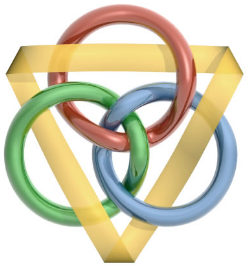
RIKEN symposium@Jan.5.2011

Majorana excitations in the superfluid 3He

Yasumasa Tsutsumi
Dept. Physics, Okayama University.

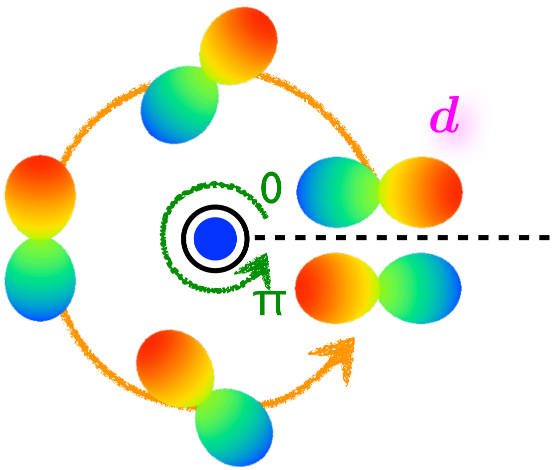
OKAYAMA UNIVERSITY

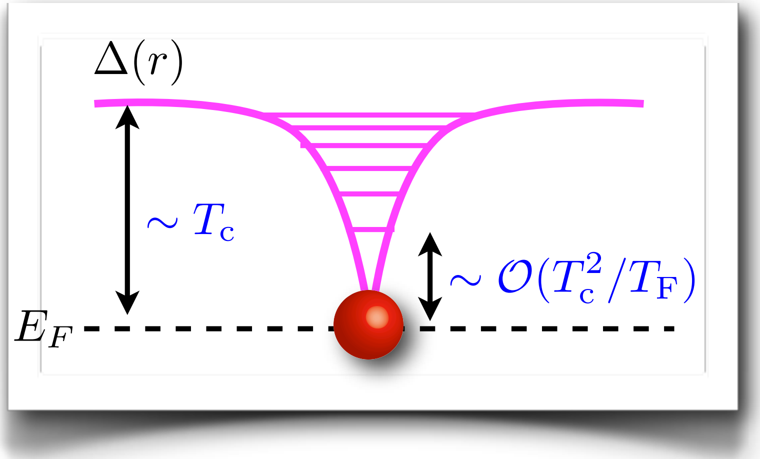


Collaborators: T. Kawakami, T. Mizushima, M. Ichioka, K. Machida

Contents

1. Half-quantum vortex





➡Majorana zero mode is bound in half-quantum vortex ⇒ non-Abelian statistics

➡stability of half-quantum vortex : strong-coupling effect ⇔ Fermi liquid correction

2. Surface Andreev bound state

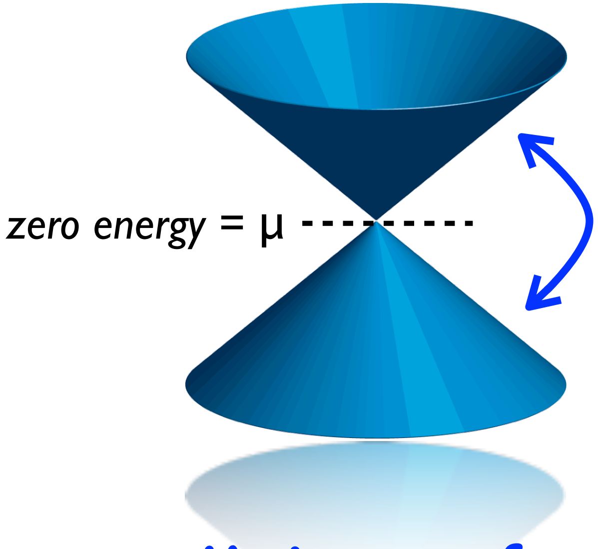
➡bulk (topologically non-trivial) ⇔ vacuum (trivial) : surface Andreev bound state

➡linear dispersion behaves as Majorana fermion

➡edge current relates to intrinsic angular momentum

Majorana Fermions

Dirac fermion



$$\Psi_{\mathrm{D}}(x,t) = \sum_{E>0} \psi_E(x) e^{-iEt} a_E + \sum_{E<0} \psi_E(x) e^{-iEt} b^{\dagger}_{-E}$$

annihilation operator for particle

creation operator for anti-particle

$$\Psi_{\mathrm{D}}(x,t) \neq \Psi^{\dagger}_{\mathrm{D}}(x,t)$$

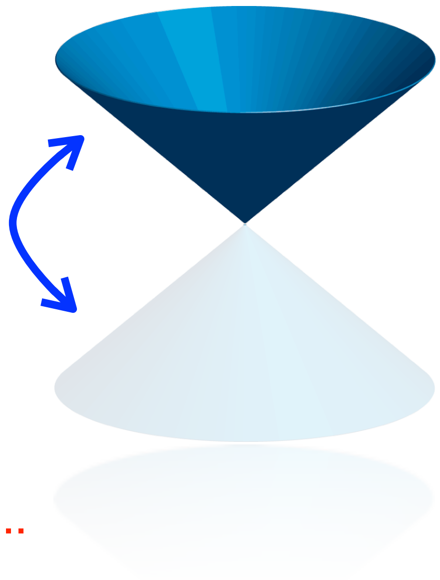
particle & anti-particle

Majorana fermion

$$\Psi_{\mathrm{M}}(x,t) = \sum_{E>0} \left[\psi_E(x) e^{-iEt} a_E + \psi^*_E(x) e^{iEt} a^{\dagger}_E \right]$$

$$a_E = a^{\dagger}_{-E}$$

$$\Psi_{\mathrm{M}}(x,t) = \Psi^{\dagger}_{\mathrm{M}}(x,t)$$



self-conjugate operator

Majorana fermion = its own anti-particle

Candidate: ^{3}He, Sr_2RuO_4, noncentro SC, cold atoms, ...

Non-Abelian Statistics

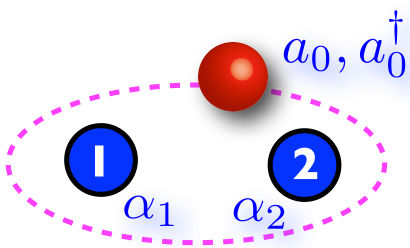
- Zero energy mode

$$\alpha \equiv a_0 + b_0^\dagger \neq \alpha^\dagger$$ Dirac zero mode = fermion $$\{\alpha, \alpha^\dagger\} = 1$$

$$\alpha \equiv a_0 + a_0^\dagger = \alpha^\dagger$$ Majorana zero mode ≠ fermion $$\alpha^2 = \frac{1}{2}$$

- Non-degenerate zero modes

complex "fermion" $a_0 = \frac{1}{\sqrt{2}}(\alpha_1 + i\alpha_2)$ 2 Majorana zero modes

