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ギャップノードのあるトポロジカル超伝導体における量子渦束縛状態の不純物効果

東大総合文化　　堤 康雅, 加藤 雄介

Y. Tsutsumi and Y. Kato, arXiv:1601.04815.

Topological superconductor

topological invariant :

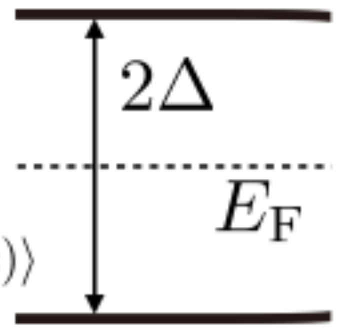
defined in gapped systems

gapless edge state = Andreev bound state

BdG Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \epsilon(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^*(\boldsymbol{k}) & -\epsilon(\boldsymbol{k}) \end{pmatrix}$$

$$\mathcal{H}(\boldsymbol{k})|u_\pm(\boldsymbol{k})\rangle = E_\pm(\boldsymbol{k})|u_\pm(\boldsymbol{k})\rangle$$



$\nu \neq 0$

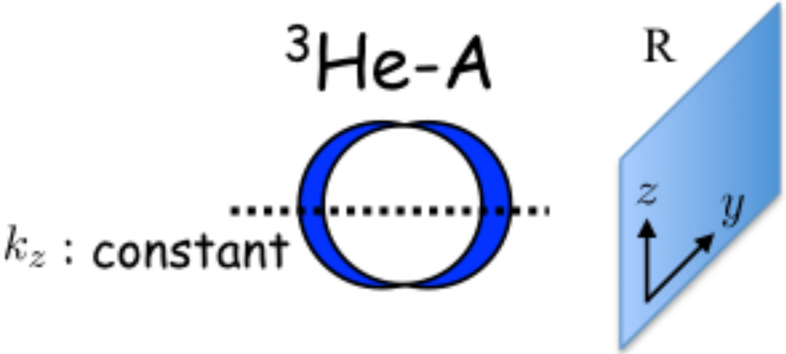
Berry connection

$$\boldsymbol{A}(\boldsymbol{k}) \equiv -i\langle u_-(\boldsymbol{k})|\boldsymbol{\nabla}_{\boldsymbol{k}}|u_-(\boldsymbol{k})\rangle$$

s-wave SC,
insulator

Chern number $\nu \equiv \dfrac{1}{2\pi}\displaystyle\int d\boldsymbol{k}[\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}(\boldsymbol{k})]_z$

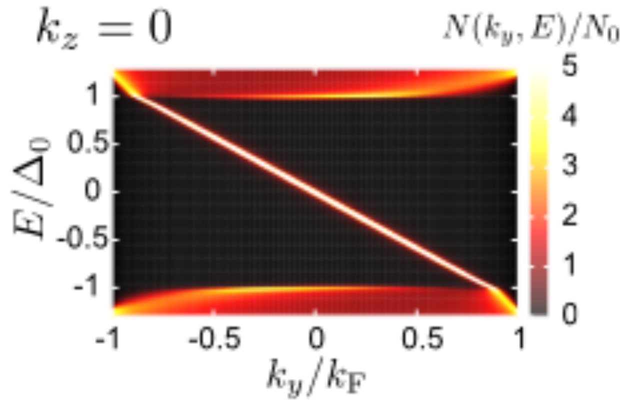
Nodal topological superconductor

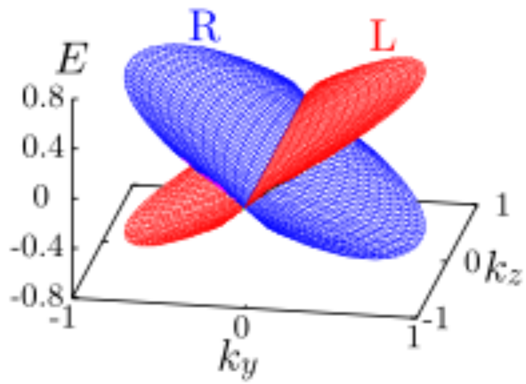


$$\Delta(\boldsymbol{k}) = \Delta_0(k_x + ik_y)$$
$$= \Delta_0 e^{i\phi_k}\sin\theta_k$$

2D chiral p-wave

Chern number: $\nu = 1$





YT *et al.*, PRB **83**, 094510 (2011).

