22pC10-6

日本物理学会２０１７年秋季大会

2017.9.22

トポロジカル超流動体である
ヘリウム３の表面状態

東大総合文化　堤 康雅

Superfluid helium-3

1971 Discovery of superfluid phase in helium-3

"Evidence for a New Phase of Solid ^{3}He",
Osheroff, Richardson, and Lee, PRL (1972)

phase transition ~ mK

^{3}He atoms make Cooper pairs in
spin-triplet p-wave state
by attraction via spin fluctuations.

Nobel Prize in 1996

D. M. Lee

D. D. Osheroff

R. C. Richardson

Nobel Prize in 2003

pioneering theoretical
contributions to superfluid ^{3}He

A. J. Leggett

Why study superfluid ^{3}He ?

Superfluid ^{3}He is "Model Matter".

- pure material
- well-known properties

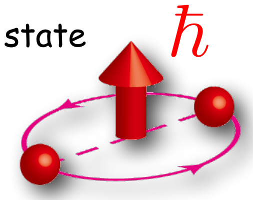
Superfluid ^{3}He
||
Topological Superfluid

Superfluid helium-3

A-phase high temp. & press.

Anderson-Brinkman-Morel (ABM) state

spin $S_z = 0$

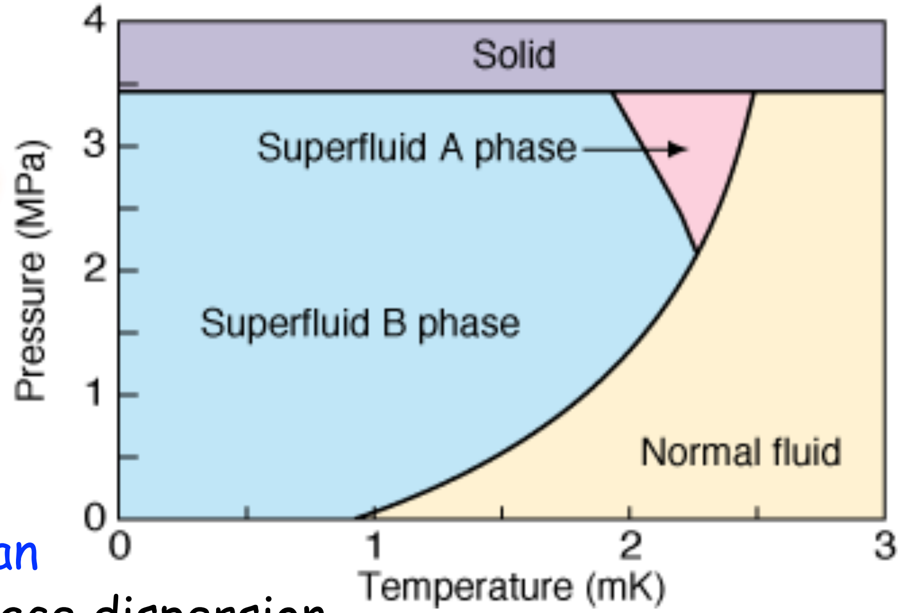


orbital $L_z = 1$ broken time reversal symmetry

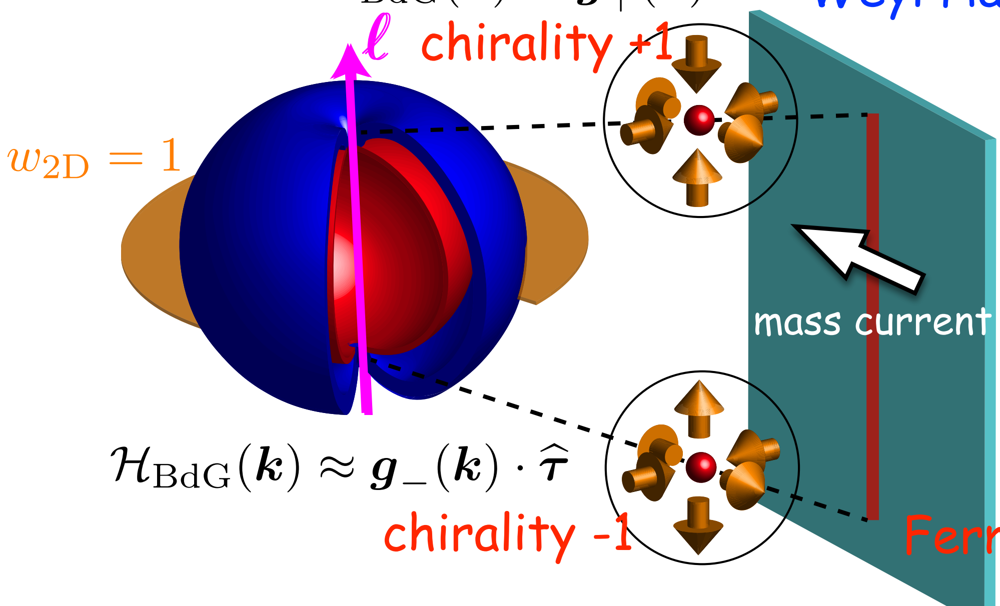
$$\hat{\Delta} \equiv \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_{\rm A}}{k_{\rm F}} \begin{pmatrix} k_x + ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$$\mathcal{H}_{\rm BdG}(\boldsymbol{k}) = \begin{pmatrix} \hat{\epsilon}(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k}) \\ \hat{\Delta}(\boldsymbol{k})^\dagger & -\hat{\epsilon}(-\boldsymbol{k})^{\rm T} \end{pmatrix}$$

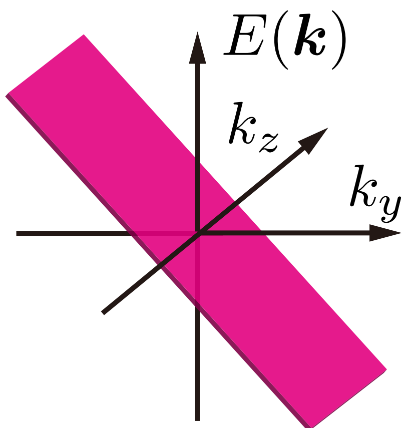
$[^3He]$ atoms make Cooper pairs in spin-triplet p-wave state.



$$\mathcal{H}_{\rm BdG}(\boldsymbol{k}) \approx \boldsymbol{g}_+(\boldsymbol{k}) \cdot \hat{\boldsymbol{\tau}}$$ Weyl Hamiltonian



surface dispersion

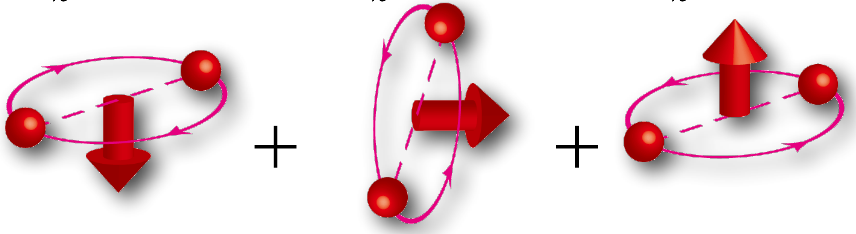


Fermi arc (zero energy states)

Superfluid helium-3

B-phase Balian-Werthamer (BW) state

$L_z = -1$ $\quad L_z = 0$ $\quad L_z = +1$



$S_z = +1$ $\quad S_z = 0$ $\quad S_z = -1$

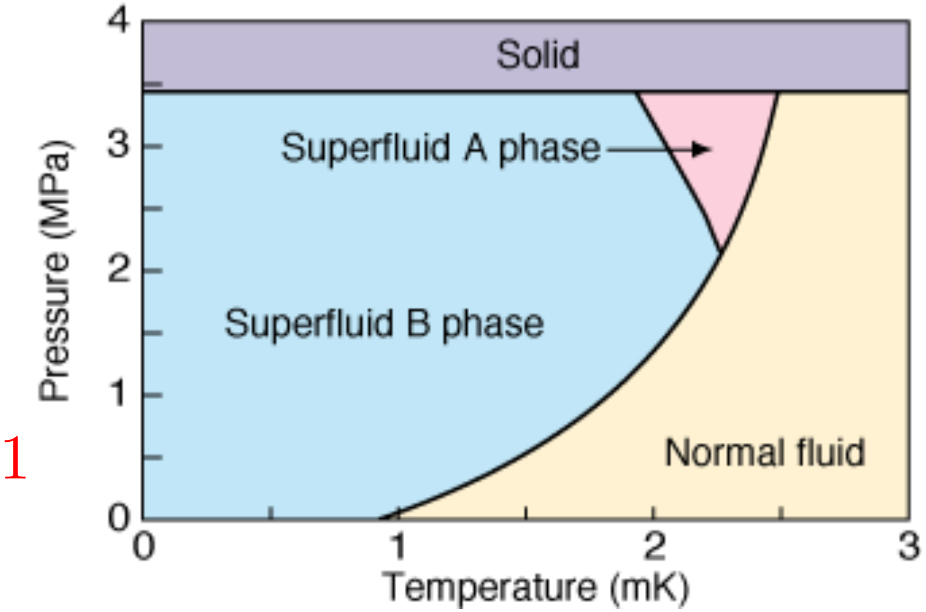
$\boldsymbol{J} = \boldsymbol{S} + \boldsymbol{L} = \boldsymbol{0}$ time reversal symmetry

$$\begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_{\mathrm{B}}}{k_{\mathrm{F}}} \begin{pmatrix} -k_x + ik_y & k_z \\ k_z & k_x + ik_y \end{pmatrix}$$

isotropic full gap topological #: $w_{3\mathrm{D}} = 1$

^{3}He atoms make Cooper pairs in

spin-triplet p-wave state.