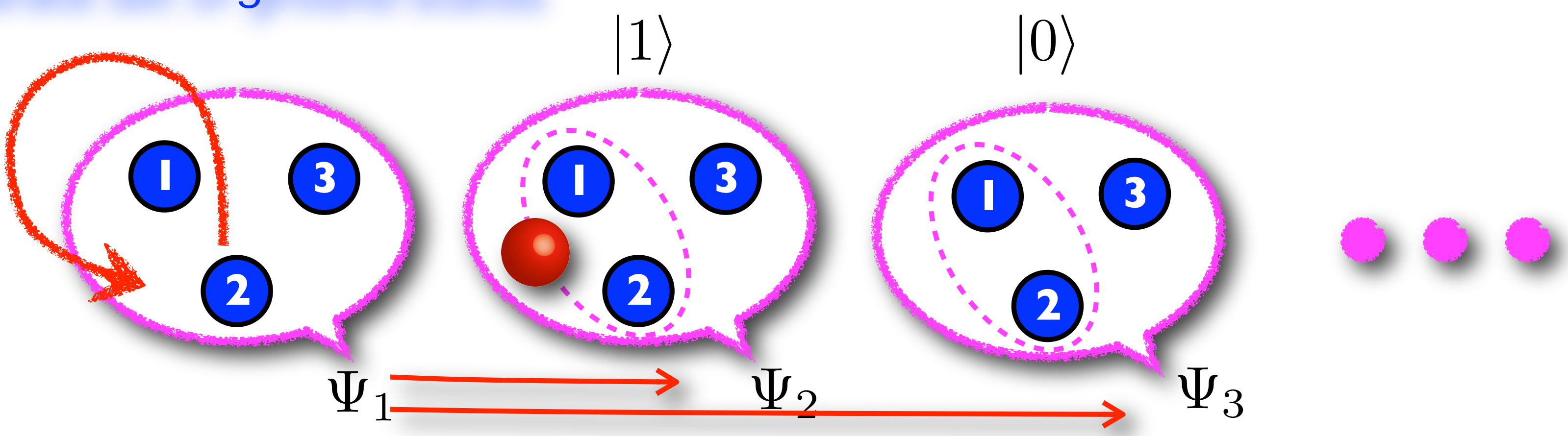


2 vortices

Non-localization of zero modes
⇒ Non-Abelian statistics

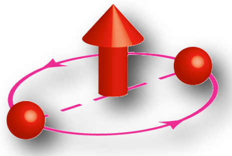
Ivanov, PRL **86**, 268 (2001)

- Degenerate set of ground states



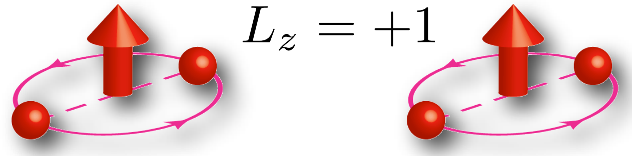
Half-Quantum Vortex

A-phase



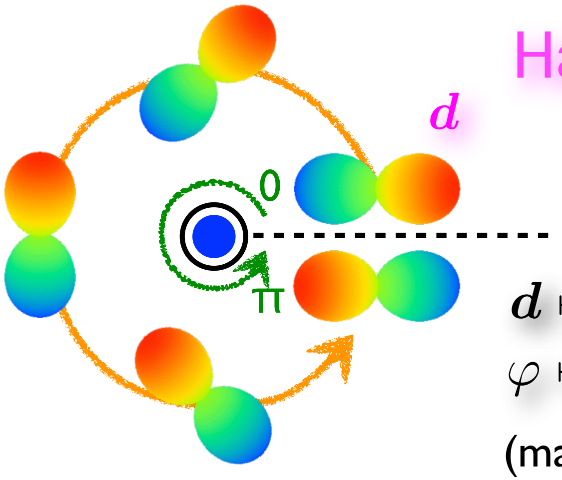
$L_z = +1$

$|\uparrow\downarrow + \downarrow\uparrow\rangle \quad S_z = 0$



$L_z = +1$

$|\uparrow\uparrow\rangle \qquad |\downarrow\downarrow\rangle$



Half-quantum vortex (HQV)

Ivanov, PRL **86**, 268 (2001)

$$\Psi = (\hat{k}_x + i\hat{k}_y)\left[e^{i\theta}\left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle\right]$$

integer vortex

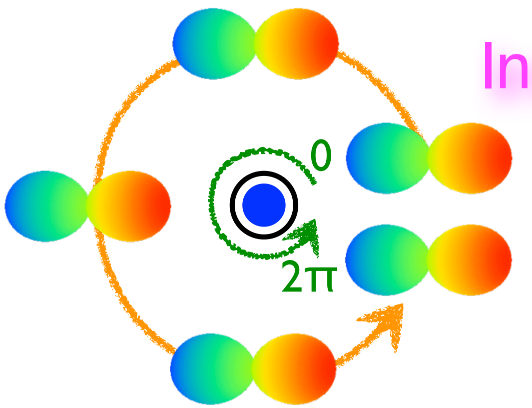
$$\boldsymbol{d} \longmapsto -\boldsymbol{d}$$

$$\varphi \longmapsto \varphi + \pi$$

(mass, spin)=(1/2, 1/2)

spin polarized zero energy mode

Non-Abelian statistics



Integer vortex

$$\Psi = (\hat{k}_x + i\hat{k}_y)\left[e^{i\theta}\left|\uparrow\uparrow\right\rangle + e^{i\theta}\left|\downarrow\downarrow\right\rangle\right]$$

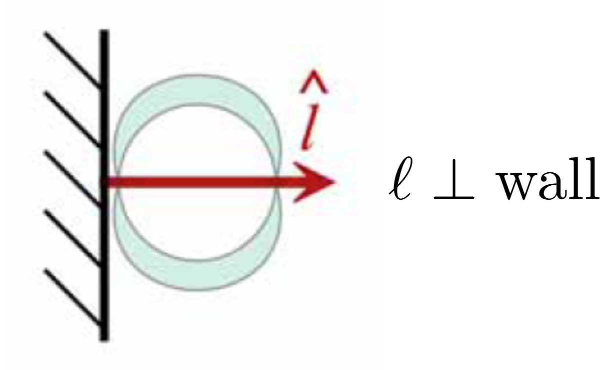
Abelian statistics

Conditions for Realization of HQV

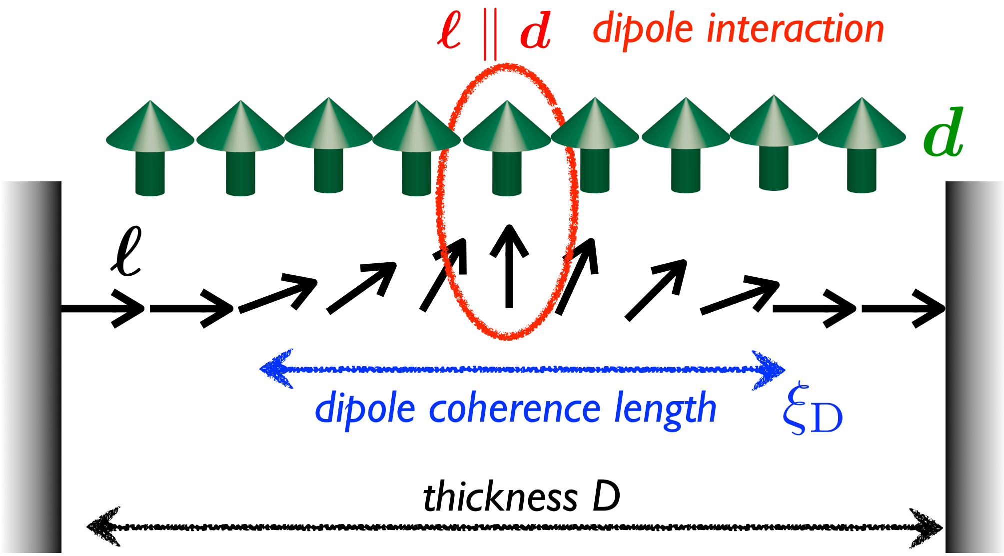
1. l-vector (=orbital) is fixed: "chiral" k_x+ik_y state

