11 Aug., 2017 @ LT28

Scattering theory on surface Majorana fermions by an impurity in ^{3}He-B

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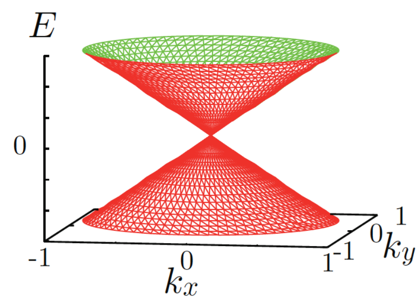
Y. Tsutsumi, Phys. Rev. Lett. **118**, 145301 (2017).

H. Ikegami, S. B. Chung, and K. Kono, J. Phys. Soc. Jpn. **82**, 124607 (2013).
K. Kono, presentation in next Half Plenary, 084.

Outline



- Surface Majorana fermions in ^{3}He B-phase

- An experiment to detect the Majorana fermions

  - ion mobility below a free surface H.Ikegami et al., JPSJ 82, 124607 (2013).

- Scattering theory on the Majorana fermions
  - ion (impurity) scattering -> ion mobility
  - Majorana fermion -> plane wave with spin & particle-hole channels

- Calculated mobility
  - mobility parallel to a free surface -> quantitative agreement with experiment
  - mobility perpendicular to a free surface -> more affected by Majorana fermions

Surface Majorana fermions in ^{3}He-B

Bogoliubov-de Gennes equation

$$\begin{pmatrix} \xi(\boldsymbol{k}) & -\hat{\Delta}(\boldsymbol{k}) \\ -\hat{\Delta}^\dagger(\boldsymbol{k}) & -\xi(\boldsymbol{k}) \end{pmatrix} \Psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}(\boldsymbol{r})$$

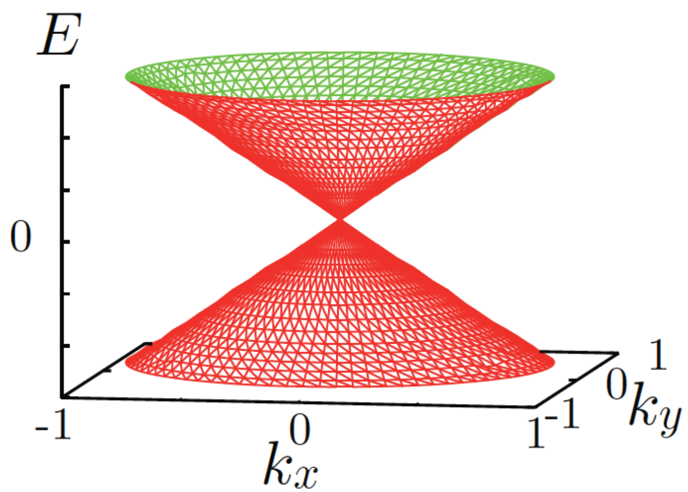
$$\xi(\boldsymbol{k}) = \hbar(\boldsymbol{k} - \boldsymbol{k}_{\rm F}) \cdot \boldsymbol{v}_{\rm F}$$ (Andreev approximation)

$$\hat{\Delta}(\boldsymbol{k}) = \frac{\Delta}{k_{\rm F}} \begin{pmatrix} -k_x + ik_y & k_z \\ k_z & k_x + ik_y \end{pmatrix}$$

quasiparticle energy

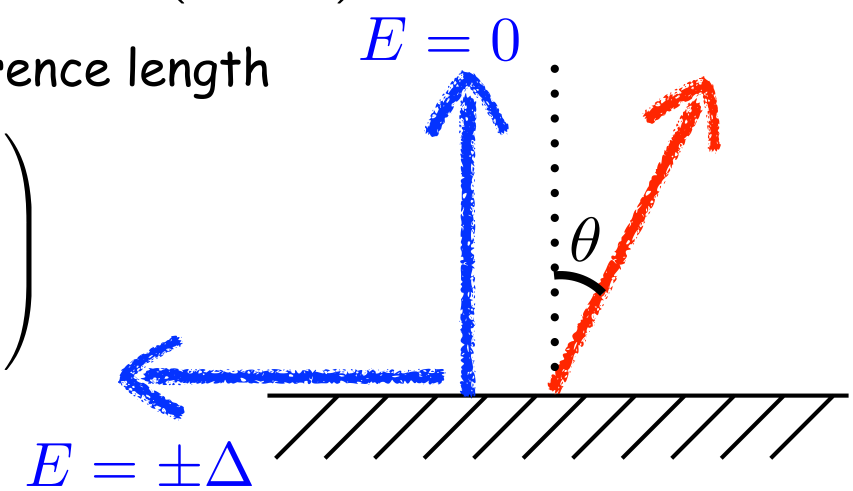
$$E^{\pm}_{\boldsymbol{k}} = \pm \Delta \sin\theta$$

$$\Psi^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_{\rm F}\cdot\boldsymbol{r}} \exp\left(-\frac{z}{2\xi_0}\right) \Phi^{\pm}_{\boldsymbol{k}_\parallel}$$



$$\xi_0 \equiv \frac{\hbar v_{\rm F}}{2\Delta}$$: coherence length

$$\Phi^{\pm}_{\boldsymbol{k}_\parallel} = \begin{pmatrix} e^{-i\phi/2} \\ \mp i e^{i\phi/2} \\ \pm e^{i\phi/2} \\ i e^{-i\phi/2} \end{pmatrix}$$



Surface Majorana fermions in ^{3}He-B

wave function of bound state

$$\Psi^{\pm}_{\boldsymbol{k}_\parallel}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_\parallel \cdot \boldsymbol{r}} \sin(k_\perp z) \exp\left(-\frac{z}{2\xi_0}\right) \Phi^{\pm}_{\boldsymbol{k}_\parallel} \qquad k_\perp = \sqrt{k_{\mathrm{F}}^2 - k_\parallel^2}$$

