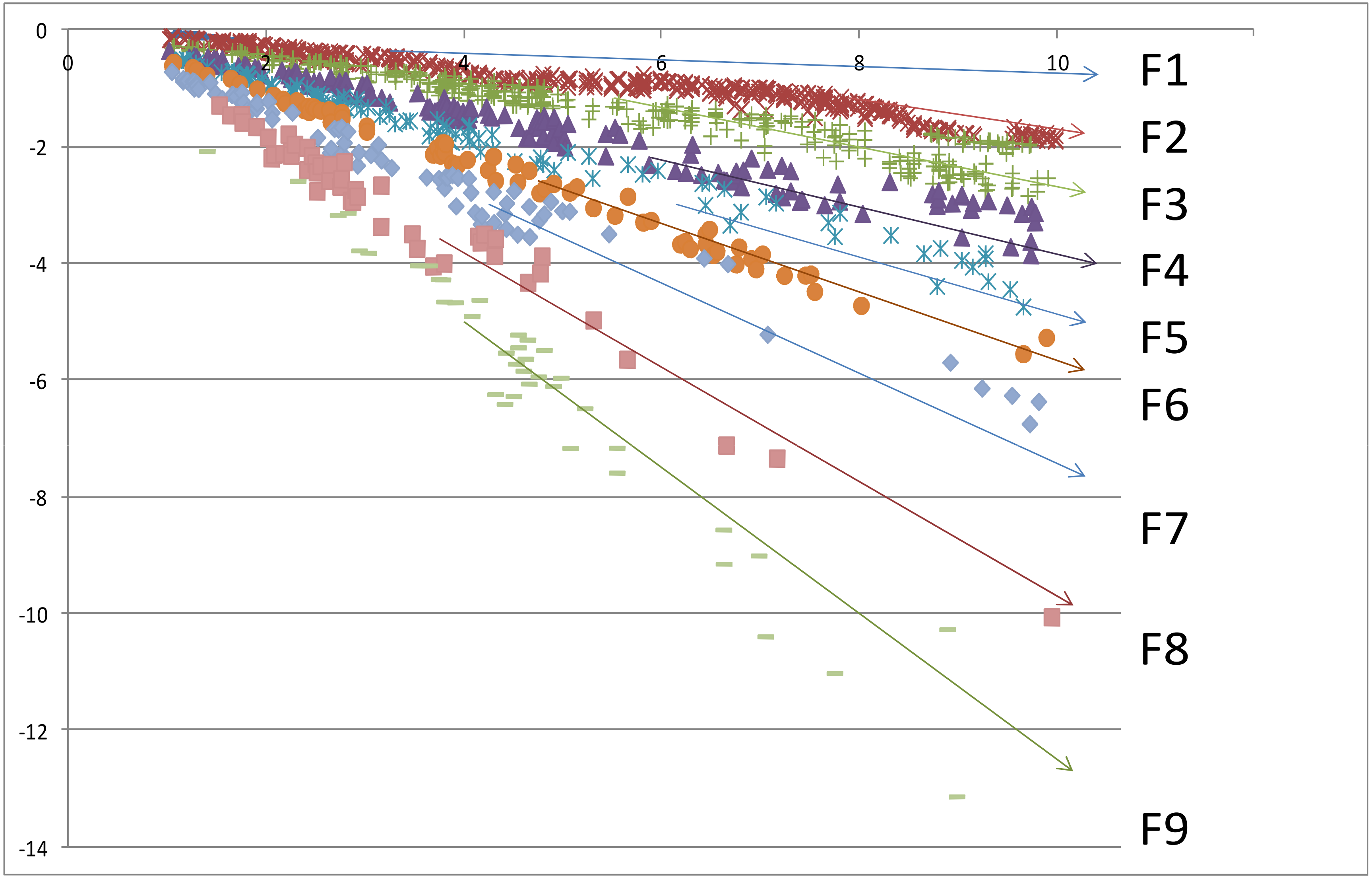
FIR(Fixed Interval Rating) based on S-CRPS measure ζ:2010.8

FIR 1			FIR 2			FIR 3			FIR 4			FIR 5		
FG	0.5Yen	#	FG	1Yen	#	FG	M1Yen	#	FG	1.5Yen	#	FG	2Yen	#
1	[-0.5,0)	10	1	[-1,0)	42	1	[-1,0)	42	1	[-1.5,0)	213	1	[-2,0)	618
2	[-1.0,-0.5)	32	2	[-2,-1)	576	2	[-2,-1)	576	2	[-3.0,-1.5)	773	2	[-4,-2)	526
3	[-1.5,-1.0)	171	3	[-3,-2)	368	3	[-3,-2)	368	3	[-4.5,-3.0)	214	3	[-6,-4)	186
4	[-2.0,-1.5)	405	4	[-4,-3)	158	4	[-4,-3)	158	4	[-6.0,-4.5)	130	4	[-8,-6)	86
5	[-2.5,-2.0)	223	5	[-5,-4)	111	5	[-5,-4)	111	5	[-7.5,-6.0)	64	5	[-10,-8)	35
6	[-3.0,-2.5)	145	6	[-6,-5)	75	6	[-6,-5)	75	6	[-9.0,-7.5)	41	6	(-∞,-10)	94
7	[-3.5,-3.0)	81	7	[-7,-6)	46	7	[-8,-6)	86	7	[-10.5,-9.0)	22			
8	[-4.0,-3.5)	77	8	[-8,-7)	40	8	[-11,-8)	48	8	(-∞,-10.5)	88			
9	[-4.5,-4.0)	56	9	[-9,-8)	19	9	[-15,-11)	40						
10	[-5.0,-4.5)	55	10	[-10,-9)	16	10	(-∞,-15)	41						
11	[-5.5,-5.0)	36	11	(-∞,-10)	94									
12	[-6.0,-5.5)	39												
13	[-7.0,-6.0)	46												
14	[-8.0,-7.0)	40												
15	[-9.0,-8.0)	19												
16	[-10.0,-9.0)	16												
17	(-∞,-10)	94												