運動量状態間の遷移がある場合にどうなるのか？

ノードのあるトポロジカル超伝導体における
量子渦束縛状態の不純物効果

f-wave, E_{1u} planar state

$$\mathcal{H}_{\rm BdG}(\boldsymbol{k}) = \begin{pmatrix} \epsilon(\boldsymbol{k}) + \mu_{\rm B}\boldsymbol{H}\cdot\boldsymbol{\sigma} & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\epsilon^*(\boldsymbol{k}) - \mu_{\rm B}\boldsymbol{H}\cdot\boldsymbol{\sigma}^* \end{pmatrix}$$

$\epsilon(\boldsymbol{k})$: reflection symmetry $\quad \boldsymbol{H} \parallel \boldsymbol{z}$

$$\Delta(\boldsymbol{k}) = i\boldsymbol{d}(\boldsymbol{k})\cdot\boldsymbol{\sigma}\sigma_y$$

particle-hole symmetry

$$\mathcal{H}_{\rm BdG}(\boldsymbol{k}) = -C\mathcal{H}_{\rm BdG}(-\boldsymbol{k})C^{-1}$$

magnetic reflection symmetry

$$\mathcal{H}_{\rm BdG}(\boldsymbol{k}) = (TM_y)\mathcal{H}_{\rm BdG}(-k_x, k_y, -k_z)(TM_y)^{-1}$$

"chiral" symmetry

$$\Gamma \equiv e^{i\chi}CTM_y, \ \Gamma^2 = 1$$

$$\{\Gamma, \mathcal{H}_{\rm BdG}(k_x, k_y=0, k_z)\} \equiv \{\Gamma, \mathcal{H}\} = 0$$

$$[\Gamma, \mathcal{H}^2] = 0$$

$$\mathcal{H}^2|u'\rangle = E^2|u'\rangle \longleftrightarrow \mathcal{H}|u\rangle = E|u\rangle$$

one-to-one correspondence

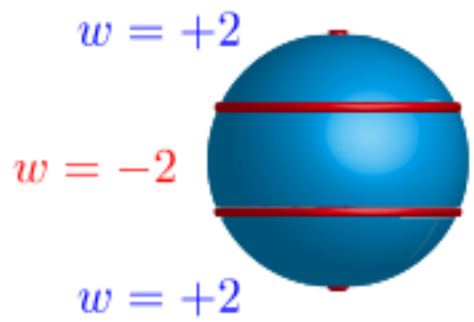
$$\Gamma|u'_\pm\rangle = \pm|u'_\pm\rangle \qquad |u'_+\rangle = c'\mathcal{H}|u'_-\rangle$$

pairs for finite energy state

1D winding number

$$w(k_z) \equiv -\frac{1}{4\pi i}\int_{\rm BZ} dk_x {\rm tr}\left[\Gamma\mathcal{H}^{-1}\partial_{k_x}\mathcal{H}\right]$$

$$= 2{\rm sgn}[\Delta(k_z)]$$



$$\boldsymbol{d}(\boldsymbol{k}) = \Delta_0(\boldsymbol{x}k_y + \boldsymbol{y}k_x)(5k_z^2 - 1)$$

Vortex bound state

BdG equation

$$\int d\boldsymbol{r}_2 \begin{pmatrix} \epsilon(\boldsymbol{r}_1,\boldsymbol{r}_2) & \Delta(\boldsymbol{r}_1,\boldsymbol{r}_2) \\ -\Delta^\dagger(\boldsymbol{r}_1,\boldsymbol{r}_2) & -\epsilon^*(\boldsymbol{r}_1,\boldsymbol{r}_2) \end{pmatrix} u_\nu(\boldsymbol{r}_2) = E_\nu u_\nu(\boldsymbol{r}_1)$$

$$\epsilon(\boldsymbol{r}_1,\boldsymbol{r}_2) = \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\frac{\hbar^2}{2m}(-\nabla^2 - k_{\rm F}^2)$$

$$\Delta(\boldsymbol{r}) = \Delta(\rho)e^{i\phi}$$

C. Caroli, P. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
N. Kopnin and M. Salomaa, PRB **44**, 9667 (1991).
T. Mizushima and K. Machida, PRA **81**, 053605 (2010).