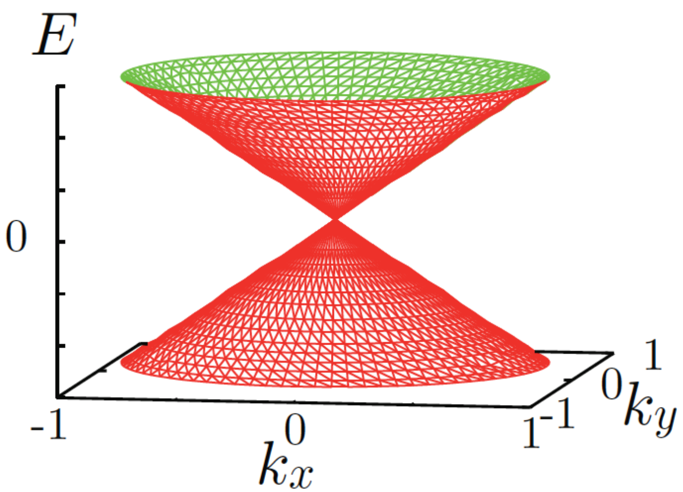
field operator

$$\hat{\Psi}(\boldsymbol{r}) = \begin{pmatrix} \hat{\Psi}_\uparrow(\boldsymbol{r}) \\ \hat{\Psi}_\downarrow(\boldsymbol{r}) \\ \hat{\Psi}^\dagger_\uparrow(\boldsymbol{r}) \\ \hat{\Psi}^\dagger_\downarrow(\boldsymbol{r}) \end{pmatrix} = \sum_{\boldsymbol{k}_\parallel} \left[\hat{\gamma}_{\boldsymbol{k}_\parallel} \Psi^{+}_{\boldsymbol{k}_\parallel}(\boldsymbol{r}) + \hat{\gamma}^\dagger_{-\boldsymbol{k}_\parallel} \Psi^{-}_{\boldsymbol{k}_\parallel}(\boldsymbol{r}) \right]$$

$$\hat{\Psi}_\sigma(\boldsymbol{r}) = \hat{\Psi}^\dagger_\sigma(\boldsymbol{r})$$ Majorana fermion

quasiparticle energy

$$E^{\pm}_{\boldsymbol{k}} = \pm \Delta \sin\theta \qquad \Psi^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_{\mathrm{F}} \cdot \boldsymbol{r}} \exp\left(-\frac{z}{2\xi_0}\right) \Phi^{\pm}_{\boldsymbol{k}_\parallel}$$



$$\xi_0 \equiv \frac{\hbar v_{\mathrm{F}}}{2\Delta}$$: coherence length

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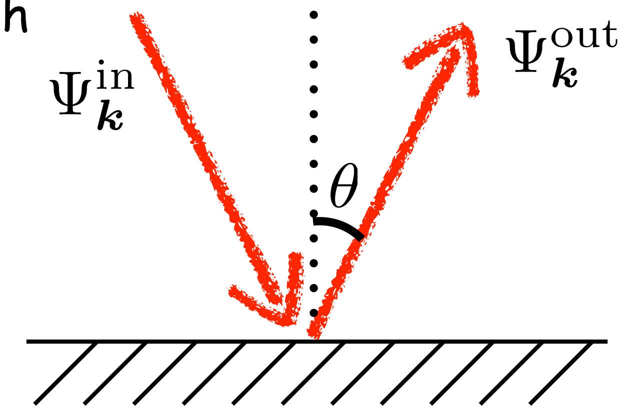
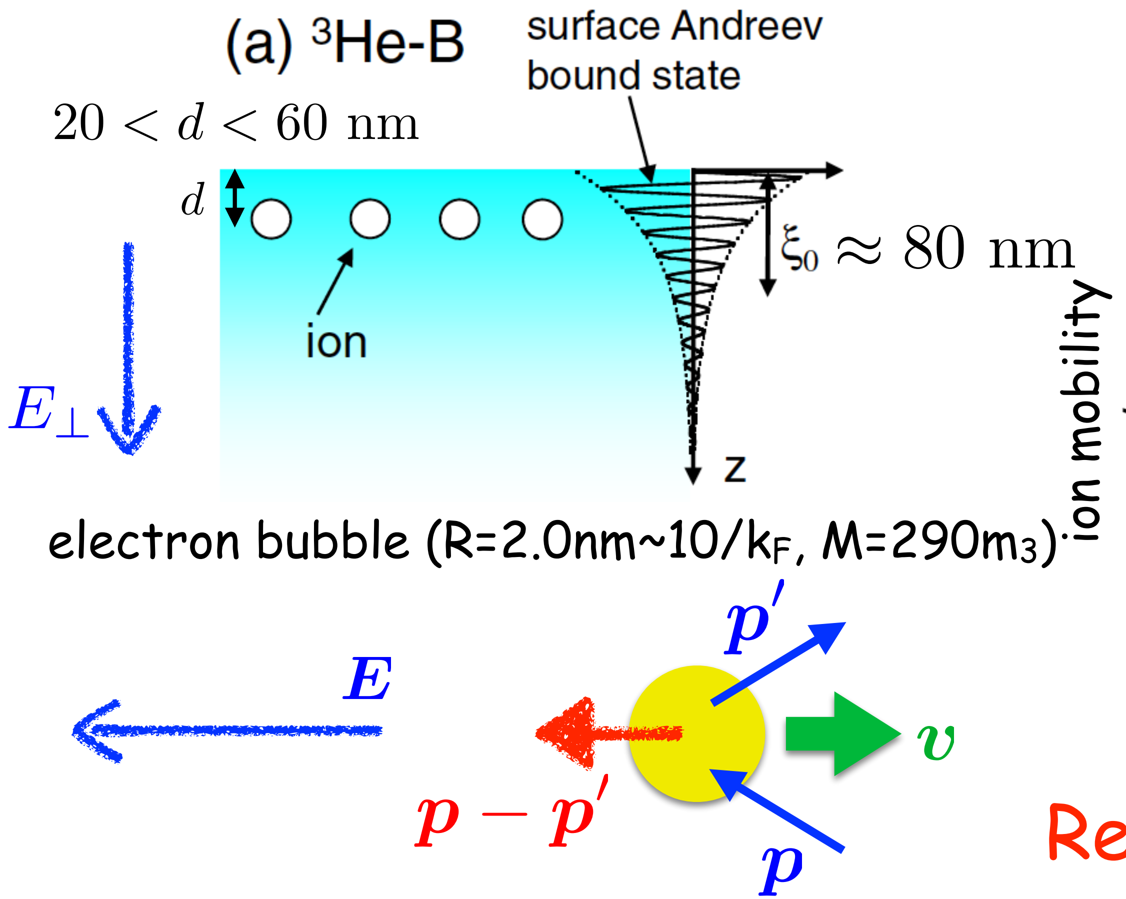
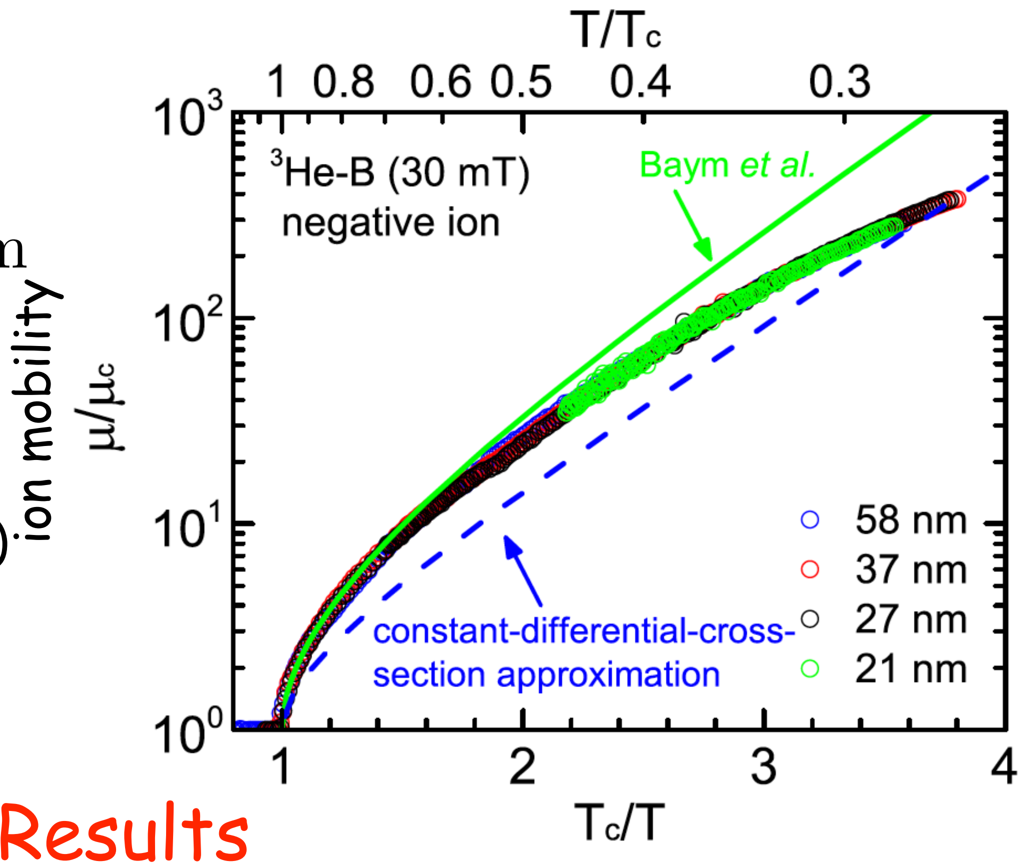