R&I	Market Grouping via FIR 3 (Total 1545 CBs)											
	F1 (0,-1]	F2 (-1,-2]	F3 (-2,-3]	F4 (-3,-4]	F5 (-4,-5]	F6 (-5,-6]	F7 (-6,-8]	F8 (-8,-11]	F9 (-11,-15]	F10 (-15,∞)	Total	# CBs
AAA	0.0	22.2	77.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100	9
AA+	6.6	**84.9**	8.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100	497
AA	0.8	**52.1**	45.4	1.7	0.0	0.0	0.0	0.0	0.0	0.0	100	119
AA-	3.5	27.9	**66.3**	1.7	0.0	0.0	0.6	0.0	0.0	0.0	100	172
A+	0.6	12.8	**52.0**	14.0	7.8	6.1	3.4	3.4	0.0	0.0	100	179
A	0.0	0.7	13.8	**46.7**	9.9	3.9	7.9	8.6	8.6	0.0	100	152
A-	0.0	0.6	8.6	25.3	**27.2**	11.1	9.9	6.2	2.5	8.6	100	162
BBB+	0.0	0.0	1.1	5.3	20.0	14.7	**27.4**	9.5	9.5	12.6	100	95
BBB	0.0	0.0	0.0	0.0	9.6	28.8	34.6	9.6	11.5	5.8	100	52
BBB-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100	100	1
CCC+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100	100	7
None	1.0	17.0	22.0	11.0	14.0	11.0	7.0	5.0	8.0	4.0	100	100
# of CBs	42	576	368	158	111	75	86	48	40	41	100	1545

Credit-homogeneous Groups via FIR grouping:2010.8



Deriving the TSDPs of C-homogeneous groups and individual firms via Kariya (2012)'s model

Recovery rate is assumed to be zero.

1)Takes a long time for settlement in the creditors meeting and so uncertainty is large

2)Chapters 7 (liquidation) and 11 (reorganization)

3) Comparison

CB Pricing Model

Expected CFs

$$V_k = \sum_{j=1}^{M(k)} \bar{C}_k(s_{kj}) D_k(s_{kj})$$

$$\bar{C}_k(s_{mj}) = C_k(s_{mj})[1 - p_k(s_{mj})] + 100\gamma_k[p_k(s_{mj}) - p_k(s_{mj-1})]$$