$$\Delta(\rho) = \Delta_0 \tanh\frac{\rho}{\xi} \qquad k_{\rm F}\xi = 5$$

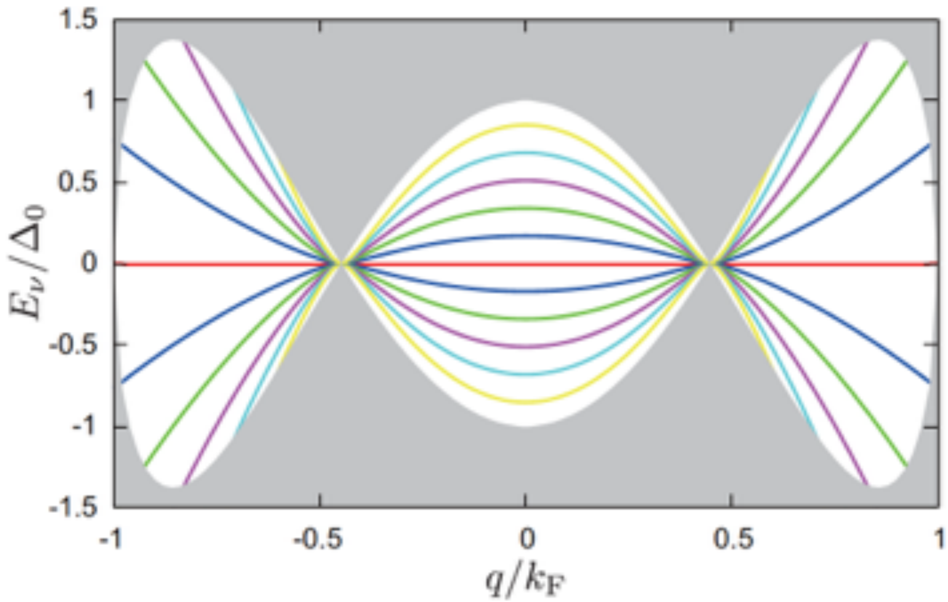
$\nu = (l,q)$ $\quad l$: angular momentum $\quad q = k_z$

spin degenerate $\quad E_\nu = -l\omega_q \qquad \omega_q \sim \Delta/E_{\rm F}^2$

$$u_\nu^\uparrow = N_\nu^\uparrow \begin{pmatrix} J_{l+1}(k_q\rho)e^{i(l+1)\phi} \\ s_q J_{l-1}(k_q\rho)e^{i(l-1)\phi} \end{pmatrix} e^{-\chi_q(\rho)+iqz}$$

$$u_\nu^\downarrow = N_\nu^\downarrow \begin{pmatrix} J_l(k_q\rho)e^{il\phi} \\ -s_q J_l(k_q\rho)e^{il\phi} \end{pmatrix} e^{-\chi_q(\rho)+iqz}$$

$$s_q \equiv {\rm sgn}[5(q/k_{\rm F})^2 - 1]$$



Self-energy

Dyson equation

$$G(\boldsymbol{r},\boldsymbol{r}',\omega_n) = G^{(0)}(\boldsymbol{r},\boldsymbol{r}',\omega_n) + \int d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 G^{(0)}(\boldsymbol{r},\boldsymbol{r}_1,\omega_n)\Sigma(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega_n)G(\boldsymbol{r}_2,\boldsymbol{r}',\omega_n)$$

$$G^{(0)}(\boldsymbol{r},\boldsymbol{r}',\omega_n) = \sum_\nu \frac{\tau_z u_\nu(\boldsymbol{r})u_\nu^\dagger(\boldsymbol{r}')}{E_\nu - i\omega_n}$$

$$\Sigma(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega_n) = \frac{\Gamma_{\rm n}}{\pi N_{\rm F}} f(\boldsymbol{r}_1 - \boldsymbol{r}_2) G(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega_n)$$