Experiment for ions trapped below a free surface

H. Ikegami *et al.*, JPSJ **82**, 124607 (2013).





Results

Mobility is smaller than the bulk value.

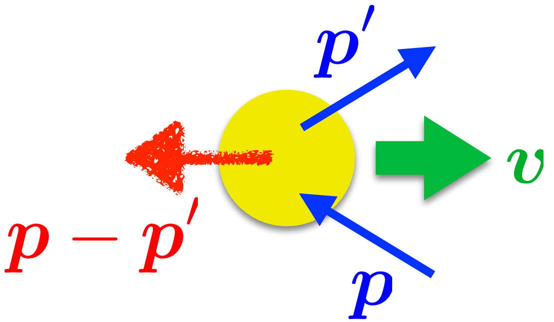
scattering by surface bound state

Mobility at d = 35 nm without magnetic field exhibits the same temperature dependence.

Mobility is independent of trapped depth.

despite spatial dependence of SBS

Equation of motion for electron bubble



$$\frac{d\boldsymbol{P}}{dt} = -\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma,\sigma'} \hbar(\boldsymbol{k}'-\boldsymbol{k})(1-f_{\boldsymbol{k}'})f_{\boldsymbol{k}}\Gamma_{\boldsymbol{v}}(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')$$

$\boldsymbol{p} = \hbar\boldsymbol{k}$: quasiparticle momentum $\quad f_{\boldsymbol{k}} = \left(e^{E_{\boldsymbol{k}}/k_{\rm B}T}+1\right)^{-1}$: Fermi distribution

σ : quasiparticle spin $\quad E_{\boldsymbol{k}}$: quasiparticle energy

first order of v

$$\frac{d\boldsymbol{P}}{dt} = -\frac{\hbar^2}{2k_{\rm B}T}\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma,\sigma'}(\boldsymbol{k}'-\boldsymbol{k})(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}(1-f_{\boldsymbol{k}})f_{\boldsymbol{k}}\Gamma(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')$$

transition rate: $\Gamma(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma') = \frac{2\pi}{\hbar}\delta(E_{\boldsymbol{k}'}-E_{\boldsymbol{k}})|t(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')|^2$

elastic scattering

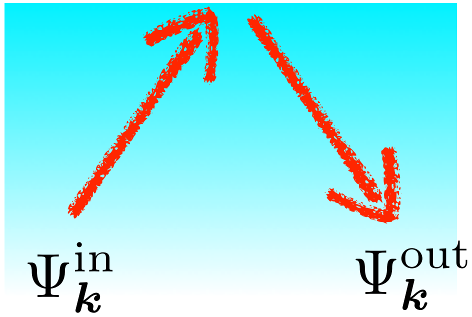
Stokes drag force: $\frac{d\boldsymbol{P}}{dt} = -\eta\boldsymbol{v}$

driving force: $e\boldsymbol{E} = \frac{e}{\mu}\boldsymbol{v}$

parallel $\quad \frac{e}{\mu_\parallel} = \eta_\parallel = \frac{\pi\hbar}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'}(k'_\parallel - k_\parallel)^2\left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'}-E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')|^2$

perpendicular $\quad \frac{e}{\mu_\perp} = \eta_\perp = \pi\hbar\sum_{\boldsymbol{k},\boldsymbol{k}'}(k'_z - k_z)^2\left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'}-E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma\to\boldsymbol{k}',\sigma')|^2$