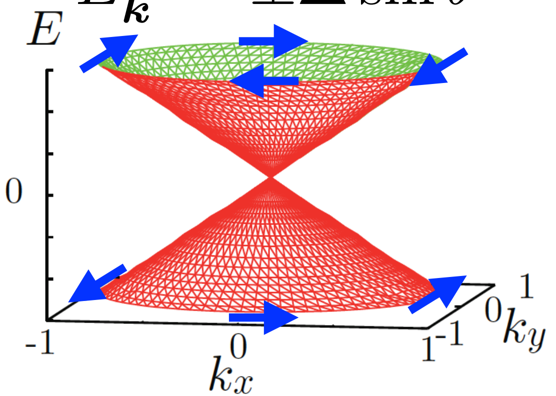


Bogoliubov-de Gennes eq.

$$\mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k})\Psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_{\boldsymbol{k}}\Psi_{\boldsymbol{k}}(\boldsymbol{r})$$

quasiparticle energy

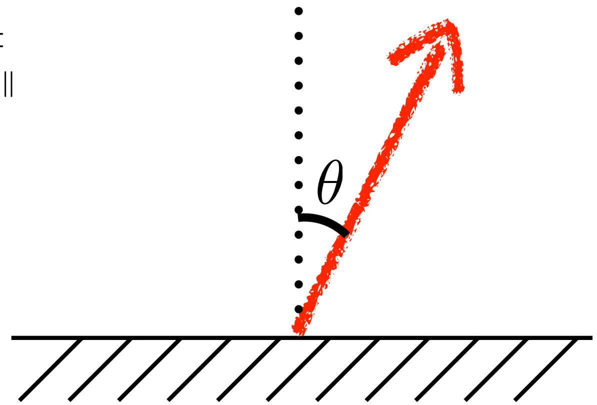
$$E^{\pm}_{\boldsymbol{k}} = \pm\Delta\sin\theta$$



$$\Psi^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}) = Ne^{i\boldsymbol{k}_{\mathrm{F}}\cdot\boldsymbol{r}} \exp\left(-\frac{z}{2\xi_0}\right) \Phi^{\pm}_{\boldsymbol{k}_\parallel}$$

$\xi_0 \equiv \dfrac{\hbar v_{\mathrm{F}}}{2\Delta}$: coherence length

$$\Phi^{\pm}_{\boldsymbol{k}_\parallel} = \begin{pmatrix} e^{-i\phi/2} \\ \mp ie^{i\phi/2} \\ \pm e^{i\phi/2} \\ ie^{-i\phi/2} \end{pmatrix}$$



Superfluid helium-3

wave function of bound state

$$\Psi^{\pm}_{\boldsymbol{k}_\parallel}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_\parallel\cdot\boldsymbol{r}} \sin(k_\perp z)\exp\left(-\frac{z}{2\xi_0}\right)\Phi^{\pm}_{\boldsymbol{k}_\parallel} \qquad k_\perp = \sqrt{k_{\rm F}^2 - k_\parallel^2}$$

field operator

$$\hat{\Psi}(\boldsymbol{r}) = \begin{pmatrix} \hat{\Psi}_\uparrow(\boldsymbol{r}) \\ \hat{\Psi}_\downarrow(\boldsymbol{r}) \\ \hat{\Psi}^\dagger_\uparrow(\boldsymbol{r}) \\ \hat{\Psi}^\dagger_\downarrow(\boldsymbol{r}) \end{pmatrix} = \sum_{\boldsymbol{k}_\parallel}\left[\hat{\gamma}_{\boldsymbol{k}_\parallel}\Psi^{+}_{\boldsymbol{k}_\parallel}(\boldsymbol{r}) + \hat{\gamma}^\dagger_{-\boldsymbol{k}_\parallel}\Psi^{-}_{\boldsymbol{k}_\parallel}(\boldsymbol{r})\right]$$

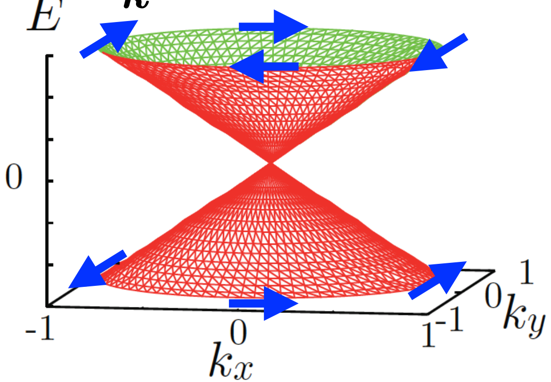
$$\hat{\Psi}_\sigma(\boldsymbol{r}) = \hat{\Psi}^\dagger_\sigma(\boldsymbol{r})$$ Majorana fermion

Bogoliubov-de Gennes eq.

$$\mathcal{H}_{\rm BdG}(\boldsymbol{k})\Psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_{\boldsymbol{k}}\Psi_{\boldsymbol{k}}(\boldsymbol{r})$$

quasiparticle energy

$$E^{\pm}_{\boldsymbol{k}} = \pm\Delta\sin\theta$$



$$\Psi^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_{\rm F}\cdot\boldsymbol{r}}\exp\left(-\frac{z}{2\xi_0}\right)\Phi^{\pm}_{\boldsymbol{k}_\parallel}$$

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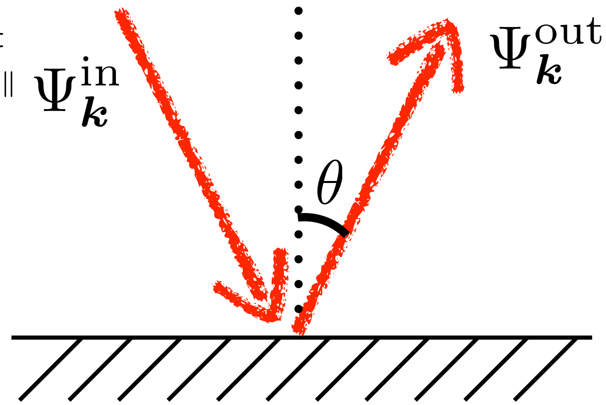
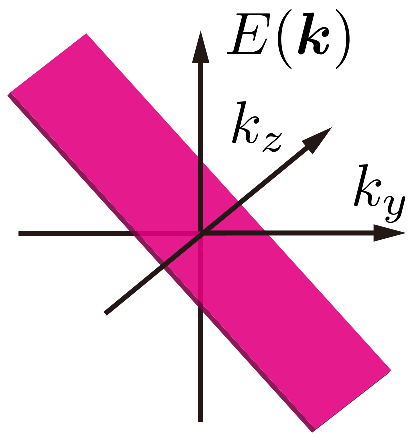
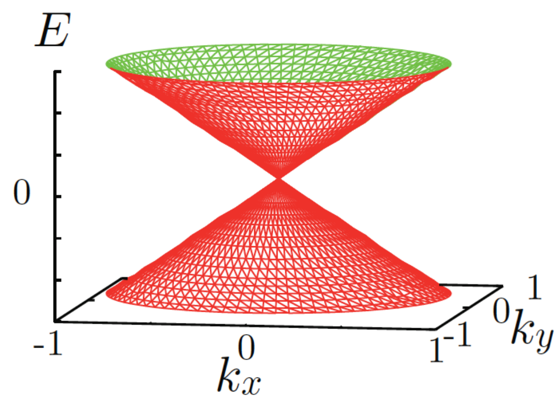


Superfluid helium-3

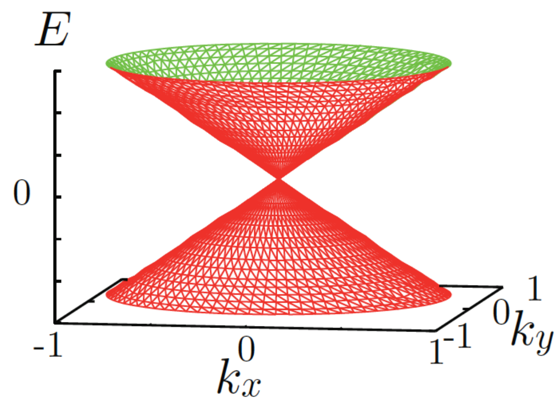
	A-phase	B-phase
	broken time reversal symmetry	time reversal symmetry
topological #	$w_{2\text{D}} = 1$	$w_{3\text{D}} = 1$
	Weyl fermion	
surface dispersion		
		Majorana fermion
surface current	mass current	spin current

Topics

- Quantitative evidence of Majorana fermions in surface bound state of B-phase

rough surface

previous work: Y. Aoki, *et al*., Phys. Rev. Lett. **95**, 075301 (2005).
H. Choi *et al*., Phys. Rev. Lett. **96**, 125301 (2006).



specular surface

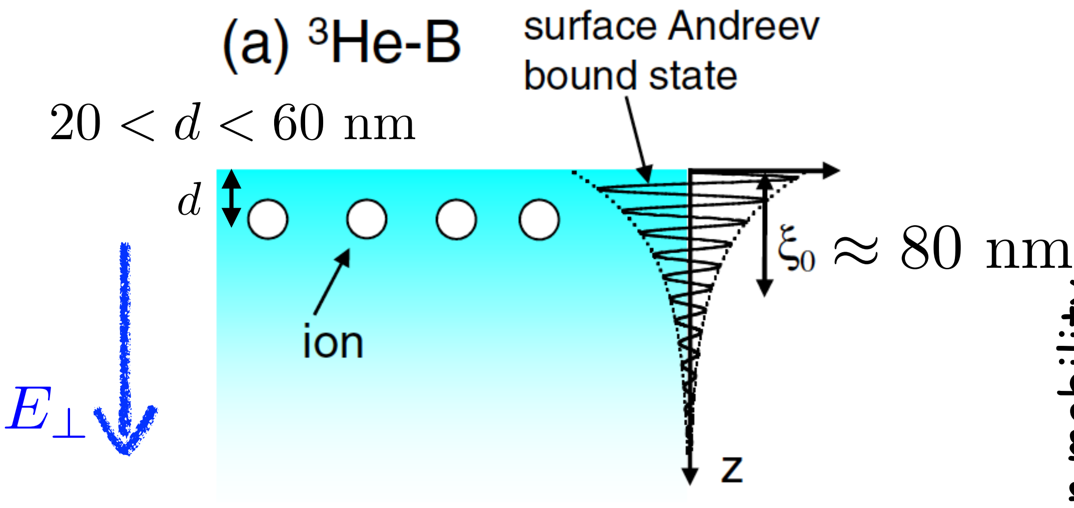
experiment: H. Ikegami, S. B. Chung, and K. Kono, J. Phys. Soc. Jpn. **82**, 124607 (2013).
theory: Y. Tsutsumi, Phys. Rev. Lett. **118**, 145301 (2017).