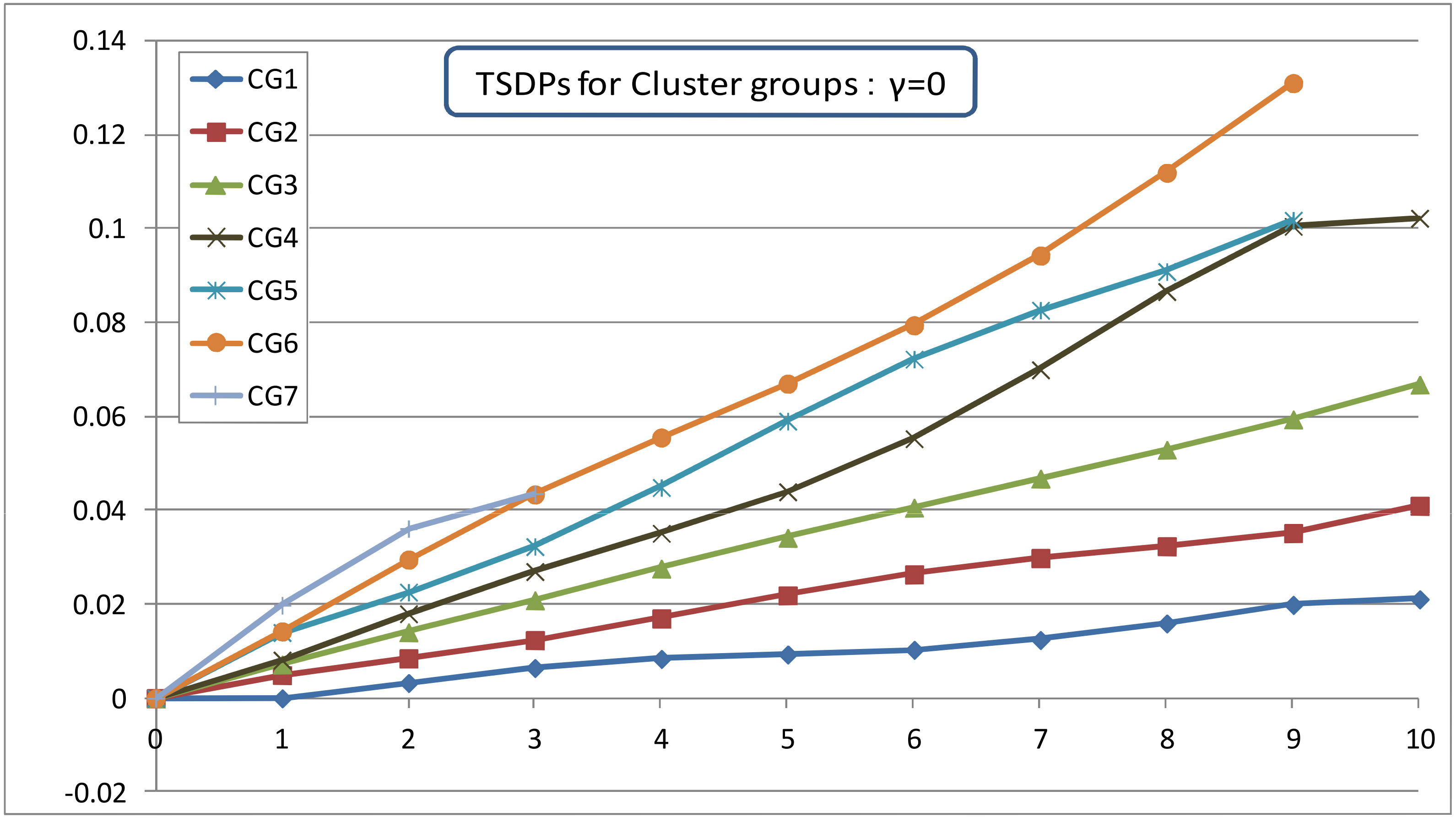
$$D_k(s) = \bar{D}_k(s) + \Delta_k(s)$$

$$p_k(s : i(k)) \equiv \sum_{j=1}^{J} w_k(j) p(s : i(k), j)$$

$$p(s : i, j) = \alpha_1^{ij} s + \alpha_2^{ij} s^2 + \cdots + \alpha_q^{ij} s^q$$

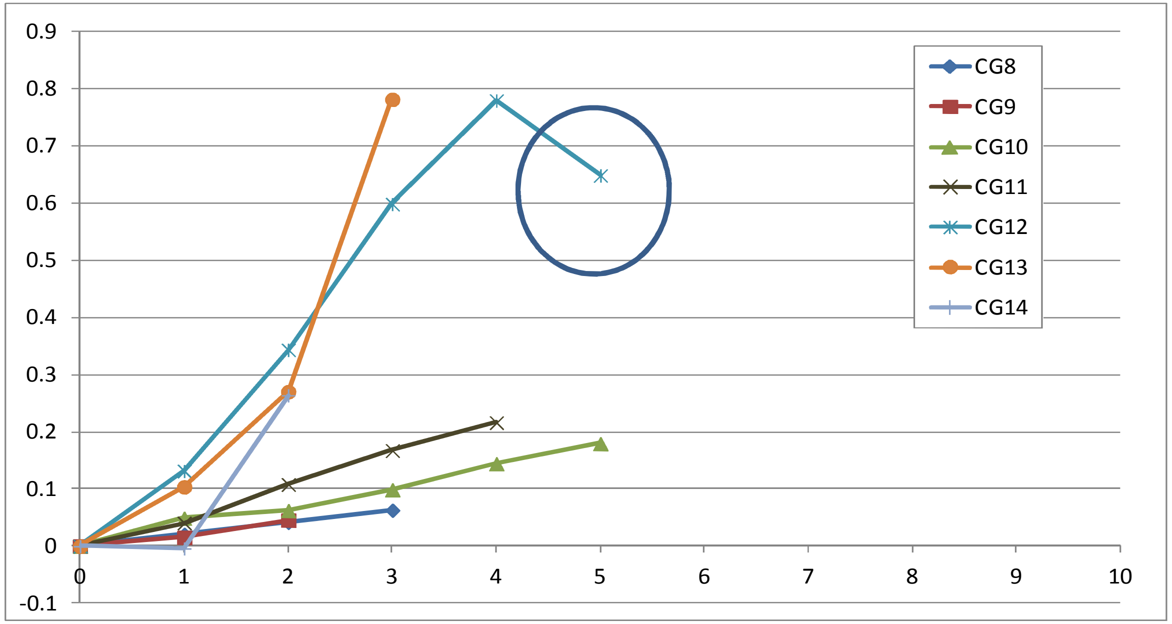
$$w_k(j) \geq 0, \quad \sum_{j=1}^{J} w_k(j) = 1$$