T-matrix



$$\sum_{\sigma,\sigma'} |t(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')|^2 = \sum_{\sigma,\sigma'=\uparrow,\downarrow} |\langle \Psi_{\boldsymbol{k}',\sigma'} | T_{\rm S} | \Psi_{\boldsymbol{k},\sigma}\rangle|^2$$

momentum eigenstate

radius of electron bubble: $R \sim k_{\rm F}^{-1} \ll \xi$

$$\langle \boldsymbol{r} | \Psi_{\boldsymbol{k},\sigma}\rangle = \Psi_{\boldsymbol{k},\sigma}(\boldsymbol{r}) = N e^{i\boldsymbol{k}_{\rm F}\cdot \boldsymbol{r}} \exp\left(-\frac{z}{2\xi}\right) \Phi_{\boldsymbol{k},\sigma}$$

$$\approx N \exp\left(-\frac{z_0}{2\xi}\right) e^{i\boldsymbol{k}_{\rm F}\cdot \boldsymbol{r}} \Phi_{\boldsymbol{k},\sigma}$$

normalization $\to \Phi_{\boldsymbol{k},\sigma} \langle \boldsymbol{r} | \boldsymbol{k}_{\rm F}\rangle$

$$\Phi_{\boldsymbol{k},\uparrow} = \frac{e^{-i\phi/2}}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ i \end{pmatrix}, \quad \Phi_{\boldsymbol{k},\downarrow} = \frac{e^{i\phi/2}}{\sqrt{2}} \begin{pmatrix} 0 \\ i \\ -1 \\ 0 \end{pmatrix}$$

T-matrix element

$$\langle \boldsymbol{k}_{\rm F}' | T_{\rm S} | \boldsymbol{k}_{\rm F}\rangle \equiv T_{\rm S}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}, E, z_0) = T_{\rm N}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}) + N_{\rm F} \int \frac{d\Omega_{\boldsymbol{k}''}}{4\pi} T_{\rm N}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}'') \left[g_{\rm S}(\hat{\boldsymbol{k}}'', E, z_0) - g_{\rm N} \right] T_{\rm S}(\hat{\boldsymbol{k}}'', \hat{\boldsymbol{k}}, E, z_0)$$

intermediate state

YT and K. Machida, JPSJ **81**, 074607 (2012).
H. Wu and J. A. Sauls, PRB **88**, 184506 (2013).

T-matrix element in normal state

$$T_{\rm N}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}) = \begin{pmatrix} t_{\rm N}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}) \hat{1} & 0 \\ 0 & -t_{\rm N}(-\hat{\boldsymbol{k}}', -\hat{\boldsymbol{k}})^* \hat{1} \end{pmatrix}, \quad t_{\rm N}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{k}}) = -\frac{1}{\pi N_{\rm F}} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\boldsymbol{k}}' \cdot \hat{\boldsymbol{k}})$$

phase shift: δ_l

hard sphere potential $\to \tan\delta_l = j_l(k_{\rm F}R)/n_l(k_{\rm F}R) \quad k_{\rm F}R = 11.17$

O. Shevtsov and J. A. Sauls, PRB **94**, 064511 (2016).

Mobility

parallel
$$\frac{e}{\mu_{\parallel}} = \frac{\pi\hbar}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'}(\boldsymbol{k}'_{\parallel} - \boldsymbol{k}_{\parallel})^2\left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'} - E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')|^2$$

perpendicular
$$\frac{e}{\mu_{\perp}} = \pi\hbar\sum_{\boldsymbol{k},\boldsymbol{k}'}(k'_z - k_z)^2\left(-\frac{\partial f_{\boldsymbol{k}}}{\partial E_{\boldsymbol{k}}}\right)\delta(E_{\boldsymbol{k}'} - E_{\boldsymbol{k}})\sum_{\sigma,\sigma'}|t(\boldsymbol{k},\sigma \to \boldsymbol{k}',\sigma')|^2$$

$$\sum_{\boldsymbol{k}} \to \int_{-\Delta}^{\Delta} dE_{\boldsymbol{k}} \int \frac{d\Omega_{\boldsymbol{k}}}{4\pi} N(\hat{\boldsymbol{k}}, E_{\boldsymbol{k}}, z_0)$$

$$\frac{e}{\mu_{\parallel,\perp}} = \frac{\pi^2}{4}\exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\mathrm{F}} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\mathrm{tr}}^{\parallel,\perp}(E, z_0)$$

DOS
$$N(E,z) = \frac{\pi}{2} N_{\mathrm{F}} \frac{|E|}{\Delta} \exp\left(-\frac{z}{\xi}\right)$$

n_3 : ^{3}He particle density

Mobility parallel to surface

$$\frac{e}{\mu_{\parallel}} = \frac{\pi^2}{4} \exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\mathrm{F}} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\mathrm{tr}}^{\parallel}(E, z_0)$$

total cross section

$$\sigma_{\mathrm{tot}}^{\parallel}(E, z) \equiv \frac{3}{2} \int_0^{2\pi} d\varphi \overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z)$$

transport cross section

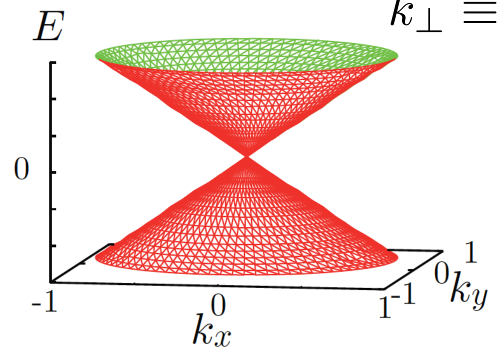
$$\sigma_{\mathrm{tr}}^{\parallel}(E, z) \equiv \frac{3}{2} \int_0^{2\pi} d\varphi (1 - \cos\varphi) \overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z)$$

$$\varphi \equiv \phi - \phi'$$

averaged differential cross section

$$\overline{\frac{d\sigma}{d\Omega}}(\varphi, E, z) = \left(\frac{\pi N_{\mathrm{F}}}{k_{\mathrm{F}}}\right)^2 \left(\frac{E}{\Delta}\right)^4 \frac{1}{4} \sum_{s, s' = \pm 1} \sum_{\sigma, \sigma'} |t(\boldsymbol{k}_{\parallel}, s k_{\perp}, \sigma \to \boldsymbol{k}'_{\parallel}, s' k_{\perp}, \sigma')|^2$$