quantitative agreement of experiment and theory

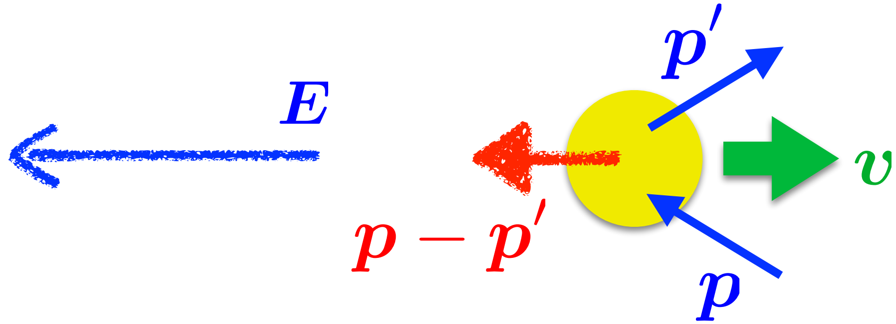
- Surface bound state in A-phase with mass current and angular momentum

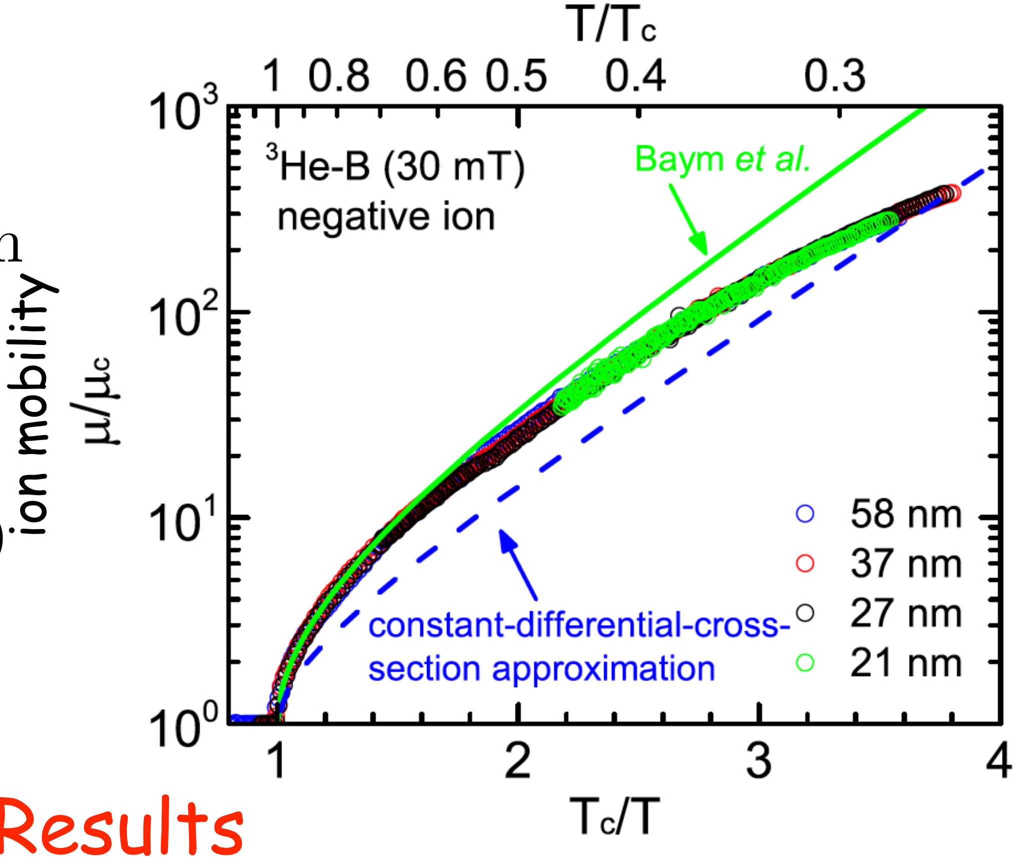
Experiment for ions trapped below a free surface

H. Ikegami *et al.*, JPSJ **82**, 124607 (2013).



electron bubble (R=2.0nm~10/k_F, M=290m_3)





Results

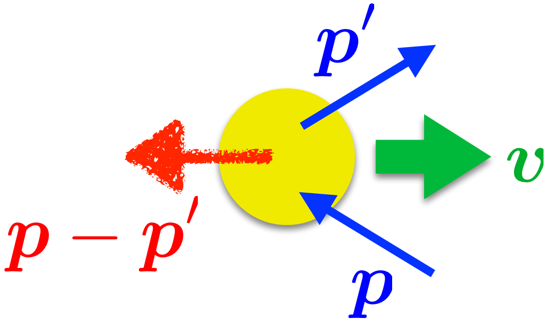
Mobility is smaller than the bulk value.

scattering by surface bound state

Mobility is independent of trapped depth.

despite spatial dependence of SBS

Equation of motion for electron bubble



$$\frac{d\boldsymbol{P}}{dt} = -\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma,\sigma'} \hbar(\boldsymbol{k}'-\boldsymbol{k})(1-f_{\boldsymbol{k}'})f_{\boldsymbol{k}}\Gamma_{\boldsymbol{v}}(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')$$

$\boldsymbol{p}=\hbar\boldsymbol{k}$: quasiparticle momentum $f_{\boldsymbol{k}}=\left(e^{E_{\boldsymbol{k}}/k_{\mathrm{B}}T}+1\right)^{-1}$: Fermi distribution

σ : quasiparticle spin $E_{\boldsymbol{k}}$: quasiparticle energy

first order of v

$$\frac{d\boldsymbol{P}}{dt} = -\frac{\hbar^2}{2k_{\mathrm{B}}T}\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma,\sigma'}(\boldsymbol{k}'-\boldsymbol{k})(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}(1-f_{\boldsymbol{k}})f_{\boldsymbol{k}}\Gamma(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')$$

transition rate: $\Gamma(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')=\frac{2\pi}{\hbar}\delta(E_{\boldsymbol{k}'}-E_{\boldsymbol{k}})|t(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')|^2$

elastic scattering

Stokes drag force: $\frac{d\boldsymbol{P}}{dt}=-\eta_{\|}\boldsymbol{v}_{\|}$

driving force: $e\boldsymbol{E}_{\|}=\frac{e}{\mu}\boldsymbol{v}_{\|}$

$$\frac{e}{\mu}=\eta_{\|}=\frac{\pi\hbar}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'}(\boldsymbol{k}'_{\|}-\boldsymbol{k}_{\|})^2\left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'}-E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')|^2$$

T-matrix

$$\sum_{\sigma,\sigma'} |t(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')|^2 = \sum_{\sigma,\sigma'=\uparrow,\downarrow} |\langle \Psi_{\boldsymbol{k}',\sigma'}|T_{\rm S}|\Psi_{\boldsymbol{k},\sigma}\rangle|^2$$

momentum eigenstate

$$|\Psi_{\boldsymbol{k},\sigma}\rangle = \Phi_\sigma |\boldsymbol{k}_{\rm F}\rangle \qquad \Phi_\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\i\end{pmatrix}, \quad \Phi_\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\i\\-1\\0\end{pmatrix}$$

Lippmann-Schwinger equation

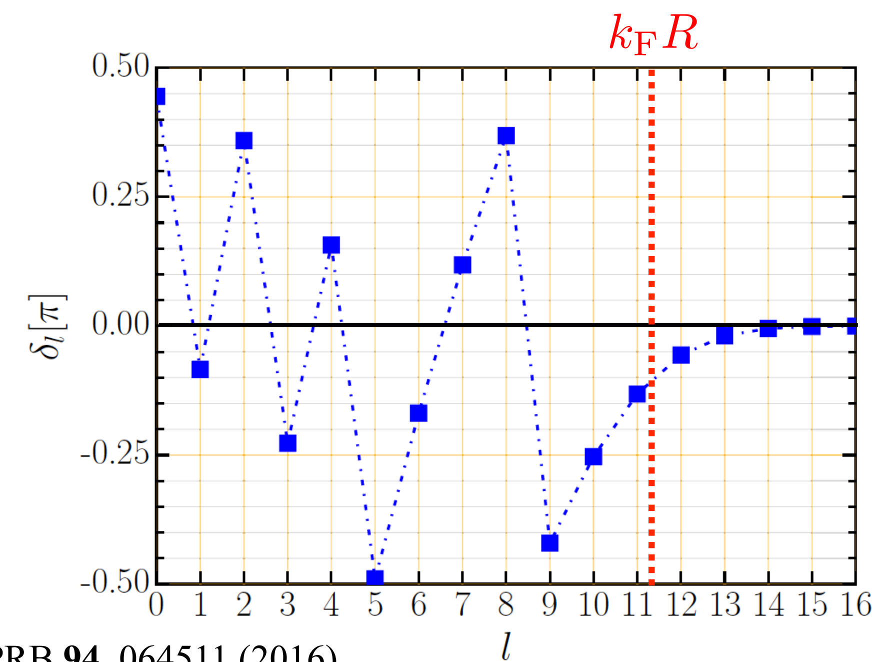
$$T_{\rm S} = V + VG_{\rm S}T_{\rm S} \qquad V \text{ :hard sphere potential} \qquad R = 11.17k_{\rm F}^{-1}$$