2. d-vector (=spin) rotates in plane perpendicular to l-vector

Boundary condition: l-vector



thickness $D \ll \xi_\mathrm{D}$

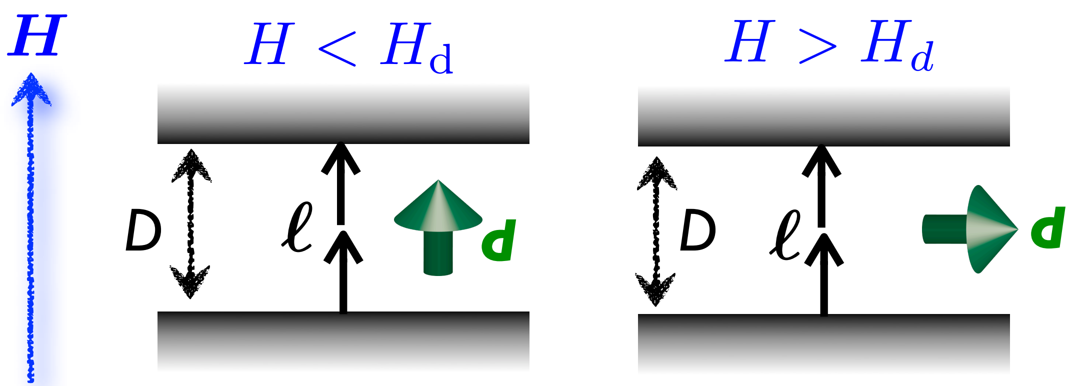


Restricted geometry

dipole interaction energy

$\ell \parallel d$

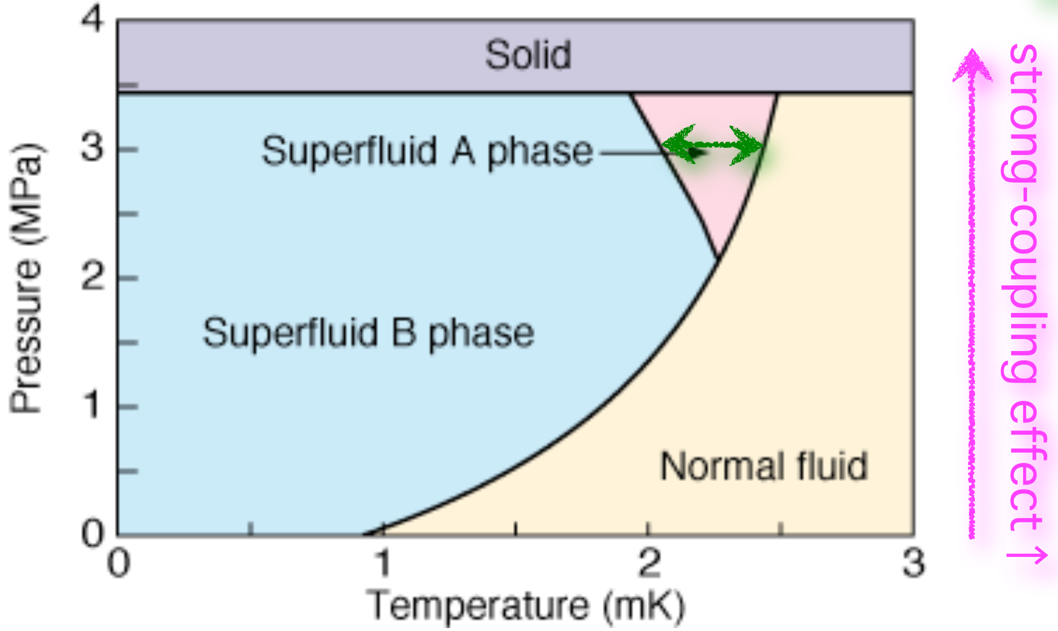
$H_\mathrm{d} \sim 2\mathrm{mT}$

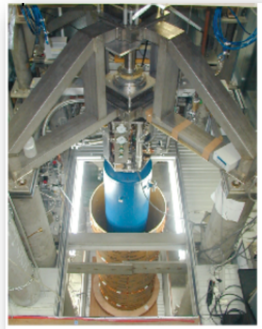


Experiment to Detect HQV

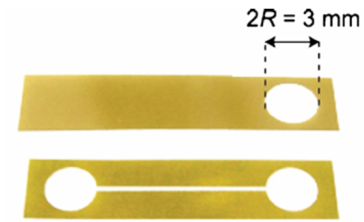
Yamashita *et al.*, PRL **101**, 025301 (2008)