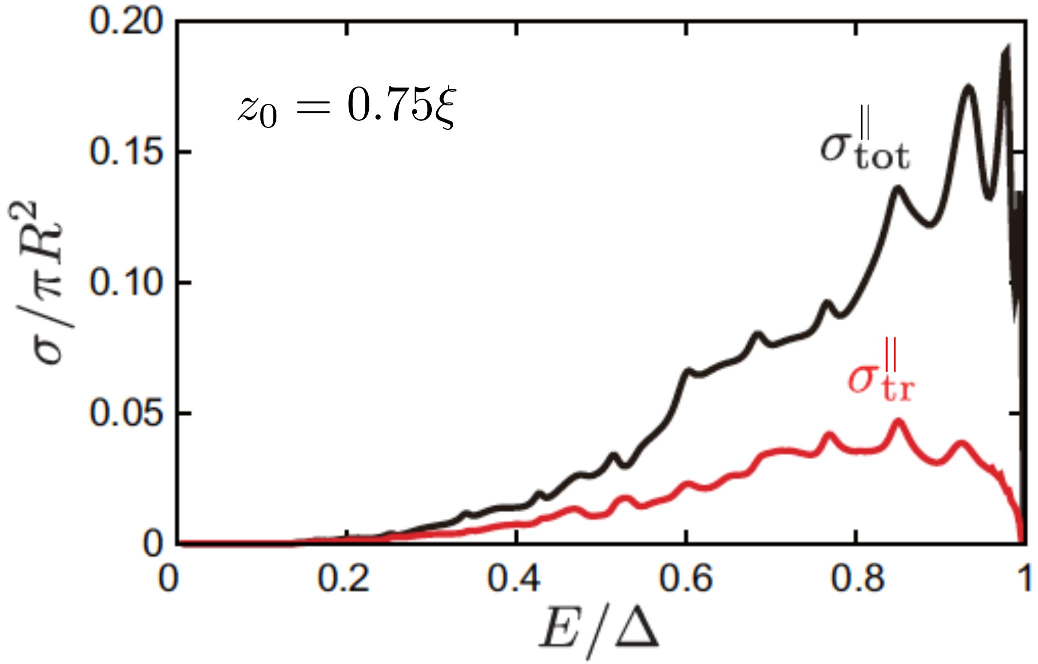
$$[k_{\parallel}(E) N(E)]^2$$



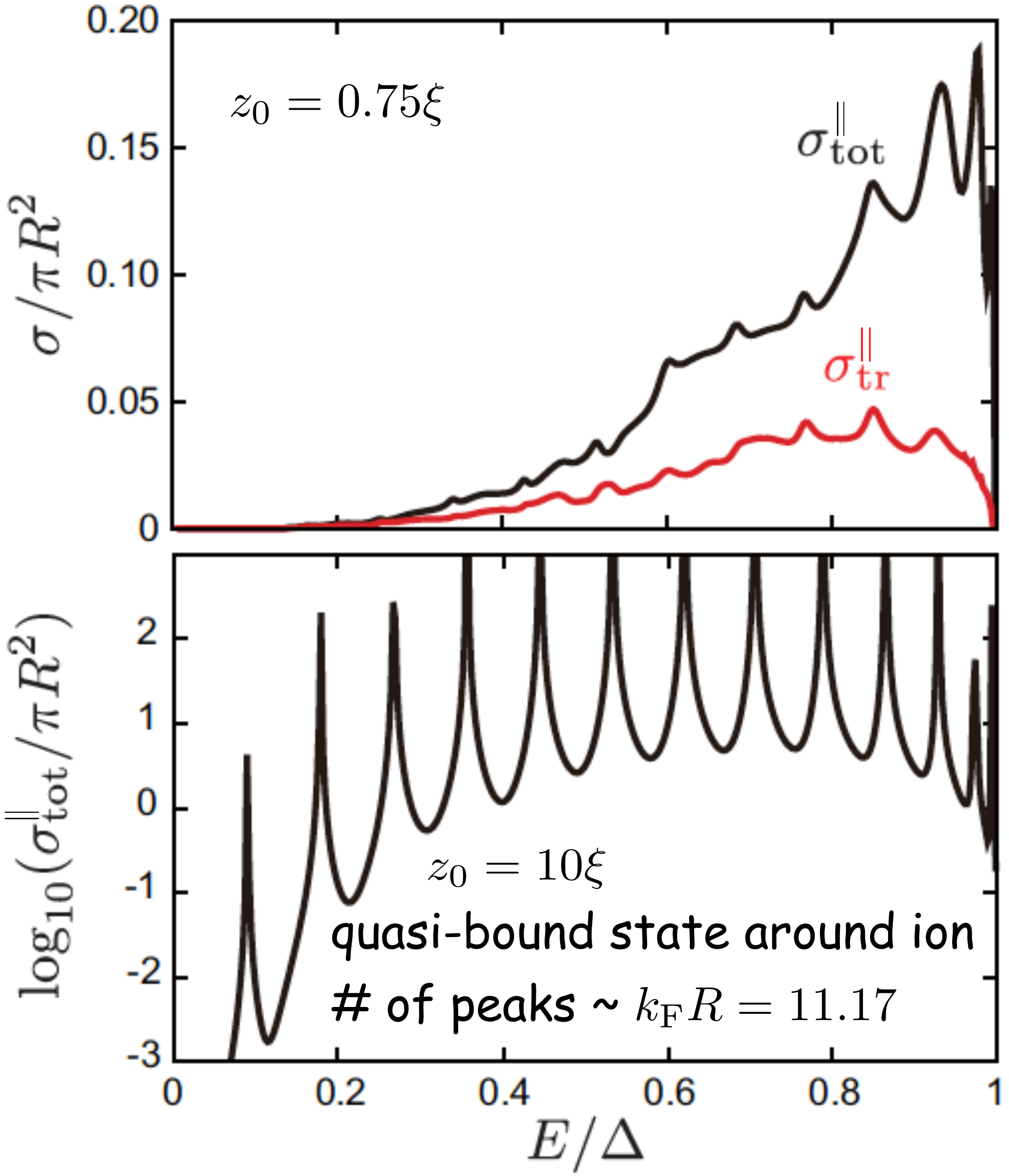
$$k_{\perp} \equiv |k_z| = \sqrt{k_{\mathrm{F}}^2 - k_{\parallel}^2}$$