T-matrix in normal state

$$T_{\rm N}(\hat{\boldsymbol{k}}',\hat{\boldsymbol{k}}) = \begin{pmatrix} t_{\rm N}(\hat{\boldsymbol{k}}',\hat{\boldsymbol{k}})\hat{1} & 0 \\ 0 & -t_{\rm N}(-\hat{\boldsymbol{k}}',-\hat{\boldsymbol{k}})^*\hat{1}\end{pmatrix},$$

$$t_{\rm N}(\hat{\boldsymbol{k}}',\hat{\boldsymbol{k}}) = -\frac{1}{\pi N_{\rm F}}\sum_{l=0}^{\infty}(2l+1)\underbrace{e^{i\delta_l}\sin\delta_l}_{\text{phase shift}} P_l(\hat{\boldsymbol{k}}'\cdot\hat{\boldsymbol{k}})$$

$$\tan\delta_l = j_l(k_{\rm F}R)/n_l(k_{\rm F}R)$$



O. Shevtsov and J. A. Sauls, PRB **94**, 064511 (2016).

Mobility

$$\frac{e}{\mu} = \frac{\pi\hbar}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'}(\boldsymbol{k}'_{\parallel} - \boldsymbol{k}_{\parallel})^2 \left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'} - E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')|^2$$

$$\sum_{\boldsymbol{k}} \to \int_{-\Delta}^{\Delta} dE_{\boldsymbol{k}} \int \frac{d\Omega_{\boldsymbol{k}}}{4\pi} N(\hat{\boldsymbol{k}}, E_{\boldsymbol{k}}, z_0)$$

$$\frac{e}{\mu} = \frac{\pi^2}{4}\exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\mathrm{F}} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right)\sigma_{\mathrm{tr}}(E, z_0)$$

DOS

$$N(E,z) = \frac{\pi}{2}N_{\mathrm{F}}\frac{|E|}{\Delta}\exp\left(-\frac{z}{\xi}\right)$$

n_3 : ^{3}He particle density

total cross section

$$\sigma_{\mathrm{tot}}(E,z) \equiv \frac{3}{2}\int_0^{2\pi} d\varphi \overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z)$$

transport cross section

$$\sigma_{\mathrm{tr}}(E,z) \equiv \frac{3}{2}\int_0^{2\pi} d\varphi (1-\cos\varphi)\overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z)$$

$$\varphi \equiv \phi - \phi'$$

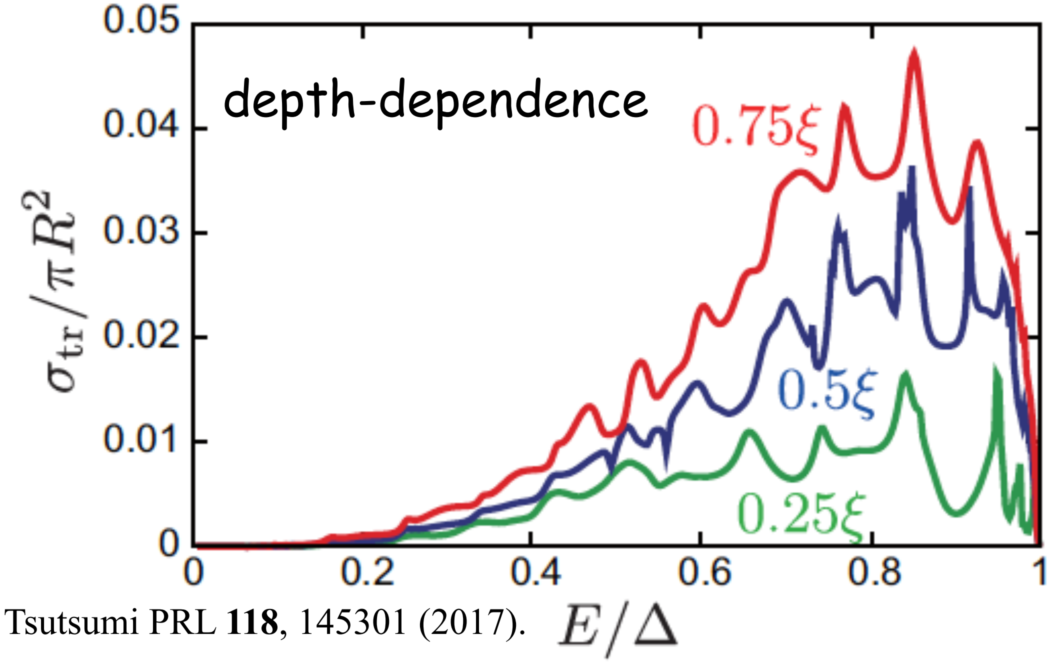
polar angle averaged differential cross section

$$\overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z) = \left(\frac{\pi N_{\mathrm{F}}}{k_{\mathrm{F}}}\right)^2 \left(\frac{E}{\Delta}\right)^4 \frac{1}{4}\sum_{s,s'=\pm 1}\sum_{\sigma,\sigma'}|t(\boldsymbol{k}_{\parallel}, sk_{\perp}, \sigma \to \boldsymbol{k}'_{\parallel}, s'k_{\perp}, \sigma')|^2$$

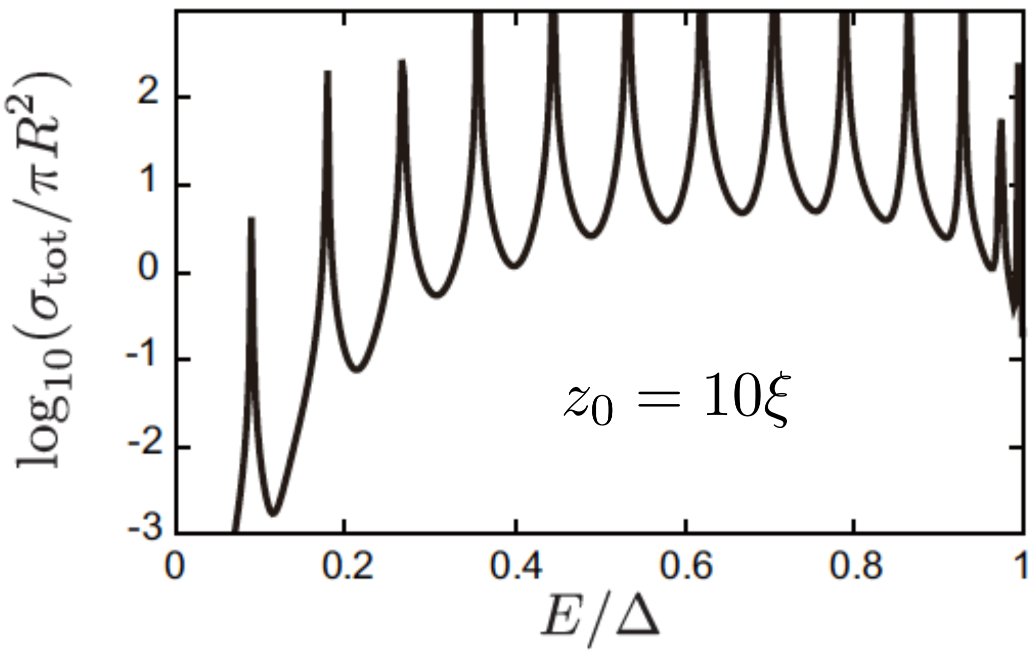
$$[k_{\parallel}(E)N(E)]^2$$

$$k_{\perp} \equiv |k_z| = \sqrt{k_{\mathrm{F}}^2 - k_{\parallel}^2}$$

Scattering cross section



Y, Tsutsumi PRL **118**, 145301 (2017).



suppressed by approaching surface

opposite dependence against DOS of surface bound state

resonance peak with bound state around ion

$$E_{b,l} = \Delta \cos(\delta_{l+1} - \delta_l)$$

E.V. Thuneberg *et al.*, Physica B **107**, 43 (1981).