thickness ~ 12.5μm





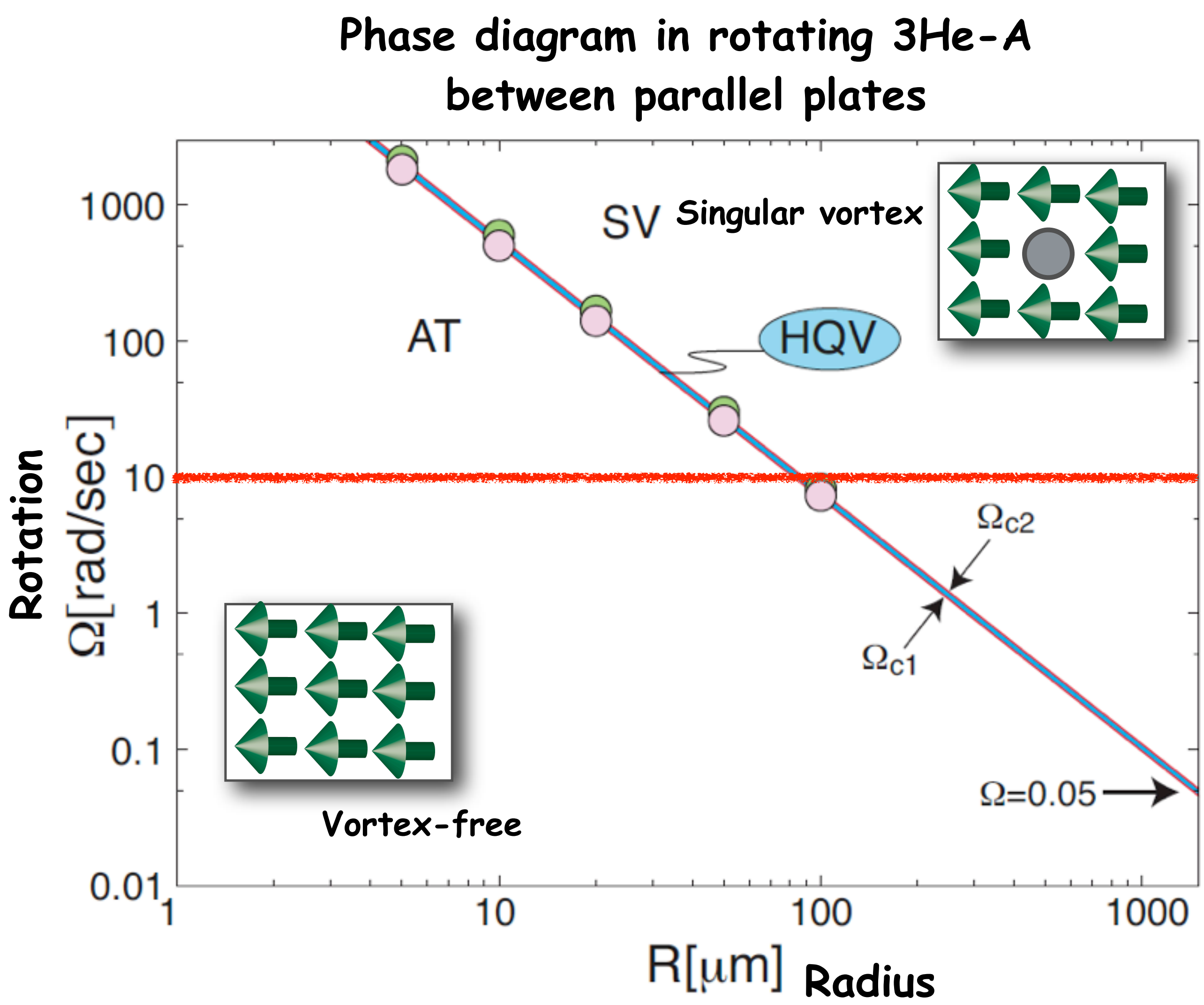
picture from Kubota lab. @ ISSP



Yamashita *et al.*, JLTP **158**, 353 (2010)

Vortex Phase Diagram

Kawakami *et al.*, PRB **79**, 092596 (2009)
Tsutsumi *et al.*, PRL **101**, 135302 (2008)



Minimizing the GL energy functional

$$T/T_c = 0.97$$
$$P = 3\mathrm{MPa}$$

strong-coupling effect ○
Fermi liquid correction ×
theory for high pressure and high temperature

rotating cryostat for sub-mK @ ISSP

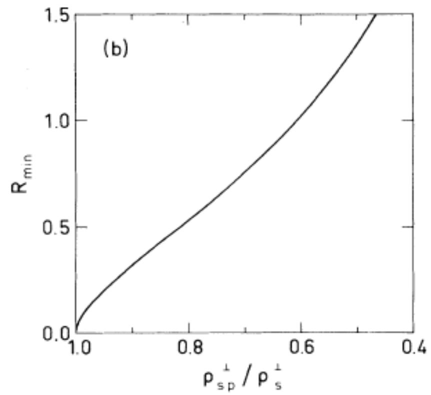
Fermi Liquid Corrections and Strong-Coupling Effect

Gradient term in GL functional

$$e^{i\Phi}\left(\boldsymbol{d}\right)_{\mu}\left(k_x + ik_y\right) \quad \boldsymbol{d} = (\cos\alpha, \sin\alpha, 0)$$

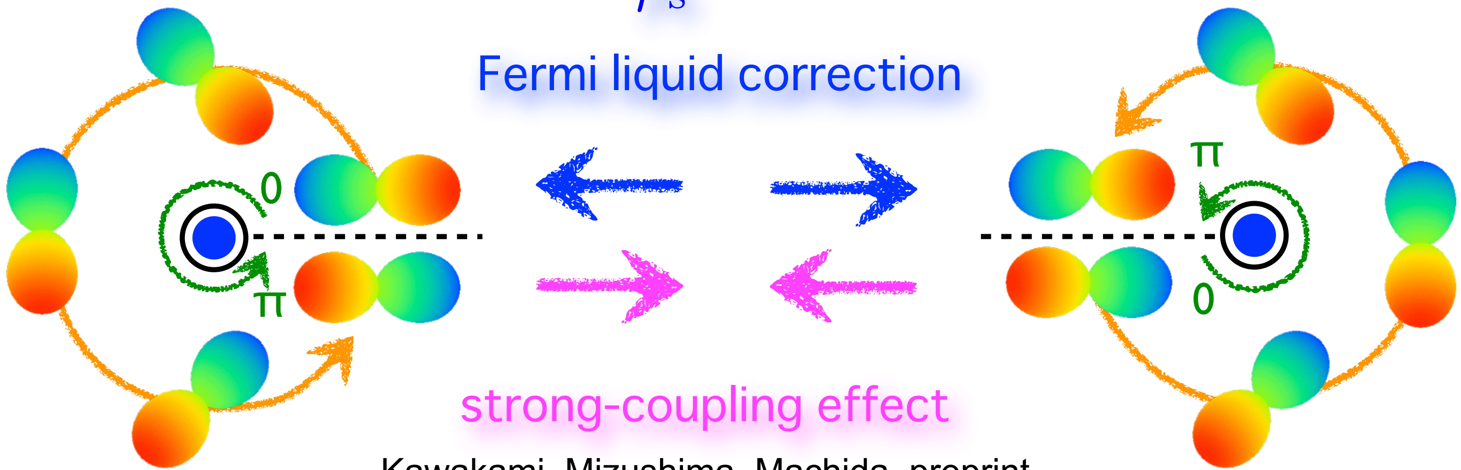
$$\mathcal{F}_{\rm grad} = C\left[\rho_{\rm s}\left(\nabla\Phi + \boldsymbol{r}\times\boldsymbol{\Omega}\right)^2 + \rho_{\rm spin}\left(\nabla\alpha\right)^2\right]$$

mass current spin current



$$\frac{\rho_{\rm spin}}{\rho_{\rm s}} < 1$$

e.g., Salomaa and Volovik, PRL '85
Vakaryuk and Leggett, PRL '09

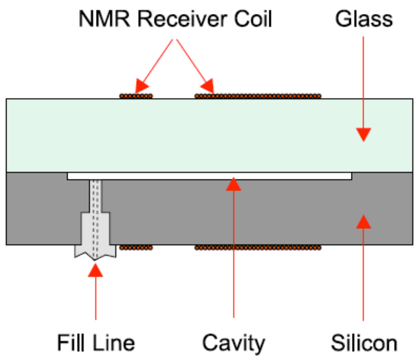


Kawakami, Mizushima, Machida, preprint

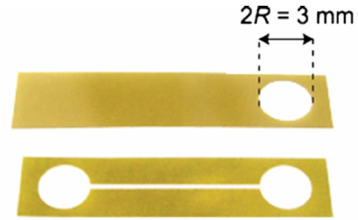
strong-coupling correction ⇒ stabilization for A-phase

HQV+HQV = singular vortex ⇒ Abelian statistics

Phase Diagram for 3He in a slab



Bennett *et al.*, JLTP **158**, 163 (2010)



Yamashita *et al.*, JLTP **158**, 353 (2010)