TSDPs for Cluster Groups:2010.8



Default Probabilities for CGs

	CG1	CG2	CG3	CG4	CG5	CG6
9yrs	2.0	3.5	5.9	10.1	10.2	13.1
10yrs	2.1	4.1	6.7	10.2		

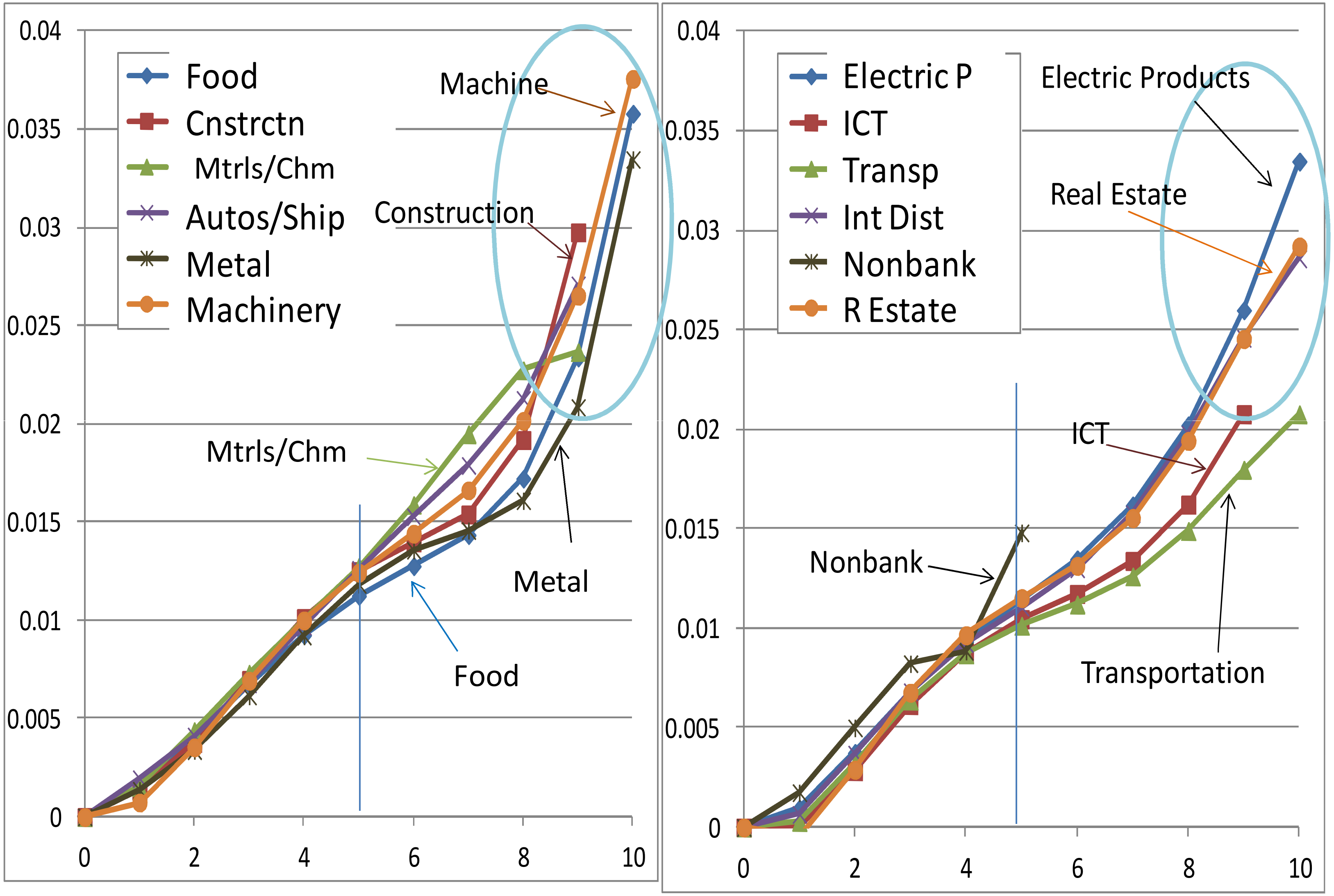


Decomposition of CG1&2 into industry groups

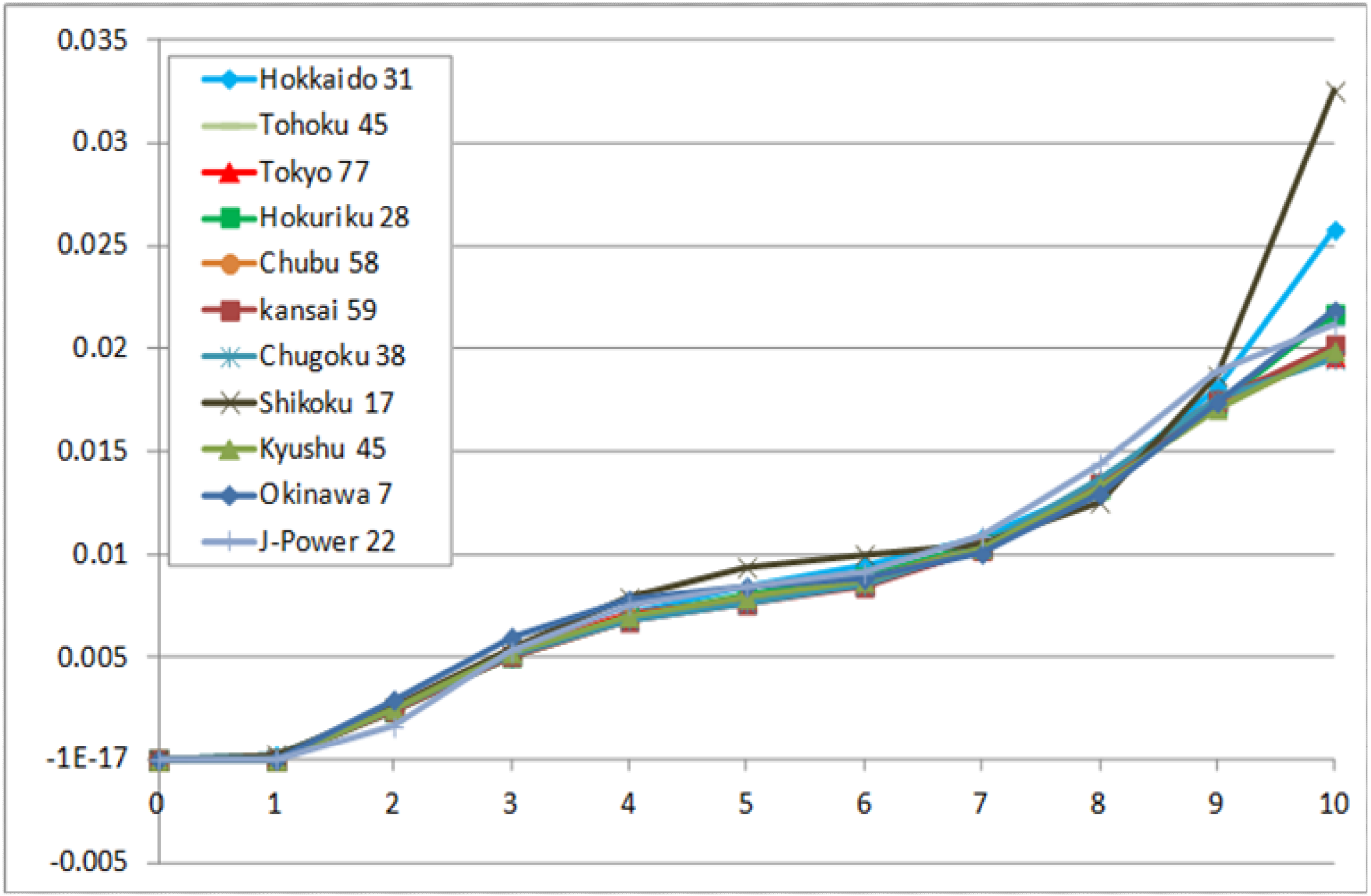
Inds	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CG1	24	16	41	30	49	19	44	53	459	124	56	6	1	14	52
CG2	18	9	46	23	30	13	27	7	2	81	42	1	18	22	12