$\Gamma_{\rm n}$: impurity scattering rate

DOS

$$N(\omega) = \int d\boldsymbol{r} \mathrm{Im}\left[\frac{1}{\pi}\mathrm{tr}\tau_z \, G(\boldsymbol{r},\boldsymbol{r},\omega_n)|_{i\omega_n\to\omega+i0^+}\right]$$

$$= -\frac{1}{\pi}\sum_{\nu,\mathrm{spin}} \mathrm{Im}\left[\frac{1}{\omega - E_\nu + \sigma_\nu^{\rm spin}(\omega) + i0^+}\right] \equiv \sum_\nu \left[N_\nu^\uparrow(\omega) + N_\nu^\downarrow(\omega)\right]$$

$$f(\boldsymbol{r}_1 - \boldsymbol{r}_2) = \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) f(z_1 - z_2)$$

neglecting continuum states

$$\sigma_\nu^{\uparrow(\downarrow)}(\omega) = -\frac{\Gamma_{\rm n}}{\pi N_{\rm F}} \sum_{\nu'} \int dq' \tilde{f}(q - q') \frac{M_{\nu,\nu'}^{\uparrow(\downarrow)}}{\omega - E_{\nu'} + \sigma_{\nu'}^{\uparrow(\downarrow)}(\omega) + i0^+}$$

Coherence factor

$$M^{\uparrow(\downarrow)}_{\nu,\nu'} = \int \rho d\rho \left| u^{\uparrow(\downarrow)\dagger}_{\nu}(\rho)\tau_z u^{\uparrow(\downarrow)}_{\nu}(\rho)\right|^2$$

$$= \int \rho d\rho \left| m^{\uparrow(\downarrow)}_{\nu,\nu'}(\rho)\right|^2 e^{-2\chi_q(\rho)-2\chi_{q'}(\rho)}$$

$$u^{\uparrow}_{\nu} = N^{\uparrow}_{\nu}\begin{pmatrix} J_{l+1}(k_q\rho)e^{i(l+1)\phi} \\ s_q J_{l-1}(k_q\rho)e^{i(l-1)\phi}\end{pmatrix} e^{-\chi_q(\rho)+iqz}$$

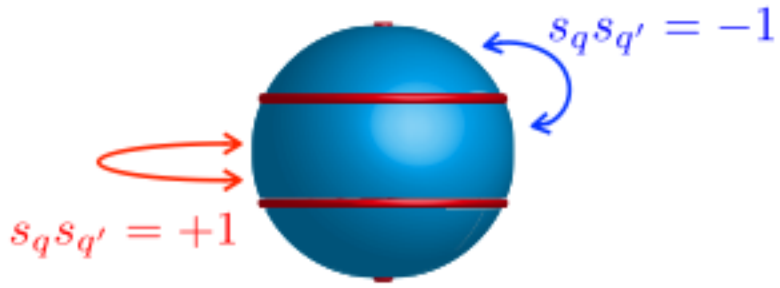
$$u^{\downarrow}_{\nu} = N^{\downarrow}_{\nu}\begin{pmatrix} J_{l}(k_q\rho)e^{il\phi} \\ -s_q J_{l}(k_q\rho)e^{il\phi}\end{pmatrix} e^{-\chi_q(\rho)+iqz}$$

$$s_q \equiv \mathrm{sgn}[5(q/k_F)^2 - 1]$$

$$m^{\uparrow}_{\nu,\nu'}(\rho) = N^{\uparrow}_{\nu}N^{\uparrow}_{\nu'}[J_{l+1}(k_q\rho)J_{l'+1}(k_{q'}\rho) - \underline{s_q s_{q'}} J_{l-1}(k_q\rho)J_{l'-1}(k_{q'}\rho)]$$

$$m^{\downarrow}_{\nu,\nu'}(\rho) = N^{\downarrow}_{\nu}N^{\downarrow}_{\nu'}[J_{l}(k_q\rho)J_{l'}(k_{q'}\rho) - \underline{s_q s_{q'}} J_{l}(k_q\rho)J_{l'}(k_{q'}\rho)]$$