increase DOS of surface bound state

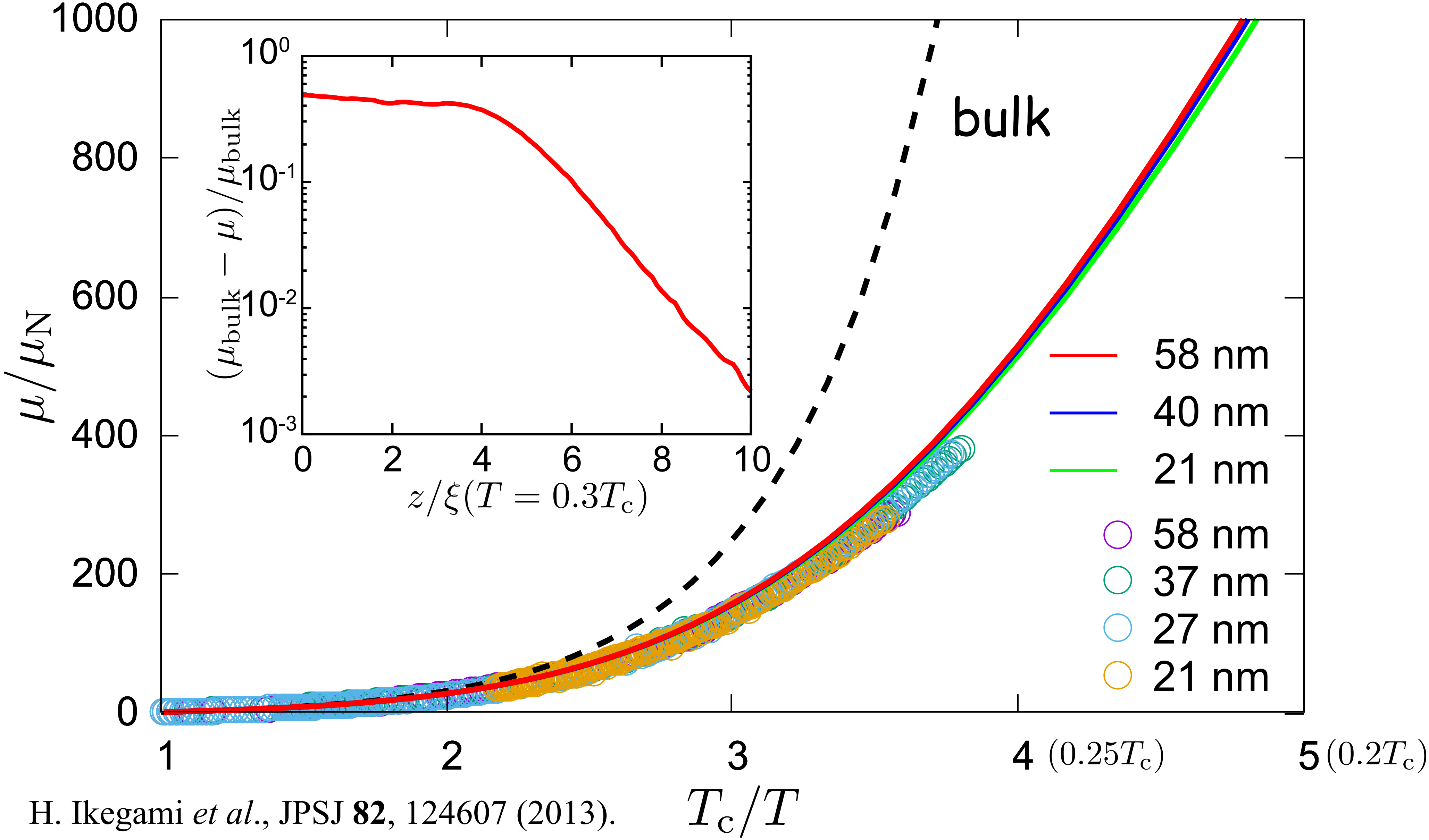
decrease DOS above gap

decrease DOS of bound state around ion

decrease scattering cross section

Mobility

$$\mu = \frac{e}{\eta_{\rm bound} + \eta_{\rm B}} \qquad \eta_{\rm bound} = \frac{\pi^2}{4} \exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\rm F} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\rm tr}(E, z_0)$$



H. Ikegami *et al.*, JPSJ **82**, 124607 (2013).

Summary (B-phase)

- Majorana fermions in surface bound state has been observed via scattering by an impurity.

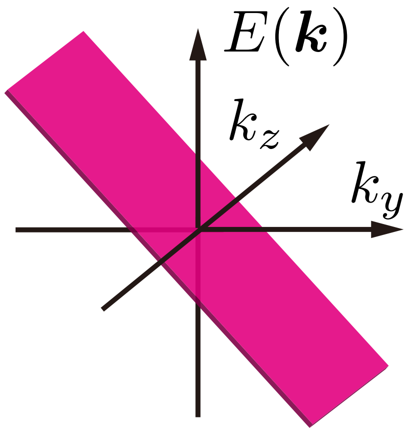
- The experimental result quantitatively agrees with theoretical calculation.

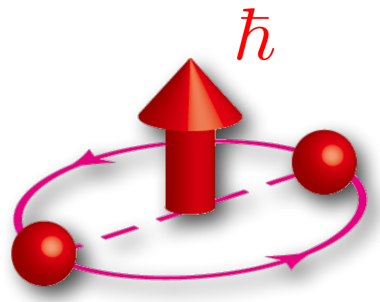
experiment: H. Ikegami, S. B. Chung, and K. Kono, J. Phys. Soc. Jpn. **82**, 124607 (2013).
theory: Y. Tsutsumi, Phys. Rev. Lett. **118**, 145301 (2017).

Topics

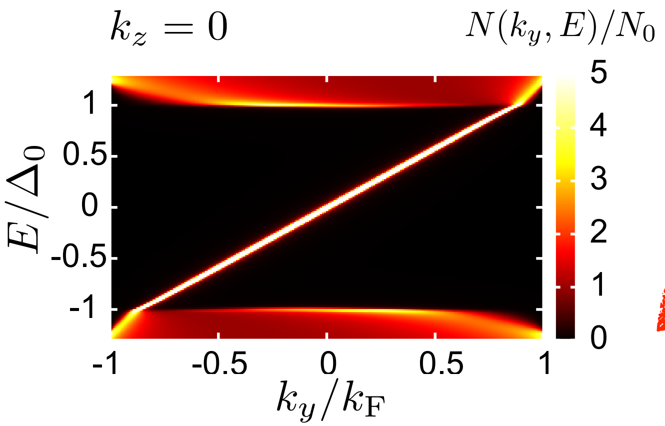
- Quantitative evidence of Majorana fermions in surface bound state of B-phase

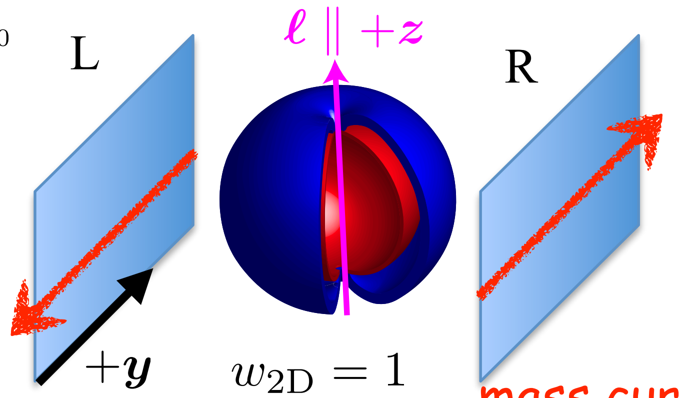
- Surface bound state in A-phase with mass current and angular momentum

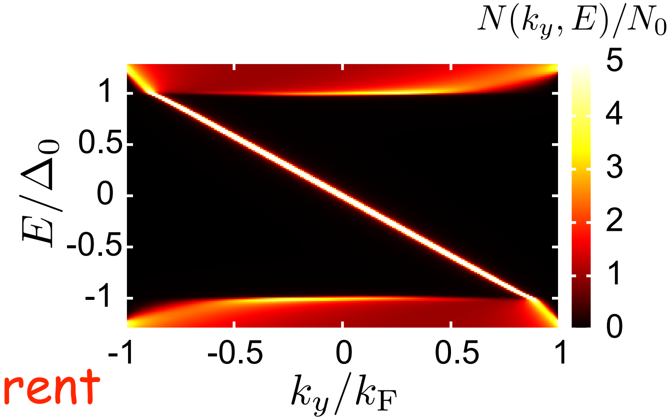




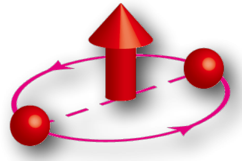
Surface bound state and mass current



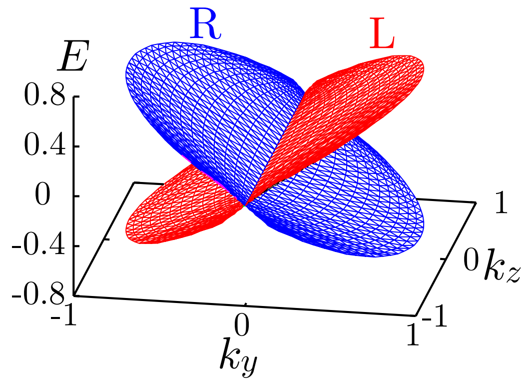




mass current



Direction of AM by the mass current corresponds to that by Cooper pairs.



Quasiclassical theory

$\Delta/E_F \ll 1$ $\quad \int d\xi_k \widehat{\sigma}_z \widehat{G}(\boldsymbol{k},\boldsymbol{r},\omega_n) \equiv \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix}$

Eilenberger equation

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F,\boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \right]$$

order parameter: $\hat{\Delta}(\boldsymbol{k}_\mathrm{F},\boldsymbol{r}) = N_0 \pi k_\mathrm{B} T \sum_n \int \frac{d\Omega_{\boldsymbol{k}'_\mathrm{F}}}{4\pi} V(\boldsymbol{k}_\mathrm{F},\boldsymbol{k}'_\mathrm{F}) \hat{f}(\boldsymbol{k}'_\mathrm{F},\boldsymbol{r},\omega_n)$