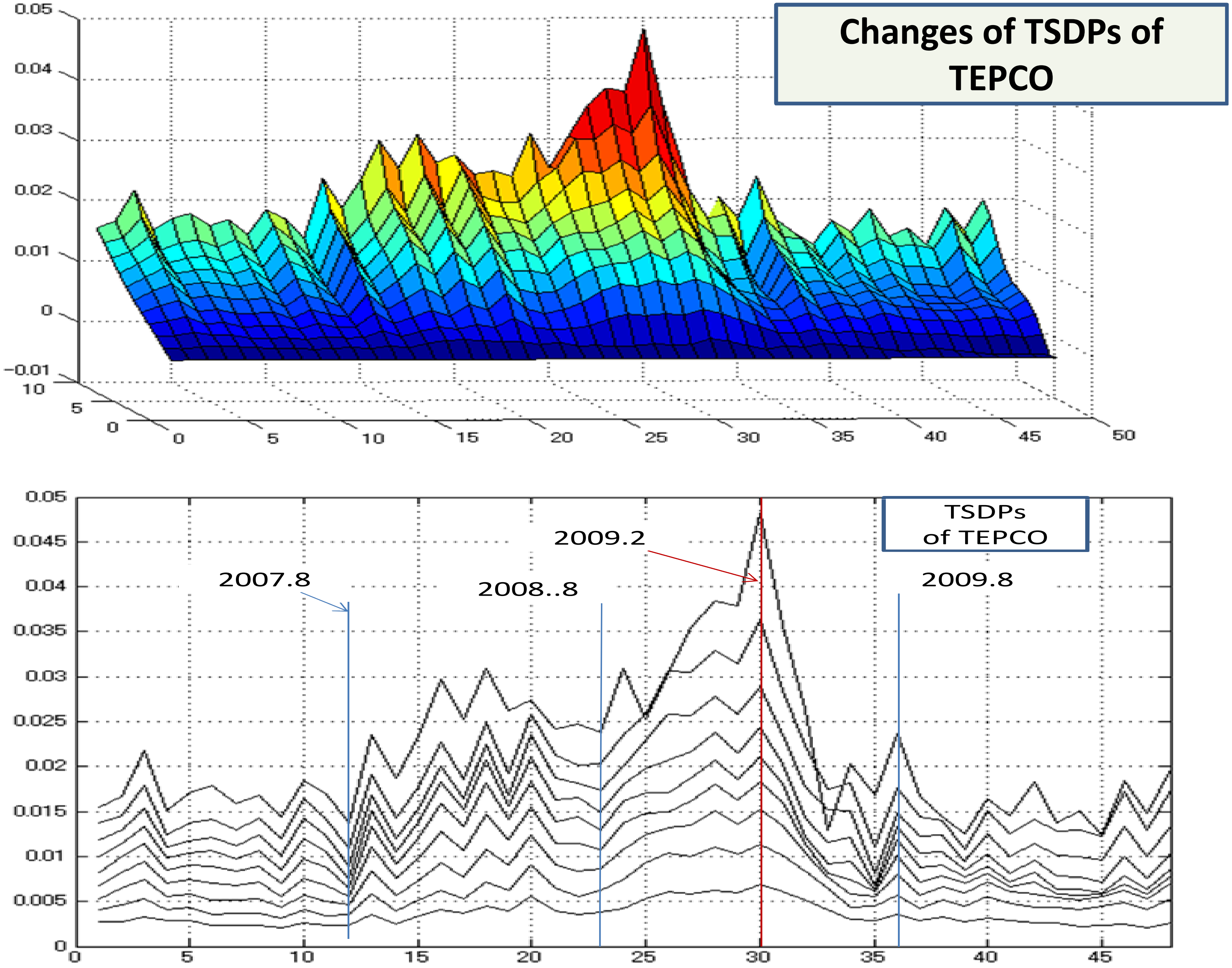
1 Foods, 2 Construction & its Materials, 3 Materials/Chemicals, 4 Transportation Equipments , 5 Steel /Non-steel/Mining, 6 Machinery, 7 Electric Appliances/Precision Instruments, 8 ICT /Services, 9 Electric Power/Gas, 10 Transportation/Distribution, 11 International Distribution (Trading), 12 Retails, 13 Banking, 14 Nonbank Financial Business, 15 Real Estate.