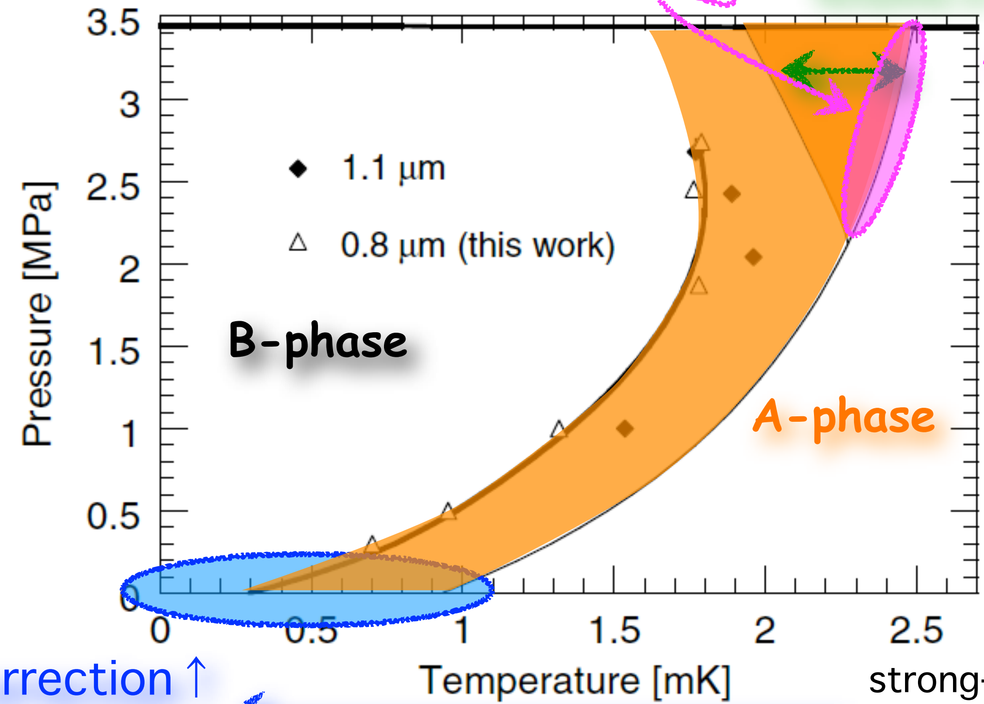
strong-coupling effect > Fermi liquid correction

HQV is unstable

Kawasaki *et al.*, PRL **94**, 105301 (2004)

Yamashita *et al.*, PRL **101**, 025301 (2008)

thickness ~ 12.5μm



strong-coupling effect ↑

Fermi liquid correction ↑

$\rho_{\rm spin} < \rho_{\rm s}$

strong-coupling effect and Fermi liquid correction can be controlled

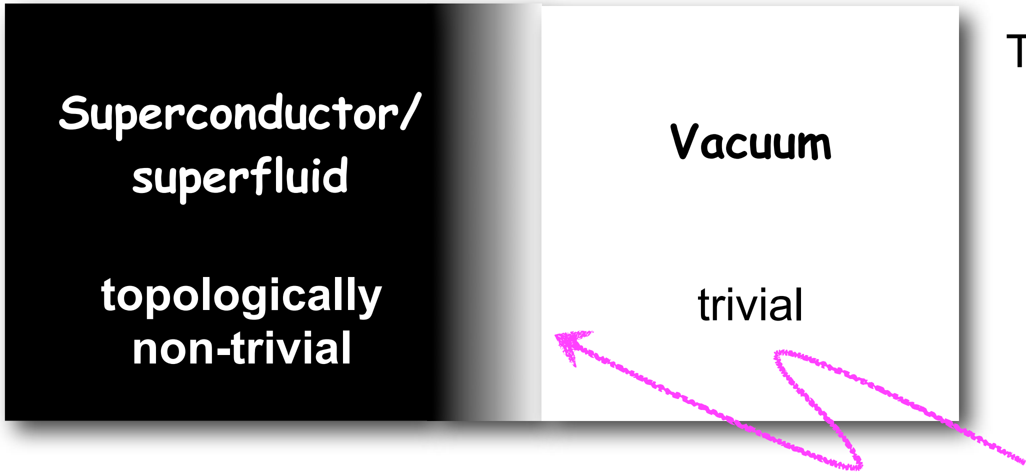
✓thickness < 1 μm ⇒ A-phase is stable in low pressure and temperature

HQV is stable by Fermi liquid correction

✓How to observe? ⇒ HQV pair may have NMR satellite peak
singular vortex has no signal

Kee and Maki, EPL **80**, 46003 (2007)

^{3}He as a Topological Superfluid



Topological # defined in momentum space

$\nu = \mathbb{Z}$ *2D p+ip*

$\nu = \mathbb{Z}_2$ *2D BW*

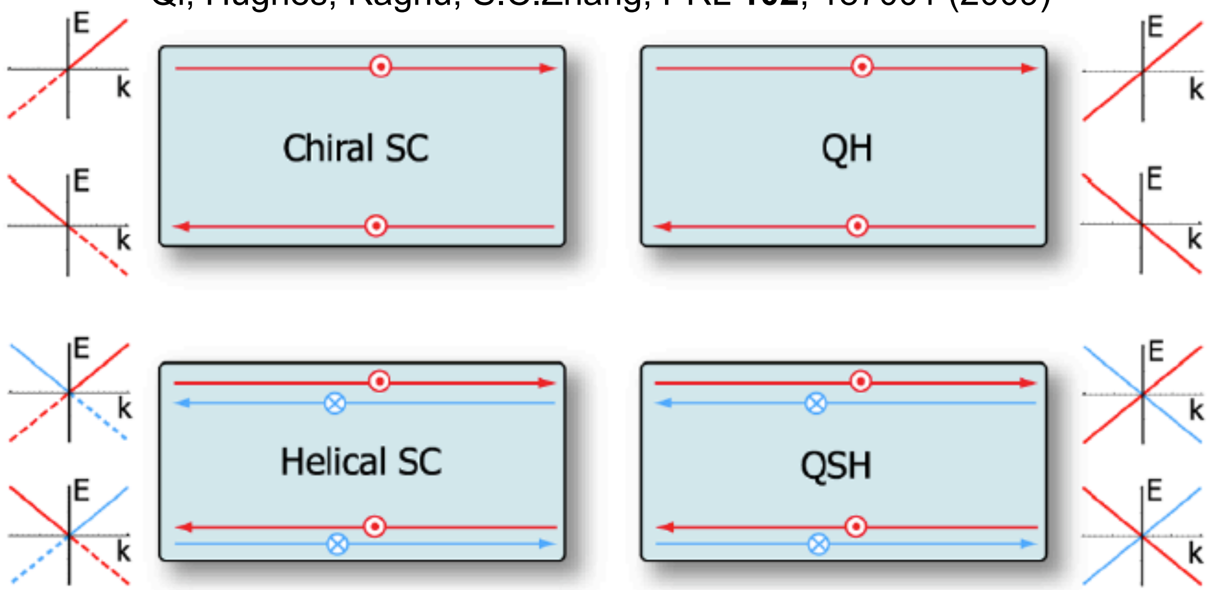
Read and Green, PRB **61**, 10267 (2000)
Schnyder *et al.*, PRB **78**, 195125 (2008)

Surface Andreev bound states
~ topological phase transition

Qi, Hughes, Raghu, S.C.Zhang, PRL **102**, 187001 (2009)