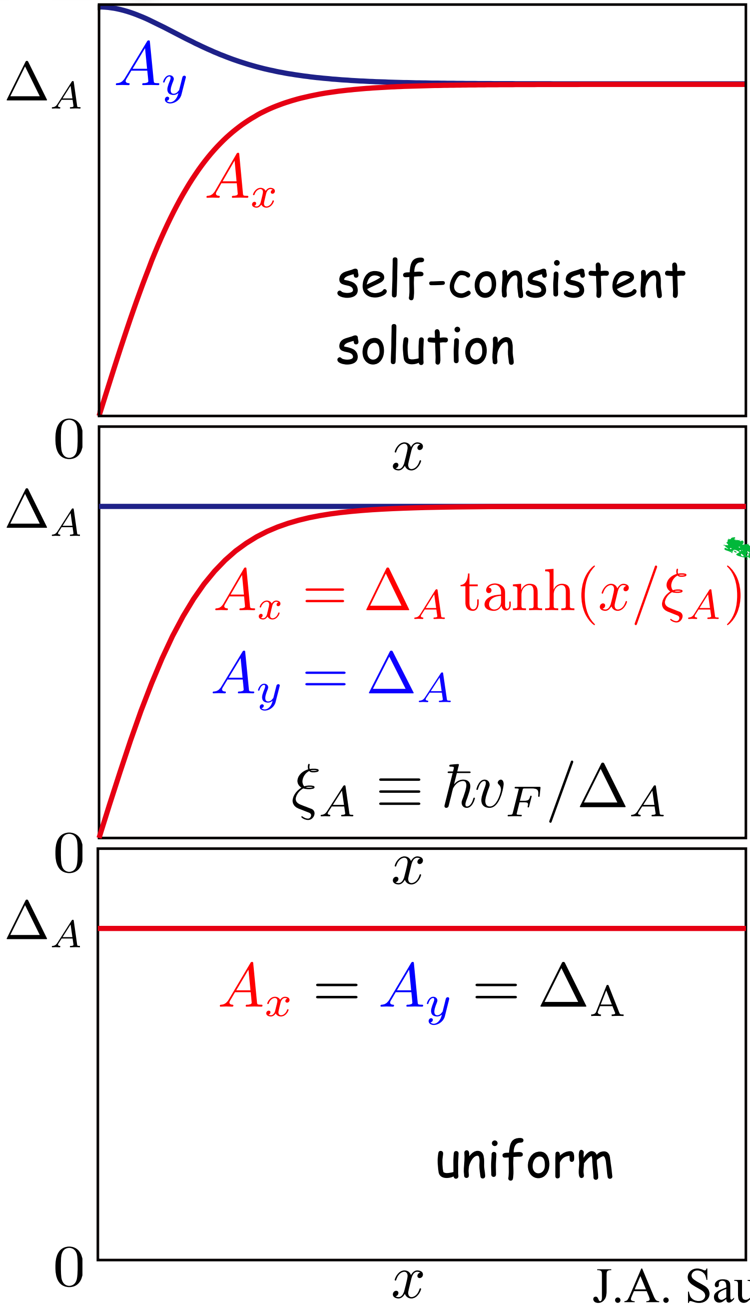
mass current: $\boldsymbol{j}(\boldsymbol{r},T) = N_0 \pi k_\mathrm{B} T \sum_n \int \frac{d\Omega_{\boldsymbol{k}_\mathrm{F}}}{4\pi} \boldsymbol{p}_\mathrm{F} g_0(\boldsymbol{k}_\mathrm{F},\boldsymbol{r},\omega_n)$

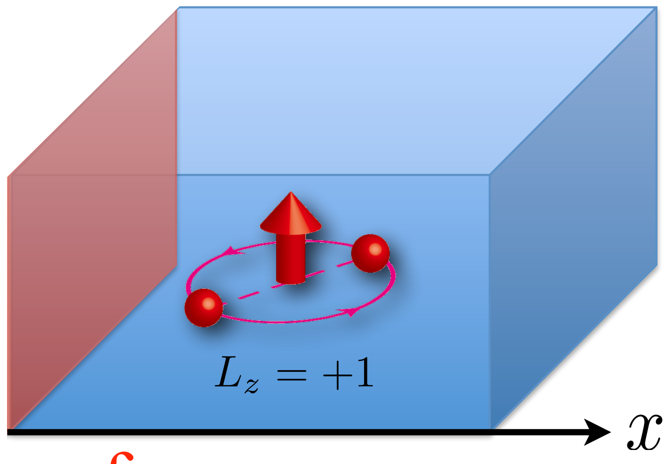
density of states: $N(\boldsymbol{k}_F,\boldsymbol{r},E) = N_0 \mathrm{Re}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n \to E+i\eta}]$

mass current spectrum: $\boldsymbol{j}(\boldsymbol{k}_\mathrm{F},\boldsymbol{r},E) = \boldsymbol{p}_\mathrm{F} N_0 \mathrm{Re}\left[g_0(\boldsymbol{k}_\mathrm{F},\boldsymbol{r},\omega_n)|_{i\omega_n \to E+i\eta}\right]$

Quasiclassical Green's function

Order parameter $\Delta = A_x k_x + i A_y k_y$





specular surface
$x = 0$

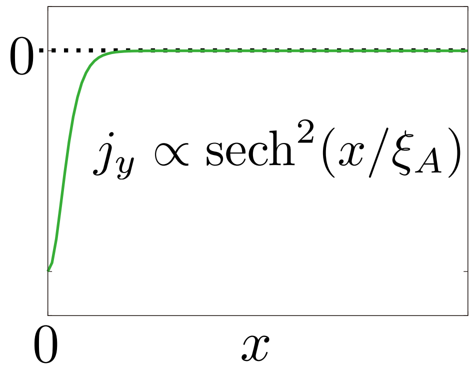
analytic solution: $(k_x = \cos\phi \sin\theta, k_y = \sin\phi \sin\theta)$

$$g_0 = \frac{1}{\sqrt{\omega_n^2 + \Delta_A^2 \sin^2\theta}} \left[\omega_n + \frac{\Delta_A^2 \sin^2\theta \cos^2\phi}{2(\omega_n + i\Delta_A \sin\theta \sin\phi)} \operatorname{sech}^2\left(\frac{x}{\xi_A}\right) \right]$$

bulk — surface state

YT and K. Machida, JPSJ **81**, 074607 (2012).

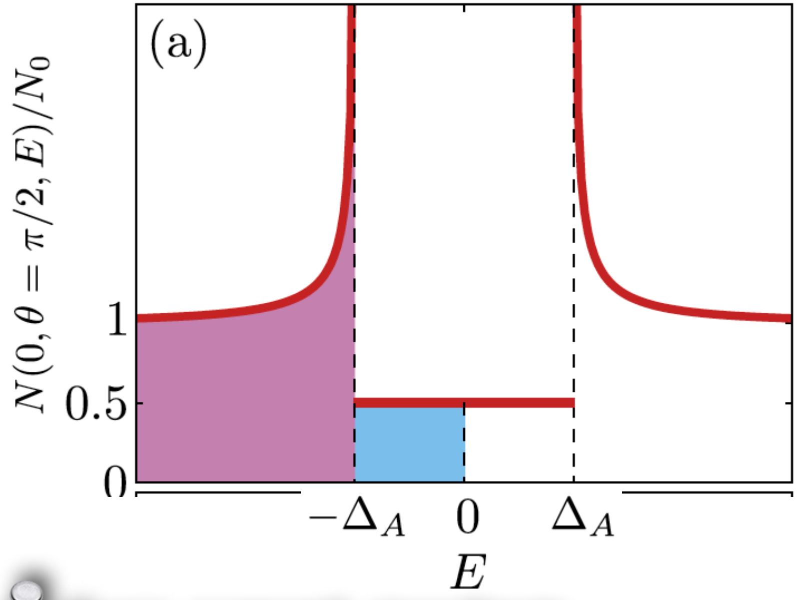
Mass current

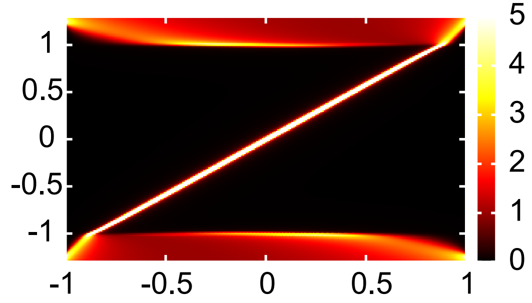


J.A. Sauls, PRB **84**, 214509 (2011).

LDOS and current spectrum

Local density of states (LDOS)



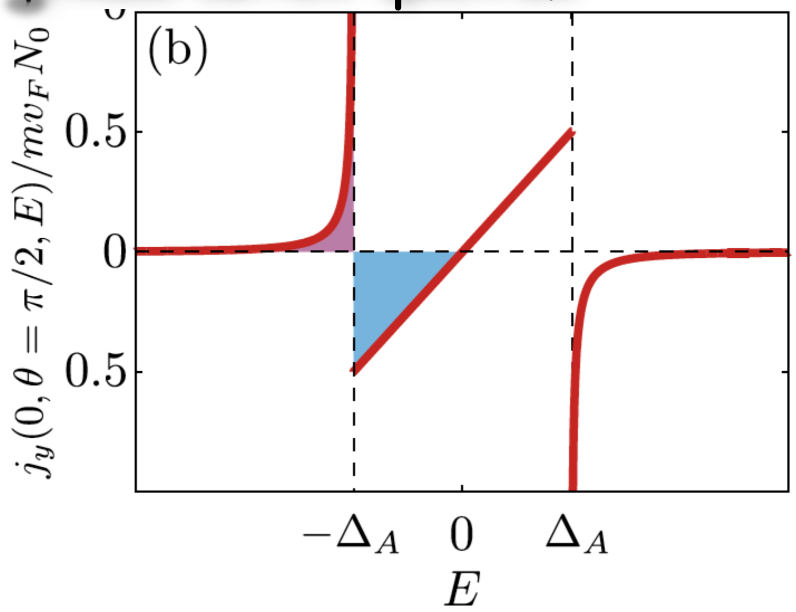


$$J_y^{\mathrm{bound}} = -\frac{n\hbar}{2} \quad J_y^{\mathrm{cont}} = \frac{n\hbar}{4}$$