Industrial Differentiation of TSDPs for CG1:2010.8



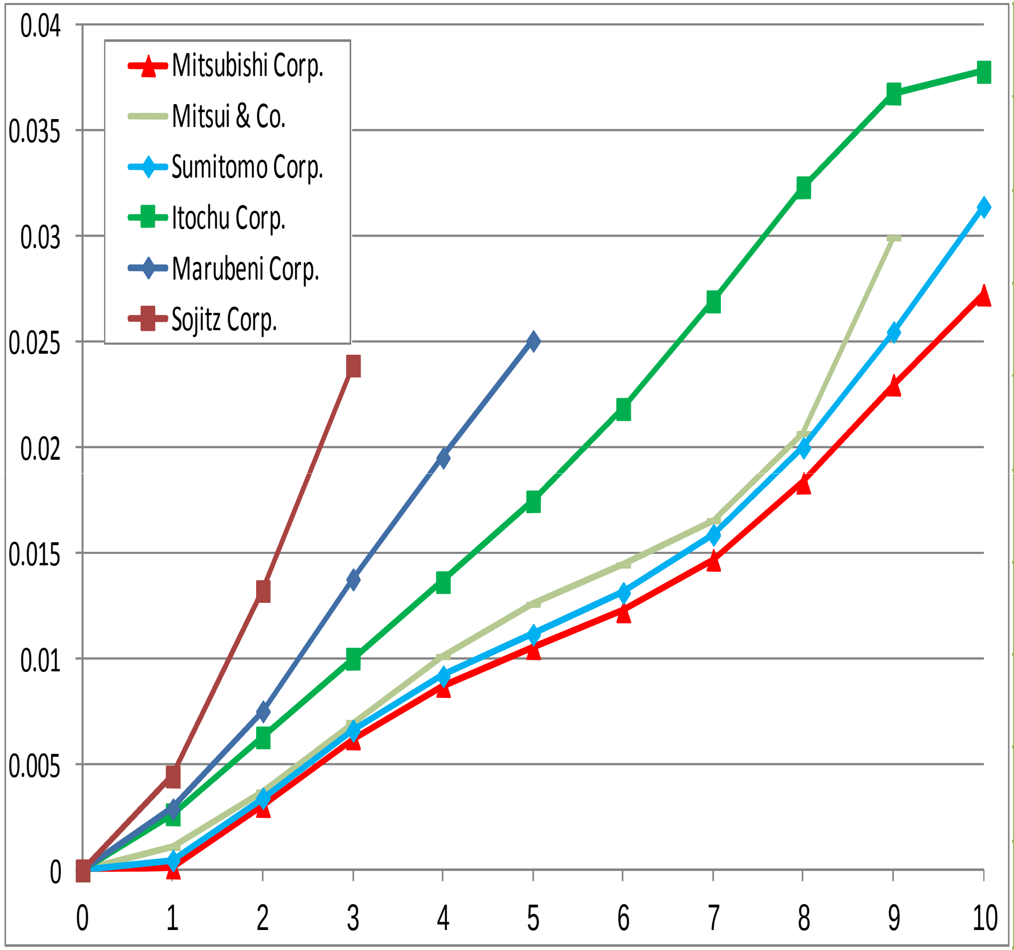
TSDPs in the Electric Power Industry:2010.8





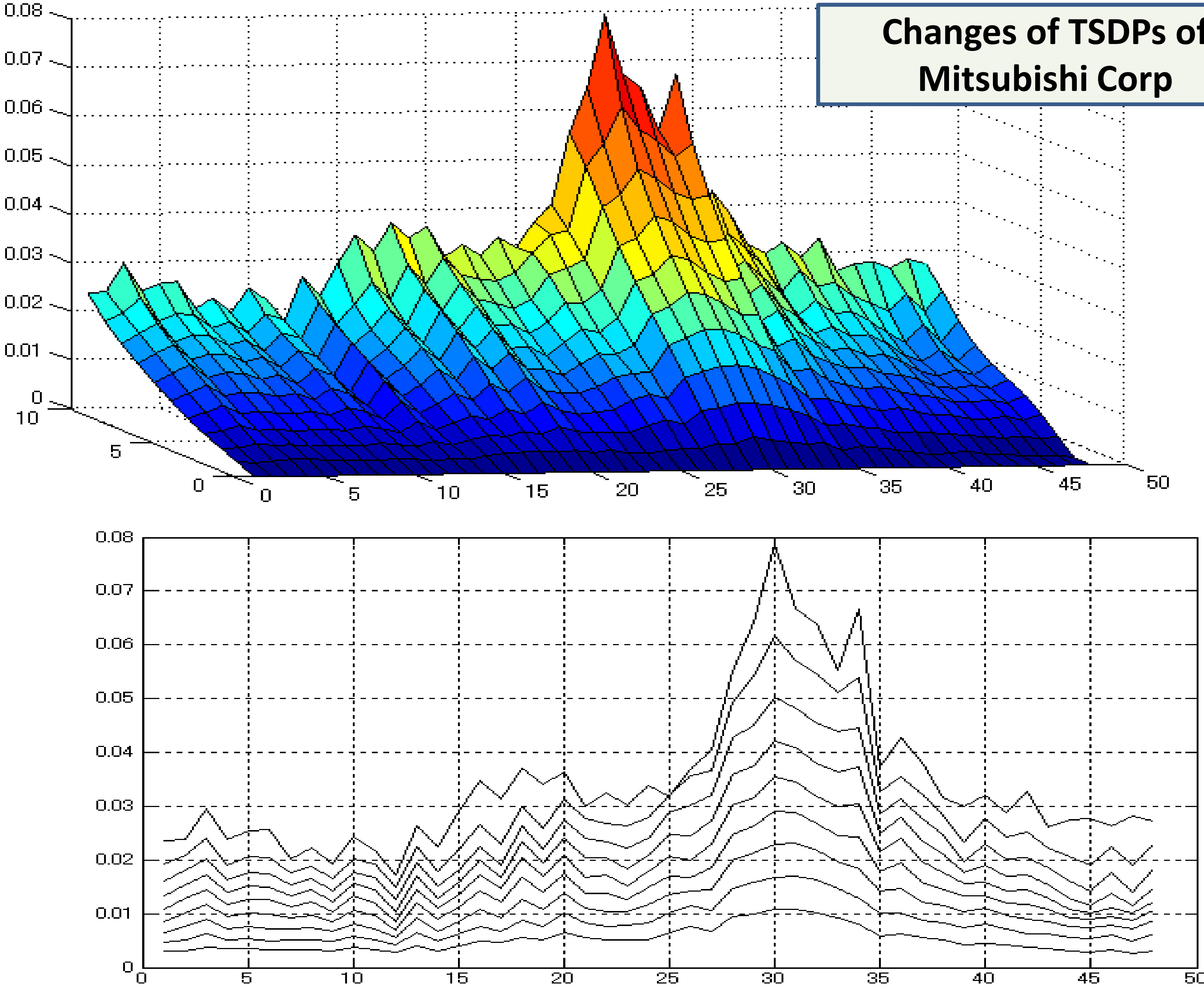
Changes of TSDPs of TEPCO
0.05
0.04
0.03
0.02
0.01
0
−0.01
10
5
0
0
5
10
15
20
25
30
35
40
45
50
TSDPs of TEPCO
2007.8
2008..8
2009.2
2009.8
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
5
10
15
20
25
30
35
40
45