$$= k_{\mathrm{F}} \sqrt{1 - \left(\frac{E}{\Delta}\right)^2}$$

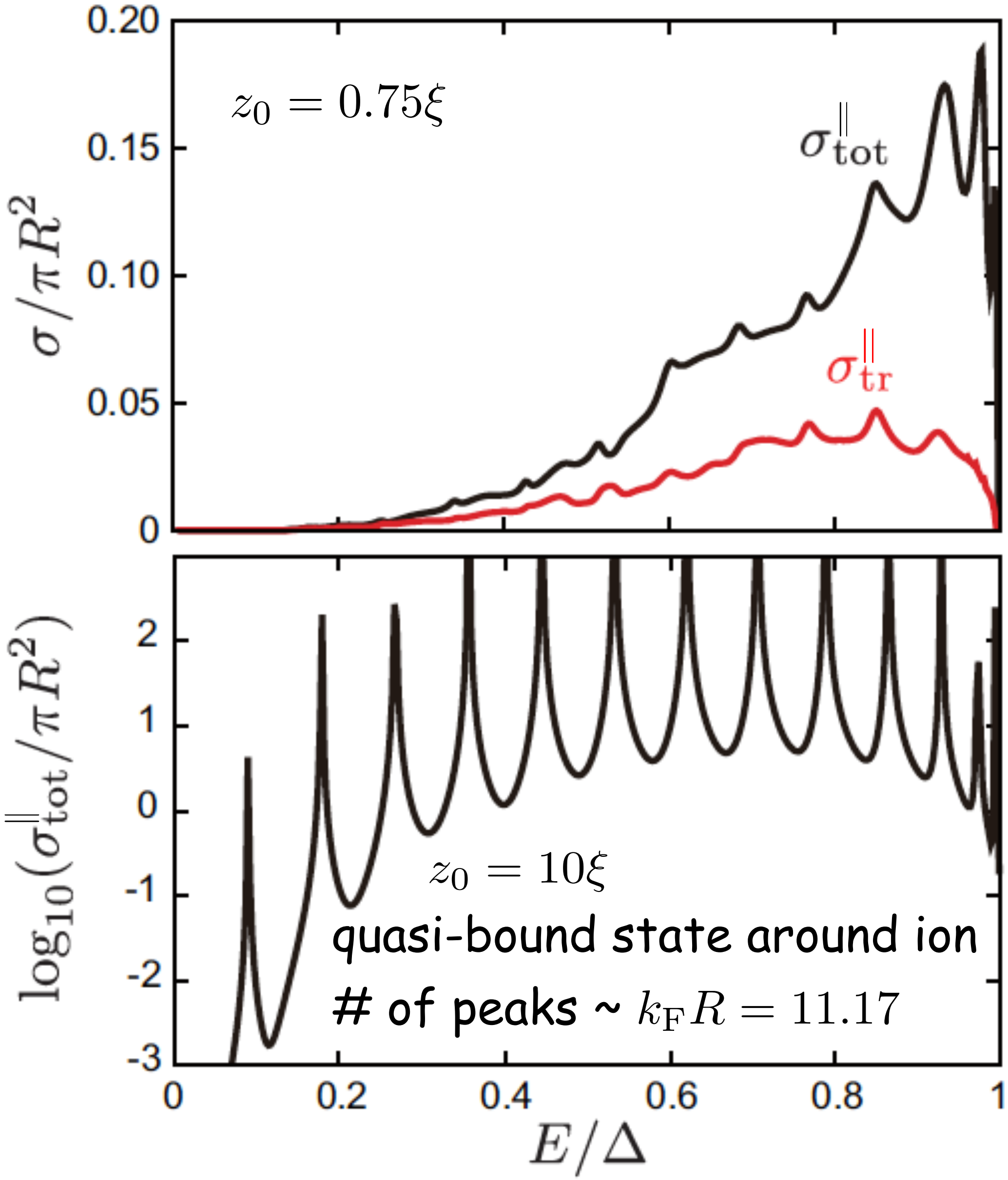
Scattering cross section



Scattering cross section

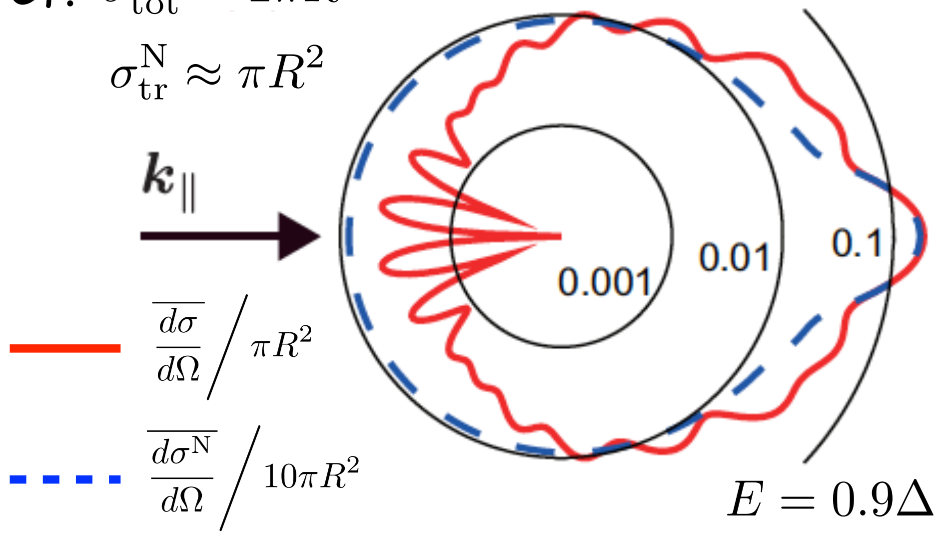


Scattering cross section



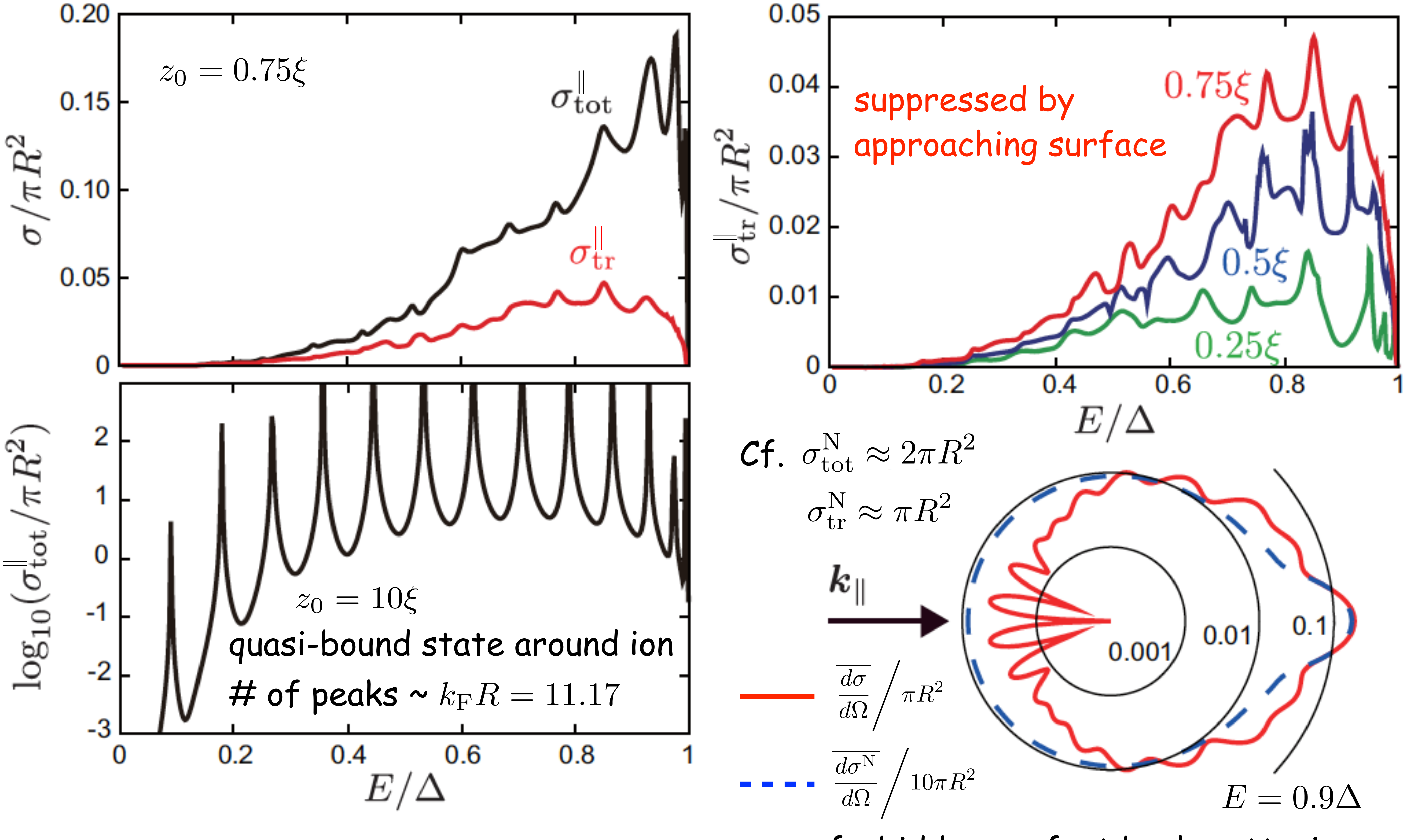
Cf. $\sigma^{\rm N}_{\rm tot} \approx 2\pi R^2$

$\sigma^{\rm N}_{\rm tr} \approx \pi R^2$



forbidden perfect backscattering

Scattering cross section



Mobility parallel to surface