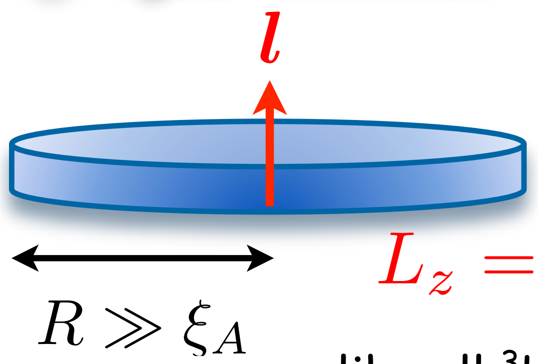
(n : density of ^{3}He atoms)

$$J_y = J_y^{\mathrm{bound}} + J_y^{\mathrm{cont}} = -\frac{n\hbar}{4}$$

Mass current spectrum



Angular momentum

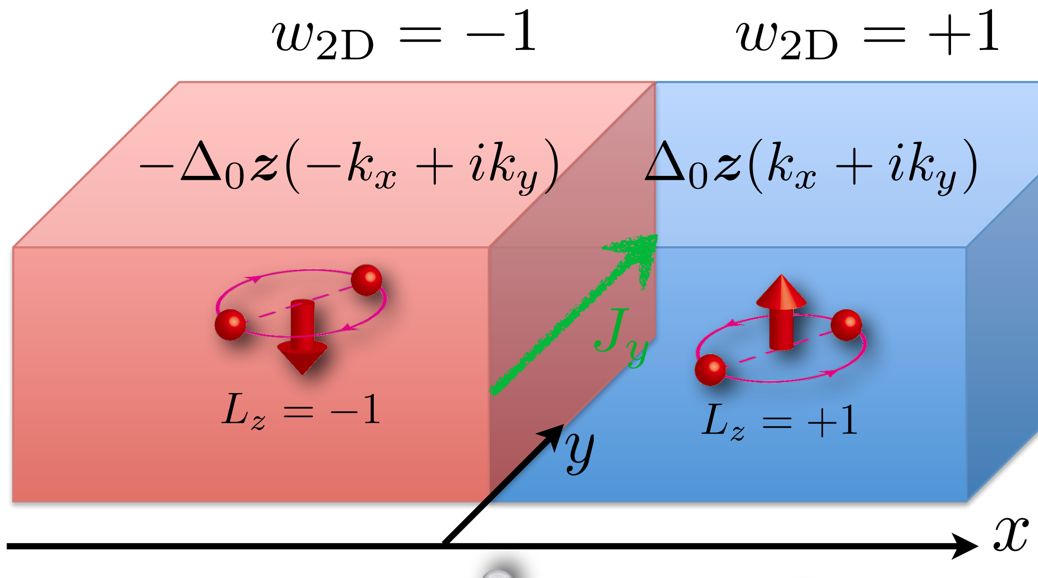


$$L_z = \frac{N\hbar}{2}$$ (N : number of ^{3}He atoms)