A-phase in 2D

B-phase in 2D



Broken time-reversal symmetry

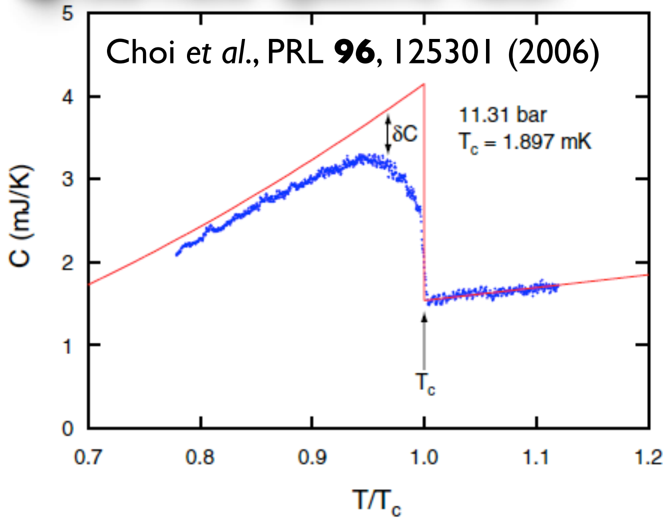
Time-reversal invariant

Majorana field

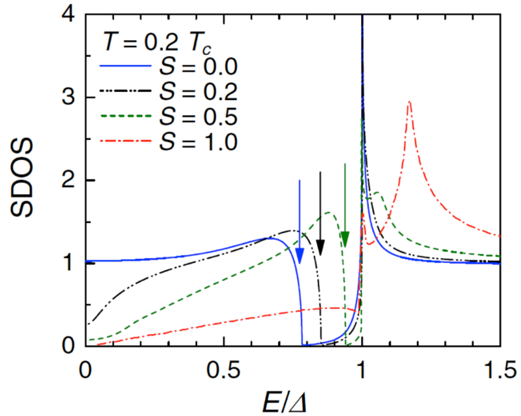
Dirac field

Observation of Surface Andreev Bound States

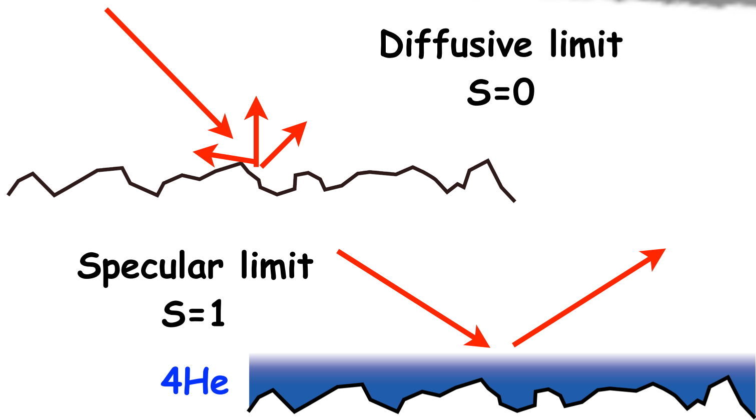
Surface specific heat



Surface density of states



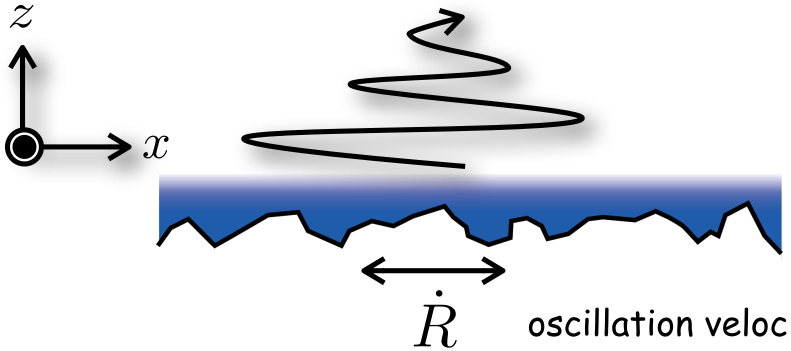
Nagato *et al.*, JLTP 1998



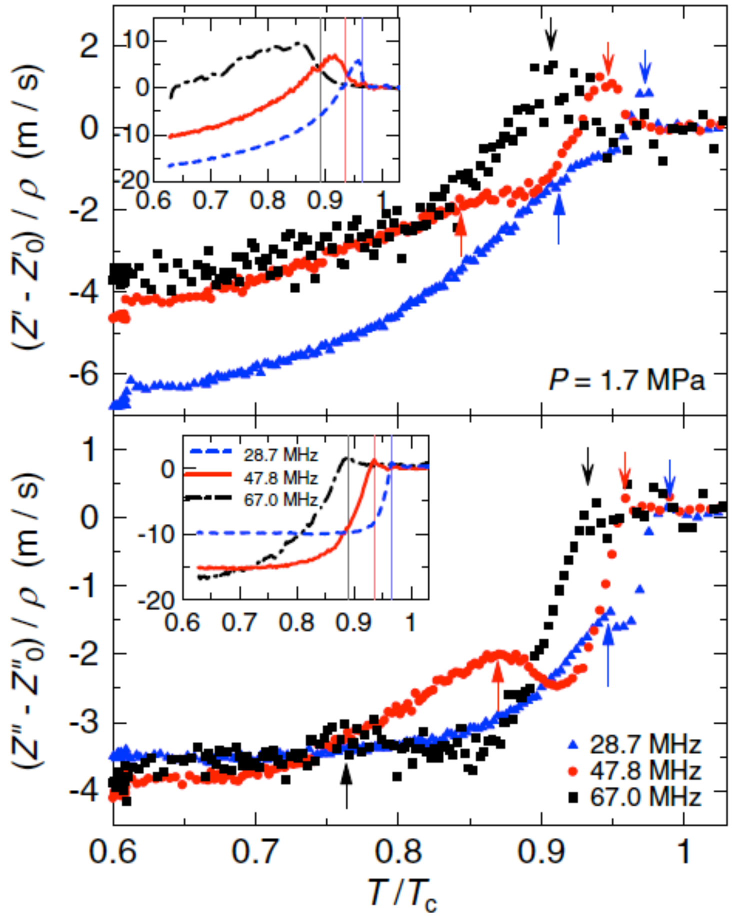
Transverse acoustic impedance

Aoki *et al.*, PRL **95**, 075301 (2005)
Murakawa *et al.*, PRL **103**, 155301 (2009)