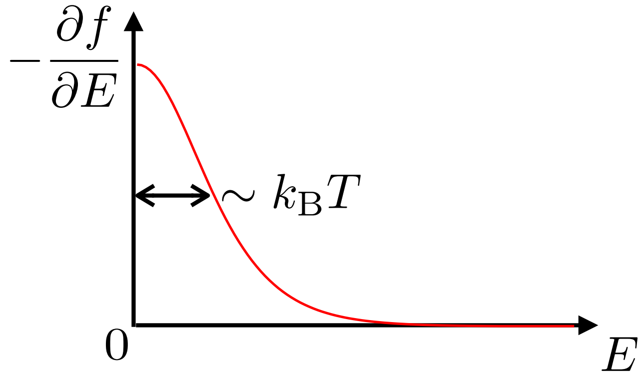
$$\mu_\parallel = \frac{e}{\eta_\parallel + \eta_{\rm cont}}$$

$$\eta_\parallel = \frac{\pi^2}{4} \exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\rm F} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\rm tr}^{\parallel}(E, z_0)$$

DOS

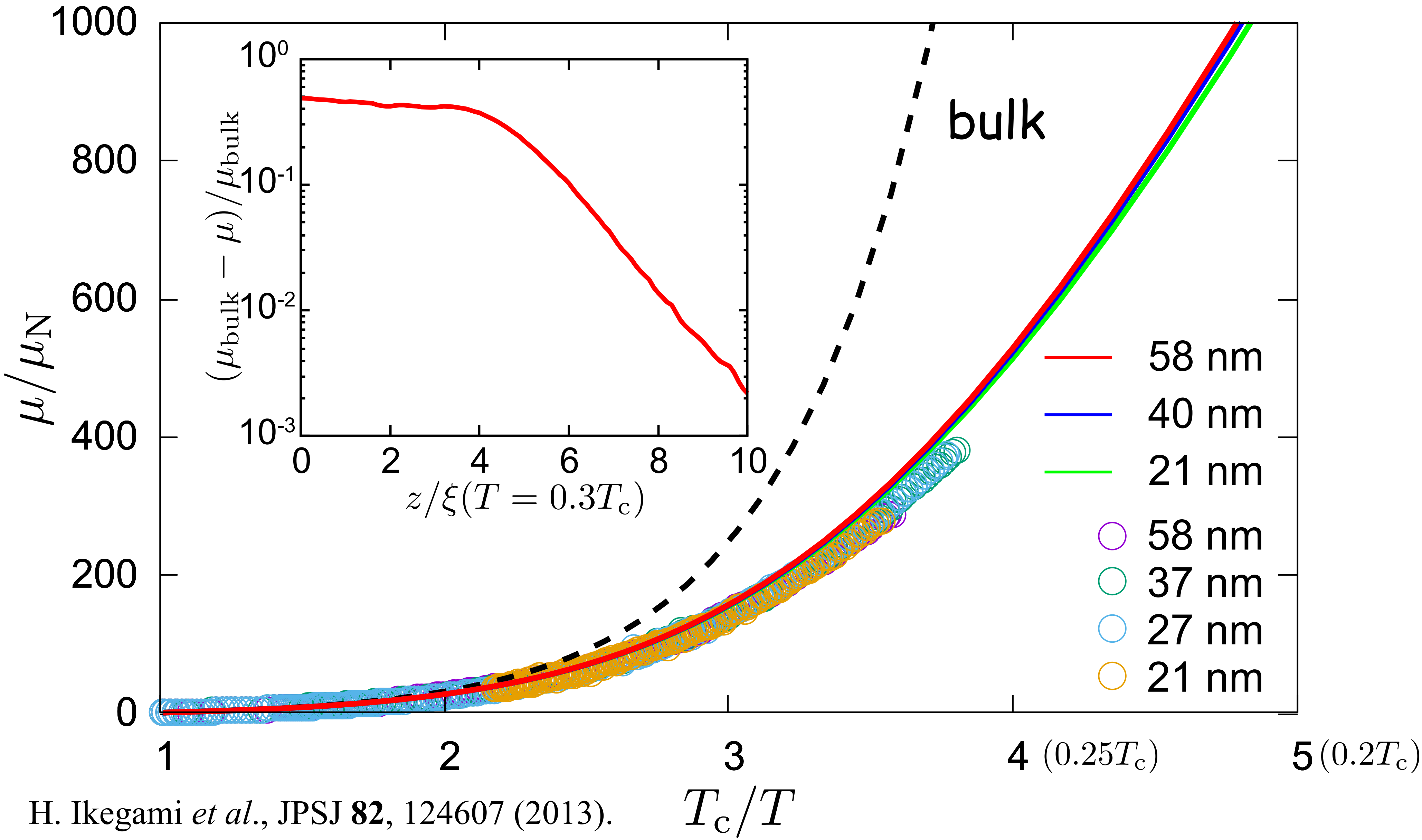
$$\eta_{\rm cont} \sim \eta_{\rm B} = 2 n_3 p_{\rm F} \int_{\Delta}^{\infty} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\rm tr}^{\rm B}(E)$$