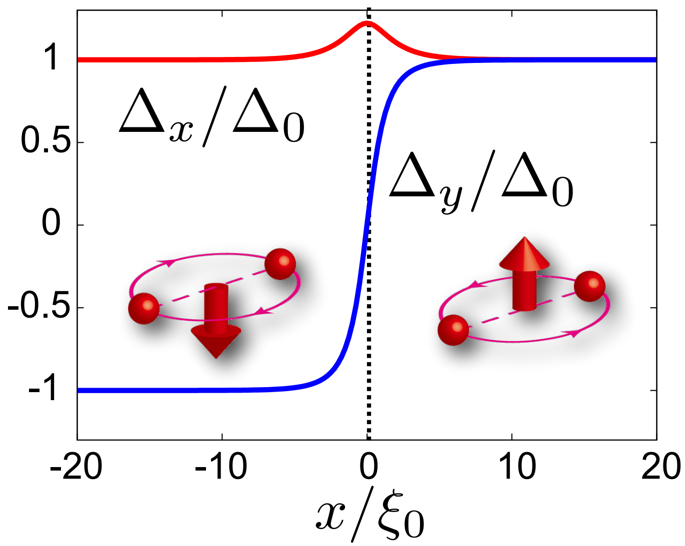
like all ^{3}He atoms form Cooper pair

macroscopic intrinsic angular momentum

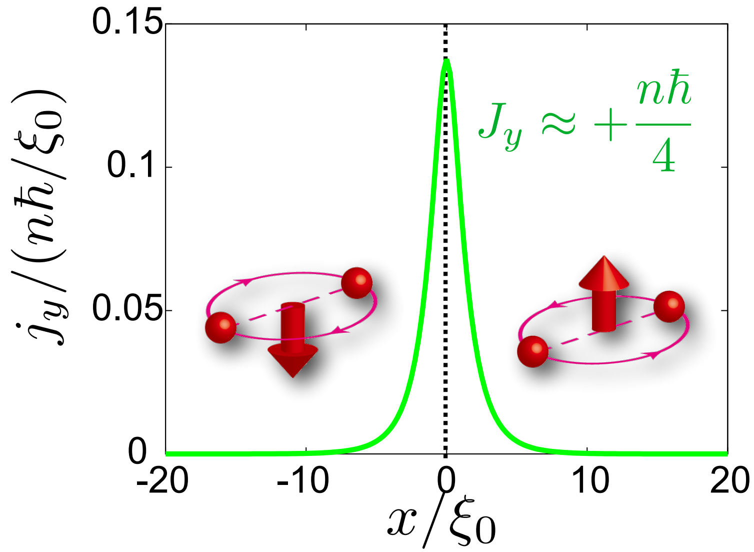
Chiral domain wall



order parameter

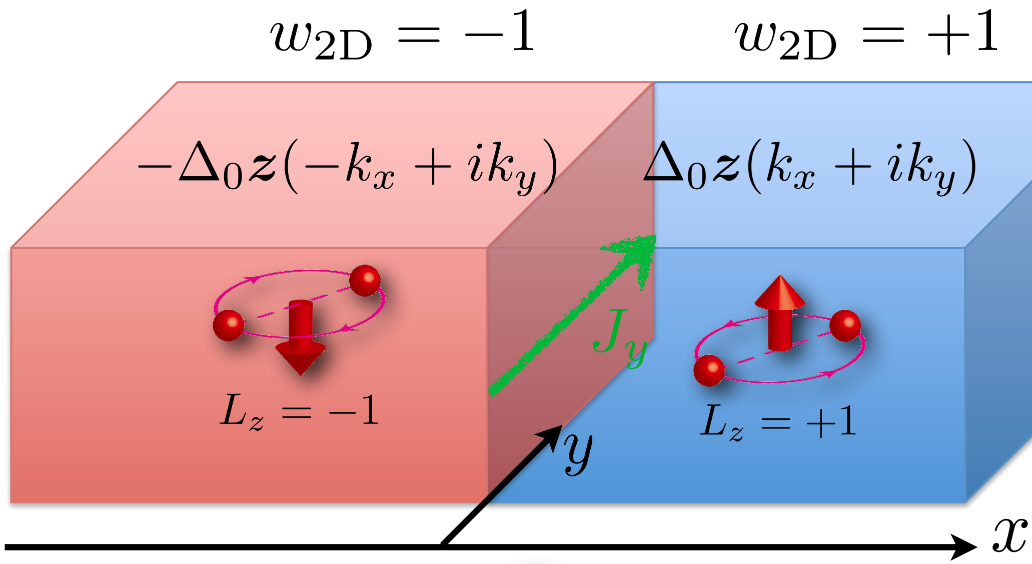


mass current

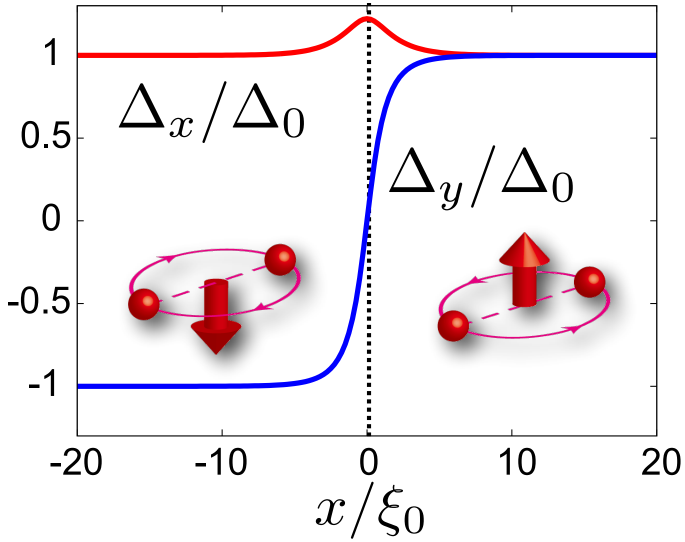


Y. Tsutsumi, J. Low Temp. Phys. **175**, 51 (2014).

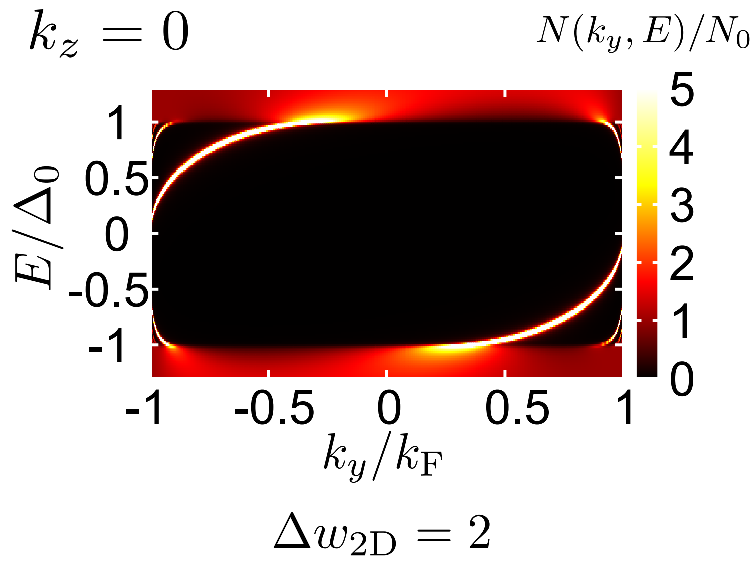
Chiral domain wall



order parameter

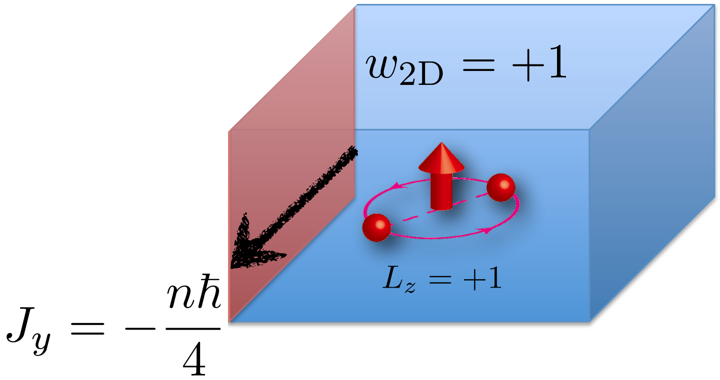


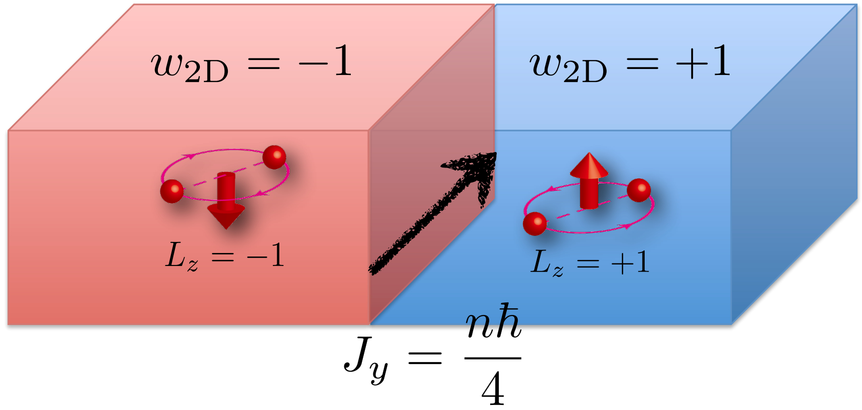
energy dispersion on domain wall

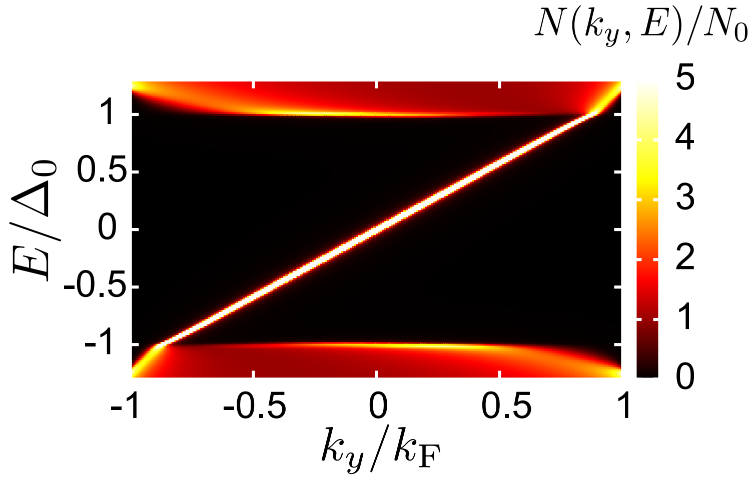


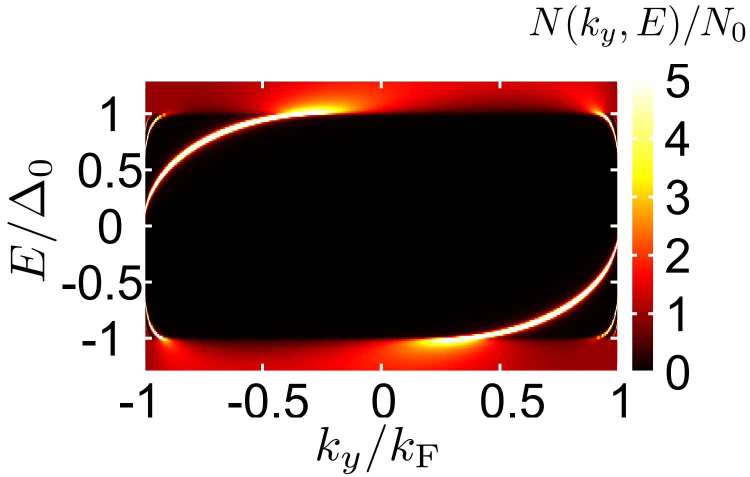
Y. Tsutsumi, J. Low Temp. Phys. **175**, 51 (2014).

Current by gapless excitations









$J_y^{\mathrm{ex}} = 0$

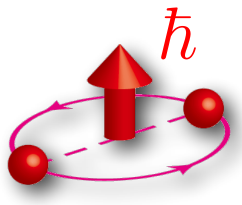
gapless excitations

G. E. Volovik, Pis'ma ZhETF **66**, 492 (1997).

$$\delta J_y^{\mathrm{ex}} = \sum_{a(E=0)} |p_{y,a}| \mathrm{sgn} \left(\left. \frac{\partial E(\boldsymbol{k})}{\partial k_y} \right|_{\boldsymbol{k}=\boldsymbol{k}_a} \right) \delta n(\boldsymbol{k}_a)$$

$$J_y^{\mathrm{ex}} = \frac{n\hbar}{2} \sum_{a(E=0)} \left(\frac{k_{y,a}}{k_{\mathrm{F}}} \right)^2 \mathrm{sgn} \left(\left. \frac{\partial E(\boldsymbol{k})}{\partial k_y} \right|_{\boldsymbol{k}=\boldsymbol{k}_a} \right) = \frac{n\hbar}{2}$$

current by Cooper pairs

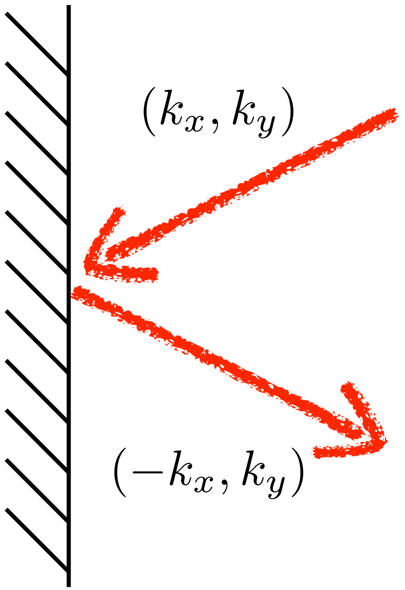


$$J_y^{\mathrm{AM}} = -\frac{n\hbar}{4}$$

Chiral state with higher AM

chiral ℓ -wave state $\quad \Delta(\boldsymbol{k}) \propto (k_x + i k_y)^{\ell}$

$$L_z = N\hbar \times O\left(\frac{\Delta}{E_{\mathrm{F}}}\right) \ll N\hbar, \quad (\ell \geq 2)$$



Y. Tada *et al.*, PRL **114**, 195301 (2015).
G.E. Volovik, JETP Lett. **100**, 742 (2014).

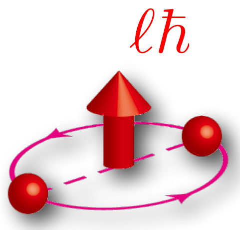
zero energy state

$$\Delta(k_{x,a}, k_{y,a}) = -\Delta(-k_{x,a}, k_{y,a})$$

$$\frac{k_{y,a}}{k_{\mathrm{F}}} = \cos\left[\left(n - \frac{1}{2}\right)\frac{\pi}{\ell}\right], \quad n = 1, 2, \cdots, \ell$$

$$J_y^{\mathrm{ex}} = \frac{n\hbar}{2} \sum_{a(E=0)} \left(\frac{k_{y,a}}{k_{\mathrm{F}}}\right)^2 \mathrm{sgn}\left(\left.\frac{\partial E(\boldsymbol{k})}{\partial k_y}\right|_{\boldsymbol{k}=\boldsymbol{k}_a}\right) = \frac{n\hbar}{4} \times \ell$$

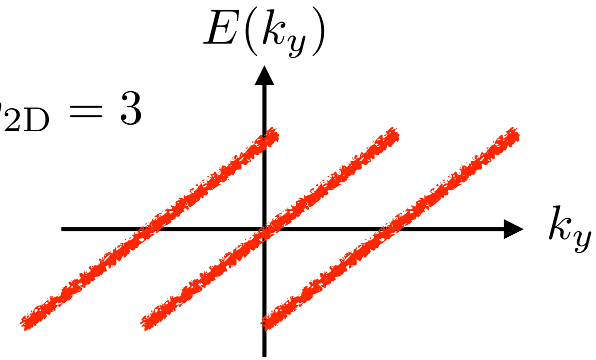
current by Cooper pairs



$$J_y^{\mathrm{AM}} = -\frac{n\hbar}{4} \times \ell$$

ex. chiral f-wave state $\quad w_{\mathrm{2D}} = 3$

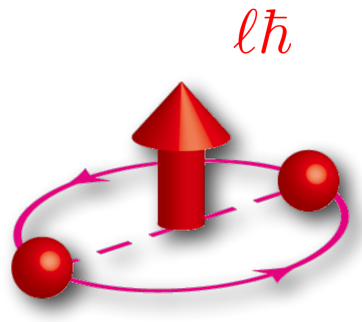
$$\frac{k_{y,a}}{k_{\mathrm{F}}} = -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}$$



Summary

$$\Delta(\boldsymbol{k}) \propto (k_x + ik_y)^\ell$$

current by Cooper pairs



$$J^{\mathrm{AM}} = -\frac{n\hbar}{4} \times \ell$$

current by gapless excitations

$$J^{\mathrm{ex}} = 0, \ (\ell = 1)$$

$$J^{\mathrm{ex}} = \frac{n\hbar}{4} \times \ell, \ (\ell \neq 1)$$

$$J^{\mathrm{surf}} = -\frac{n\hbar}{4}, \ (\ell = 1)$$

$$J^{\mathrm{surf}} \approx 0, \ (\ell \neq 1)$$

- ^{3}He-A is a good system to observe surface current.

pure system, large chiral domain, charge neutrality

Summary

- Quantitative evidence of Majorana fermions in surface bound state of B-phase
- Surface bound state in A-phase with mass current and angular momentum