acoustic impedance $$Z' + iZ'' = \frac{\Pi_{xz}}{\dot{R}}$$ stress tensor of 3He

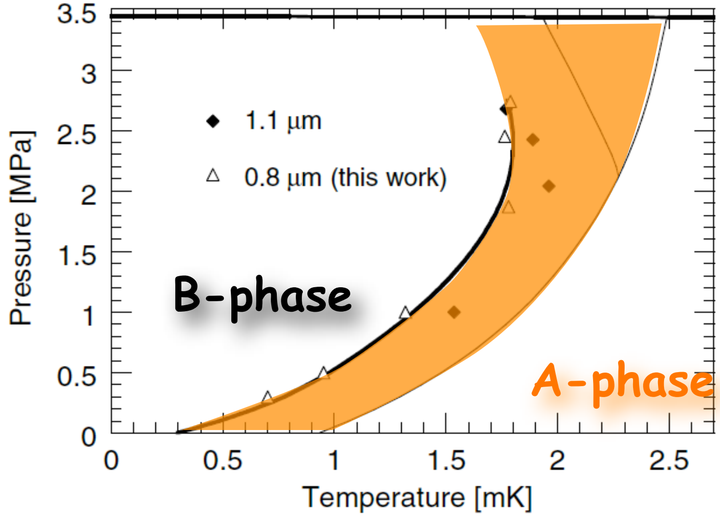


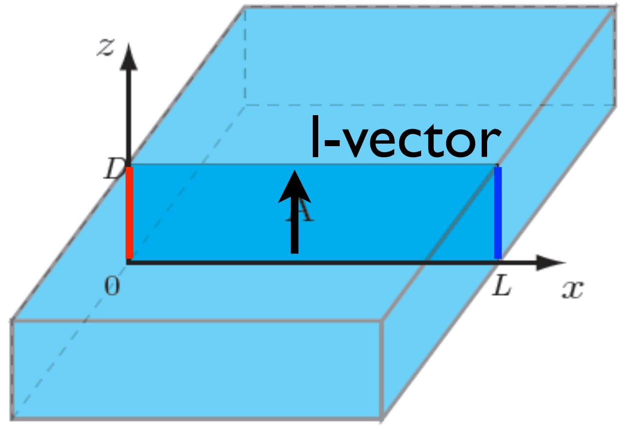
oscillation velocity of the wall

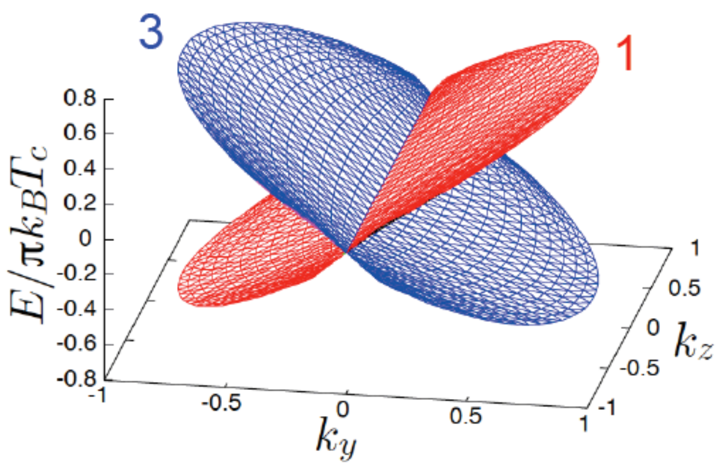


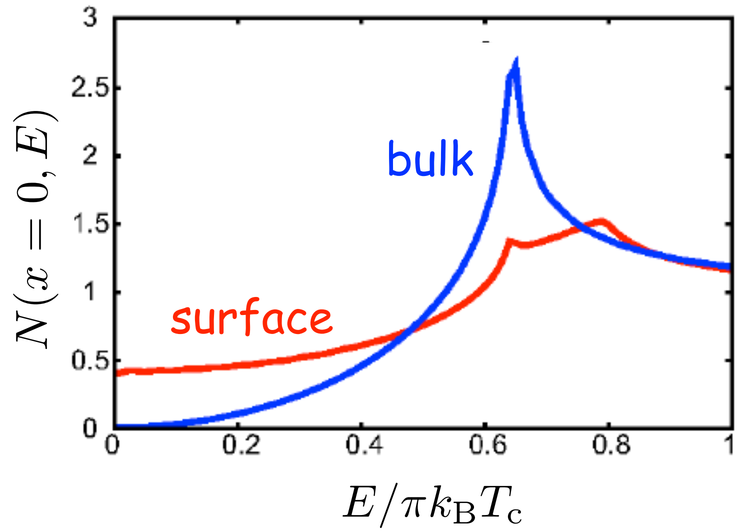
Surface Andreev Bound State in A-phase

Kawasaki *et al.*, PRL **94**, 105301 (2004)









Dispersionless (flat band) zero energy states

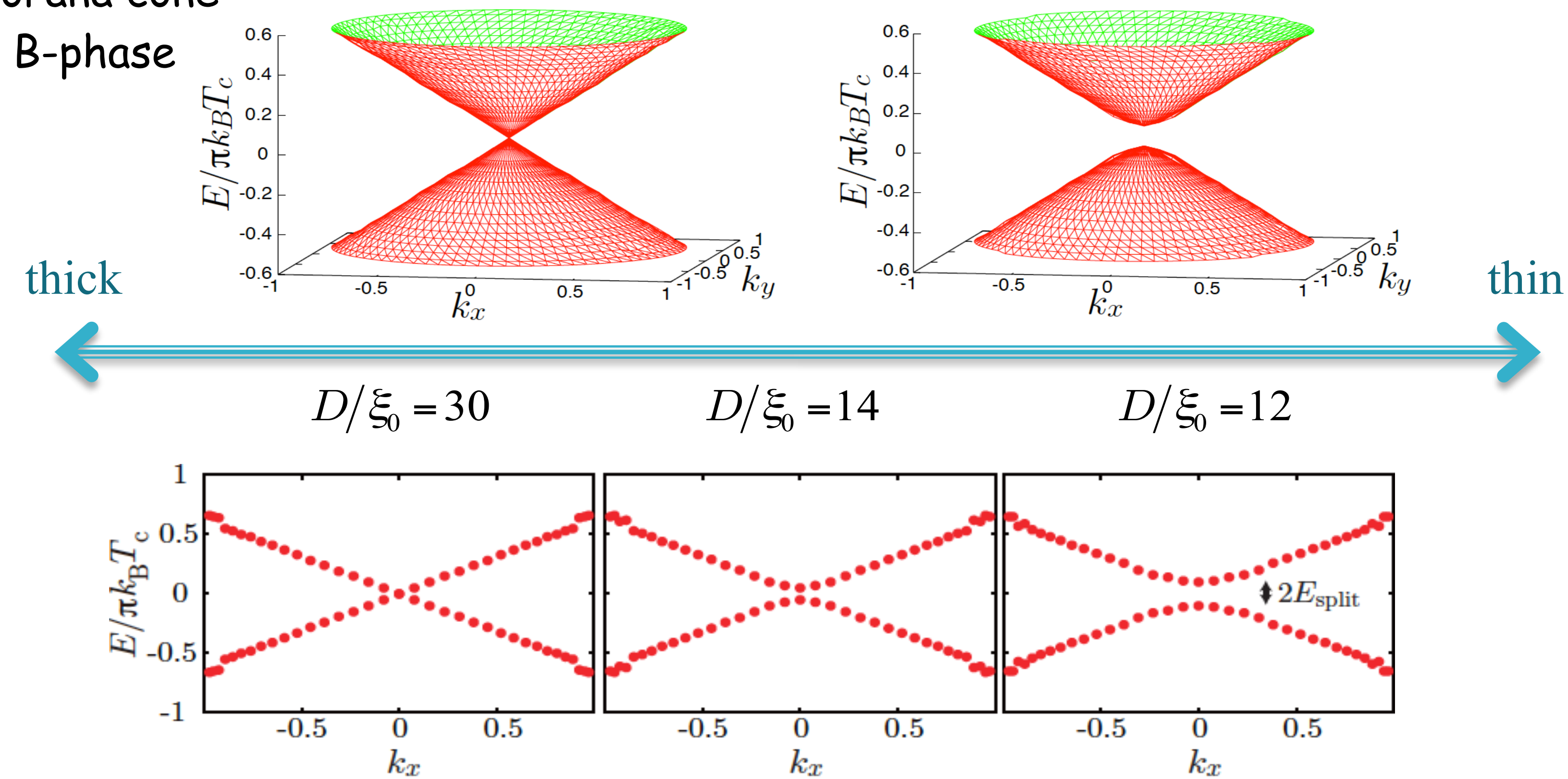
$$\Psi = \Psi_{\rm edge} + \Psi_{\rm node} + \Psi_{\rm gap} \approx \Psi_{\rm edge}$$