$s_q s_{q'} = +1$	$s_q s_{q'} = -1$
$m^{\uparrow}_{(0,q),(0,q')}(\rho) = 0$	$m^{\uparrow}_{\nu,\nu'}(\rho) \neq 0$
$m^{\downarrow}_{\nu,\nu'}(\rho) = 0$	$m^{\downarrow}_{\nu,\nu'}(\rho) \neq 0$



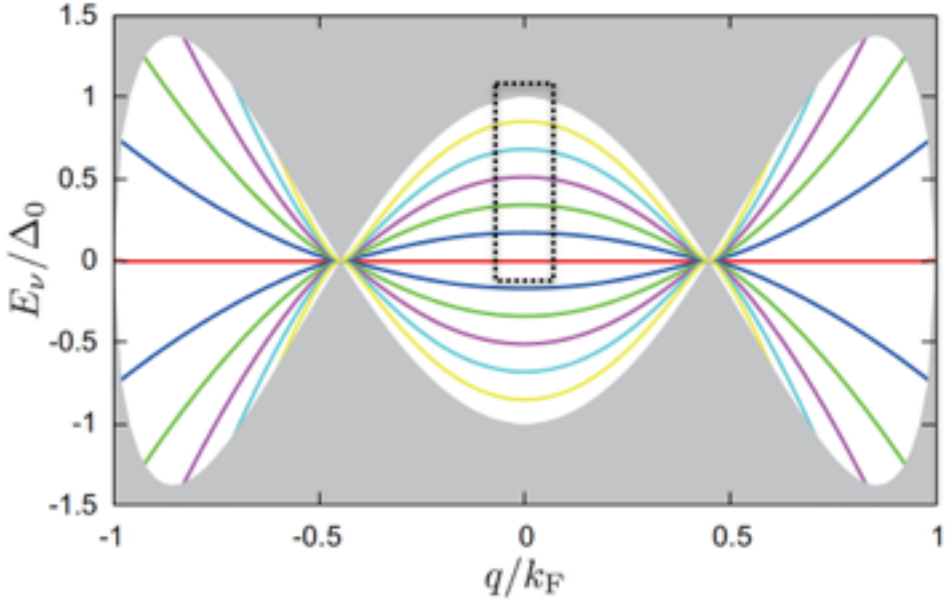
DOS of vortex bound state

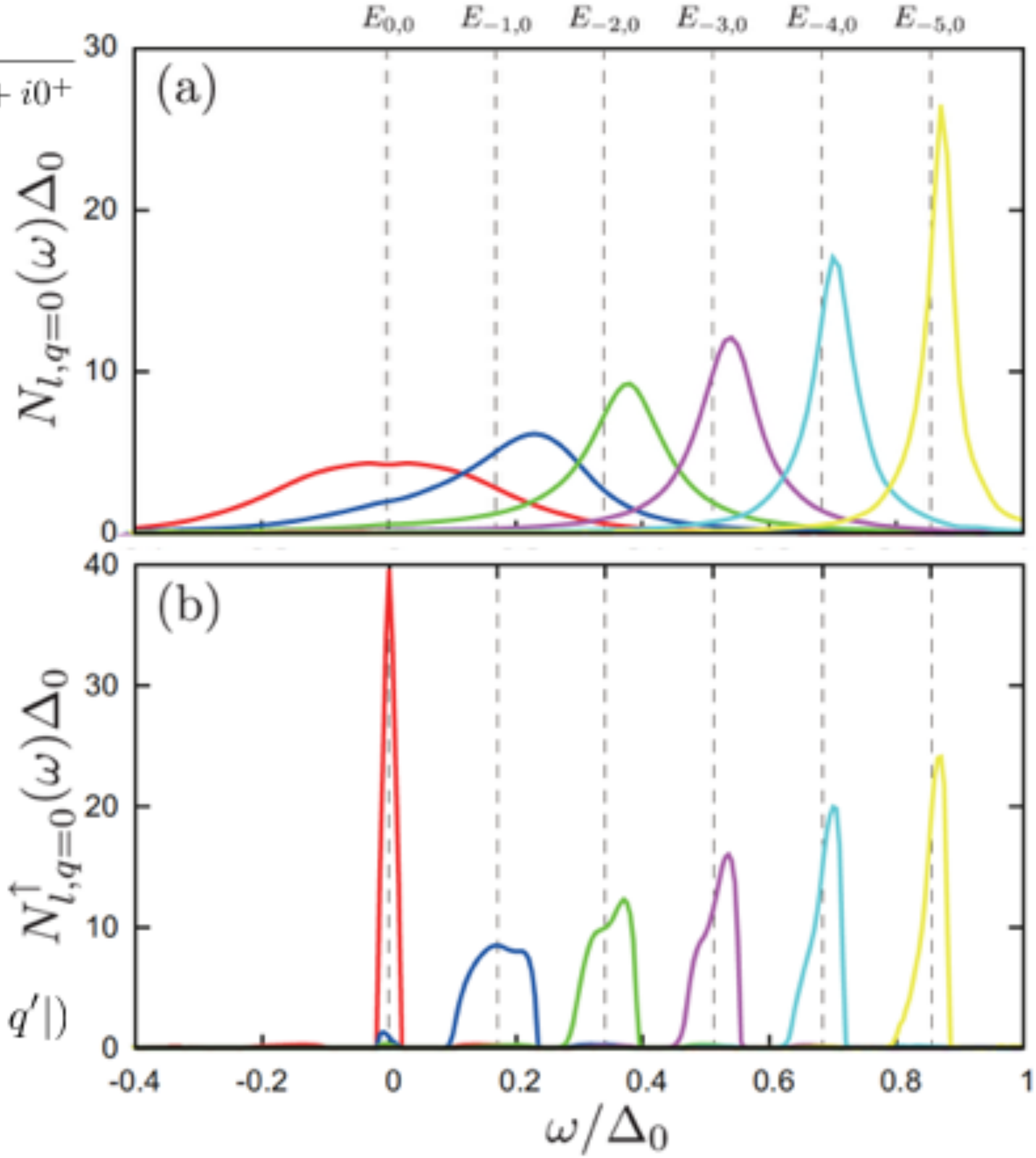
$$N_\nu^{\uparrow(\downarrow)}(\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\frac{1}{\omega - E_\nu + \sigma_\nu^{\uparrow(\downarrow)}(\omega) + i0^+}\right]$$

$$\sigma_\nu^{\uparrow(\downarrow)}(\omega) = -\frac{\Gamma_\mathrm{n}}{\pi N_\mathrm{F}}\sum_{\nu'}\int dq' \tilde{f}(q-q')\frac{M_{\nu,\nu'}^{\uparrow(\downarrow)}}{\omega - E_{\nu'} + \sigma_{\nu'}^{\uparrow(\downarrow)}(\omega) + i0^+}$$

$$\Gamma_\mathrm{n} = 0.2\pi\Delta_0$$

$$\tilde{f}(q-q') = 1$$





$$\tilde{f}(q-q') = \theta(k_\mathrm{F}/10 - |q-q'|)$$

columnar defects

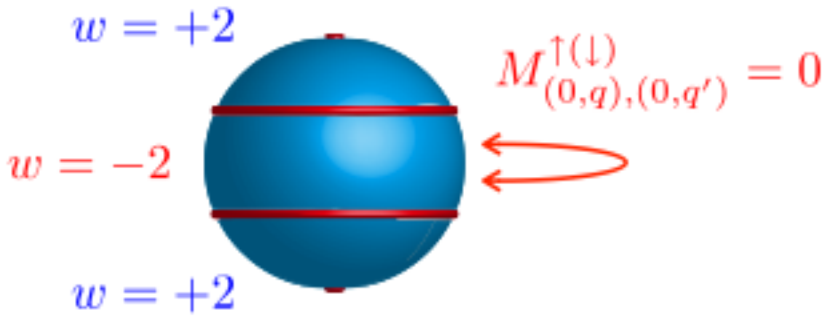
Summary

Topological invariants of nodal topological superconductors defined in a particular momentum space are effectual even with momentum transfers.

topology and symmetry of Hamiltonian → chirality

wave function of zero energy modes → coherence factor

chirality → coherence factor



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