TSDPs of individual firms in the Trading Industry:2010.8

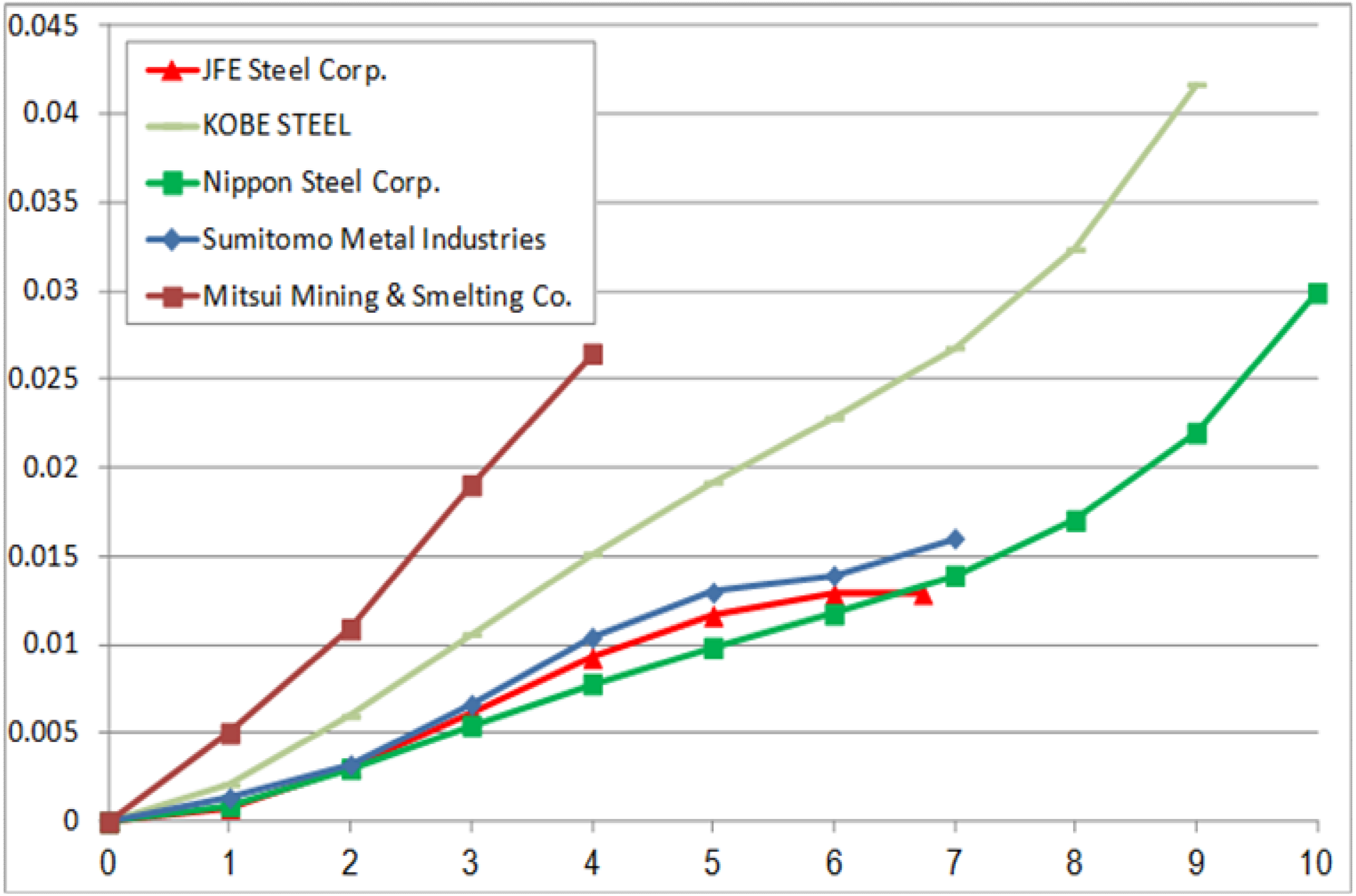


	MBS	SuS	MiT	ITo	MBN	SJT
2yrs	0.30	0.34	0.37	0.63	0.75	1.32
3yrs	0.62	0.66	0.70	1.00	1.38	2.39
4yrs	0.87	0.92	1.01	1.37	1.95	
5yrs	1.05	1.12	1.26	1.75	2.51	
6yrs	1.23	1.31	1.45	2.19		
7yrs	1.47	1.59	1.66	2.69		
8yrs	1.84	2.00	2.07	3.23		
9yrs	2.30	2.55	3.00	3.68		
10yrs	2.73	3.14		3.78	%	