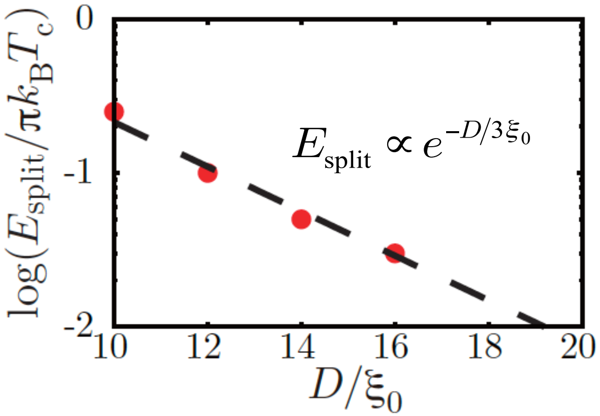
Self-hermitian fermions

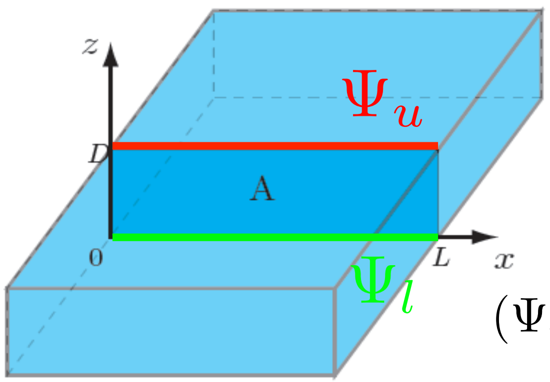
Tsutsumi, Mizushima, Ichioka, Machida, JPSJ **79**, 113601 (2010)

Surface Andreev Bound State in B-phase

Majorana cone
in B-phase





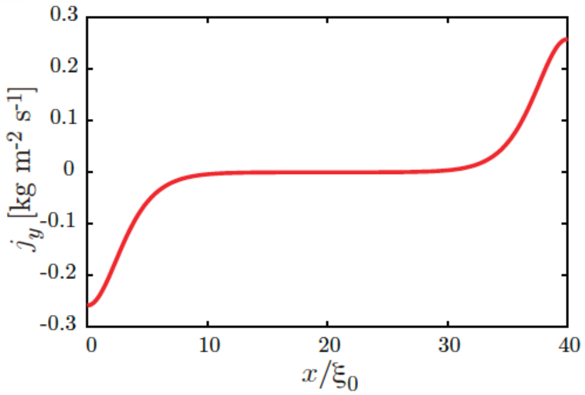


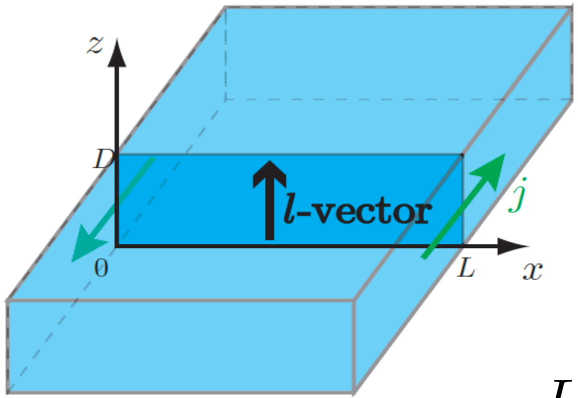
Energy split by
superposition of
the Majorana bound state

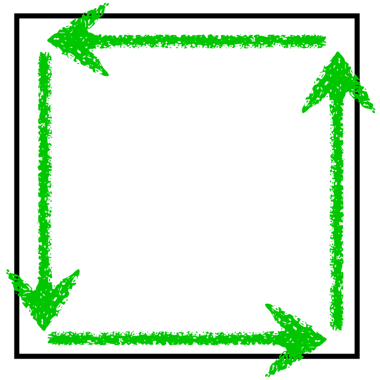
$$\left(\Psi_u \pm i\Psi_l\right)/\sqrt{2} \rightarrow E_{\pm}$$

Edge Current

A-phase : mass current





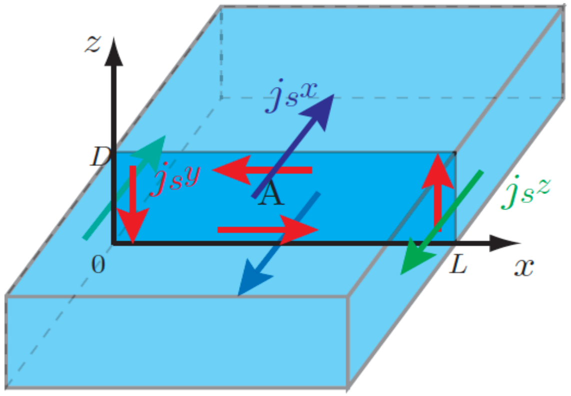


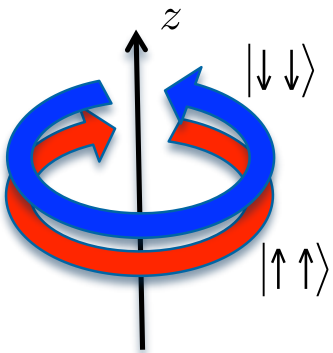
$$L_z \approx 0.42 N\hbar \; (T = 0.2T_c)$$

$$L_z = \frac{1}{2} N\hbar \; (T = 0)$$

Intrinsic angular momentum?

B-phase : spin current





Summary

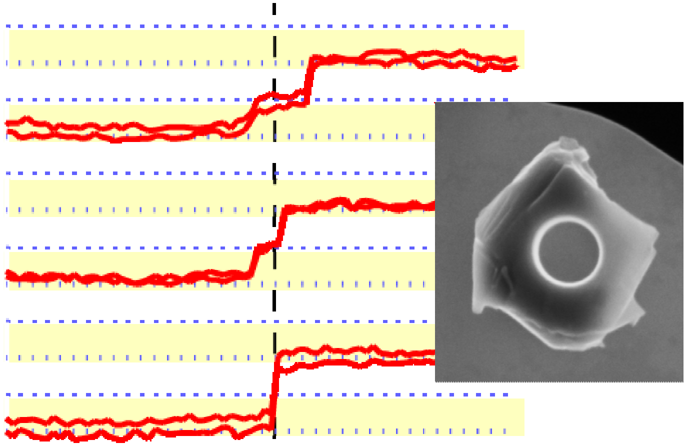
Half-quantum vortex (HQV)

➡Majorana zero mode in HQV ⇒ Non-Abelian statistics

➡Strong-coupling effect makes HQV unstable

How to observe HQV?
⇒ Quasi-classical theory for HQV pair

HQV in Sr_2RuO_4



Budakian-Maeno group, 2010

Surface Andreev bound state

➡Non-trivial topological invariant in bulk makes surface state

➡Linear dispersion behaves as Majorana fermion

➡Thickness of sample ⇒ variation of surface state

Edge mass current in A-phase ⇔ Intrinsic angular momentum
?