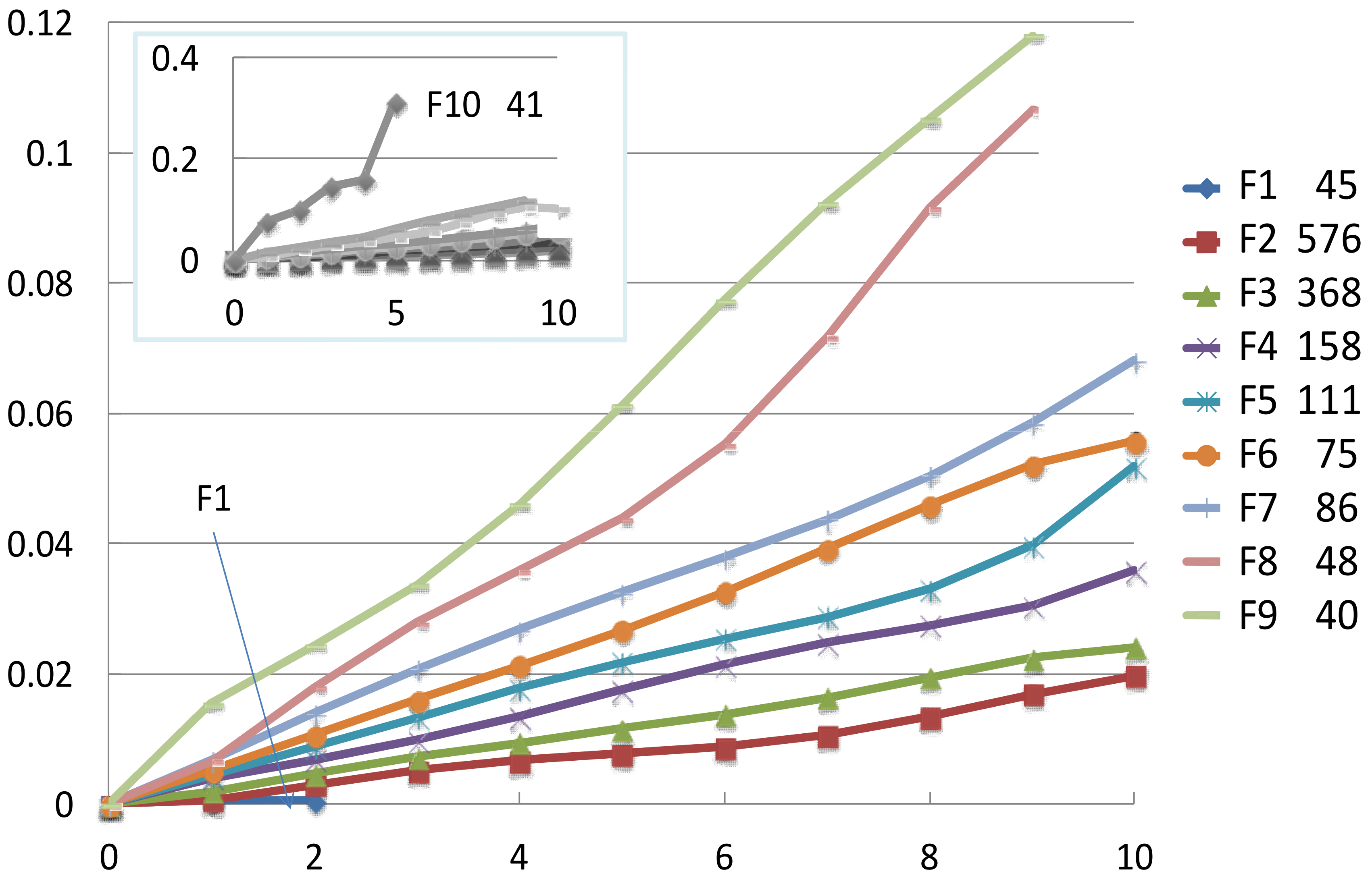
Changes of TSDPs of Mitsubishi Corp



TSDPs for individual firms in the Metal Industry:2010.8



TSDPs for Fixed Interval Rating Groups:2010.8



Summary

- Using Kariya,et al (2012)'s bond pricing model, we tested & **rejected H of no attribute effect**, and show the effectiveness of our GB-equivalent CB price and **CRPS measure** to analyze the C- structure.
- Showed ineffectiveness of the R&I rating scheme to analyze C-homogeneous groups in view of CRPS
- Proposed **FIR method** for Market Rating of individual CBs based on the standardized CRPS, on which Cluster groups are derived and analyzed
- TSDPs of cluster groups, FIR groups, E-power industry, TEPCO, Mitsubishi Corp are derived via Kariya(2012)'s model

Selected References

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Attribute-independent spot rate approach for given r0

$$P_g(1) = \sum_{j=1}^{M(g)} C_g(s_{gj})\overline{\overline{D}}(s_{gj}), \qquad \overline{\overline{D}}(s_{gj}) = E_0[\exp(-\int_0^{s_{gj}} r_u du)] \equiv H(r_0, s_{gj}, \theta)$$

Zero yield $$R_s = -\frac{1}{s}\log H(r_0, s, \theta)$$

Attribute-dependent formulation for interest rates

$$\overline{\overline{D_g}}(s_{gj}) = E_0[\exp(-\int_0^{s_{gj}} r_{gs} ds)] \ \textit{conditional on } r_{g0}$$

Attribute-independent forward rate approach

$$D(s_{gj}) = \exp(-\int_0^{s_{gj}} f_s ds)$$

Attribute-dependent forward rate approach (our view)

$$D_g(s_{gj}) = \exp(-\int_0^{s_{gj}} f_{gs} ds)$$