$$\left(-\frac{\partial f}{\partial E}\right) \propto {\rm sech}^2 \left(\frac{E}{2k_{\rm B}T}\right)$$



Mobility parallel to surface

$$\mu_{\parallel} = \frac{e}{\eta_{\parallel} + \eta_{\mathrm{cont}}} \qquad \eta_{\parallel} = \frac{\pi^2}{4} \exp\left(-2\frac{z_0}{\xi}\right) n_3 p_{\mathrm{F}} \int_{-\Delta}^{\Delta} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{\mathrm{tr}}^{\parallel}(E, z_0)$$

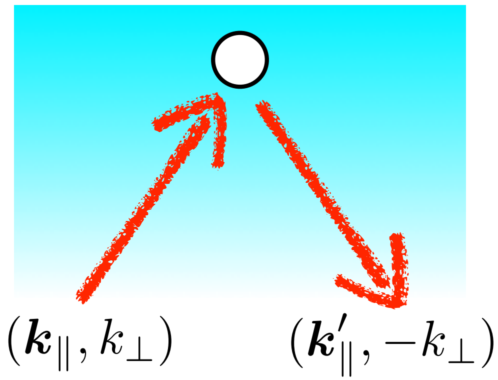


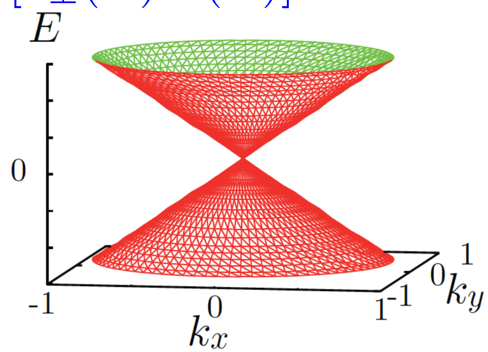
H. Ikegami *et al.*, JPSJ **82**, 124607 (2013).

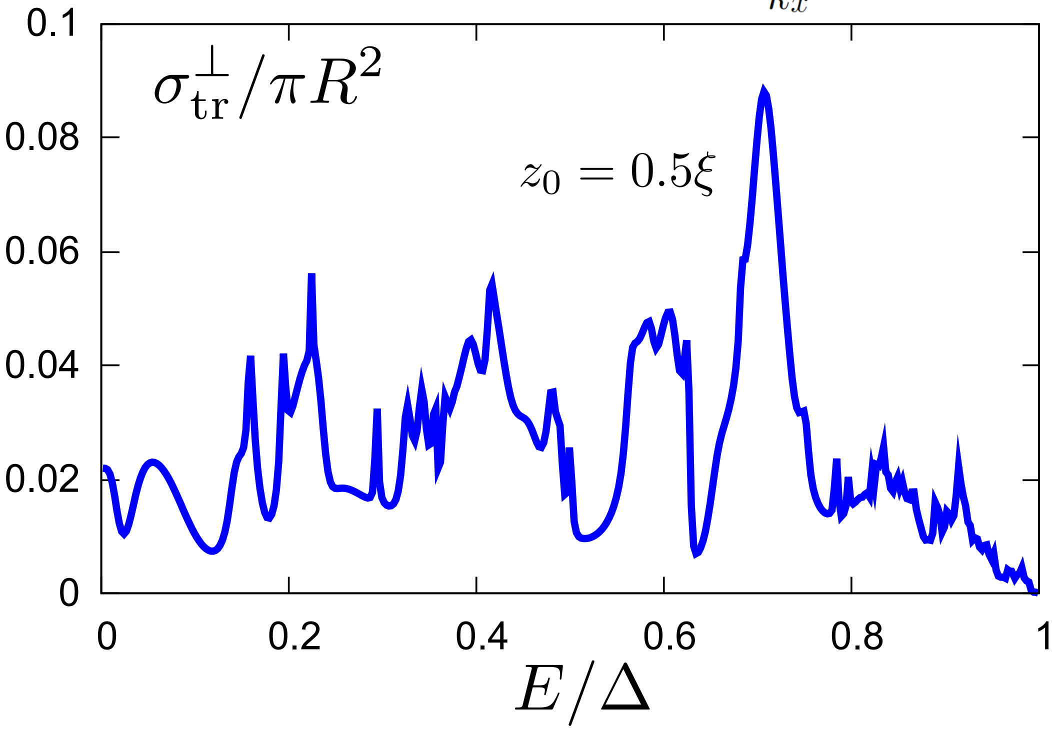
Cross section on perpendicular mobility

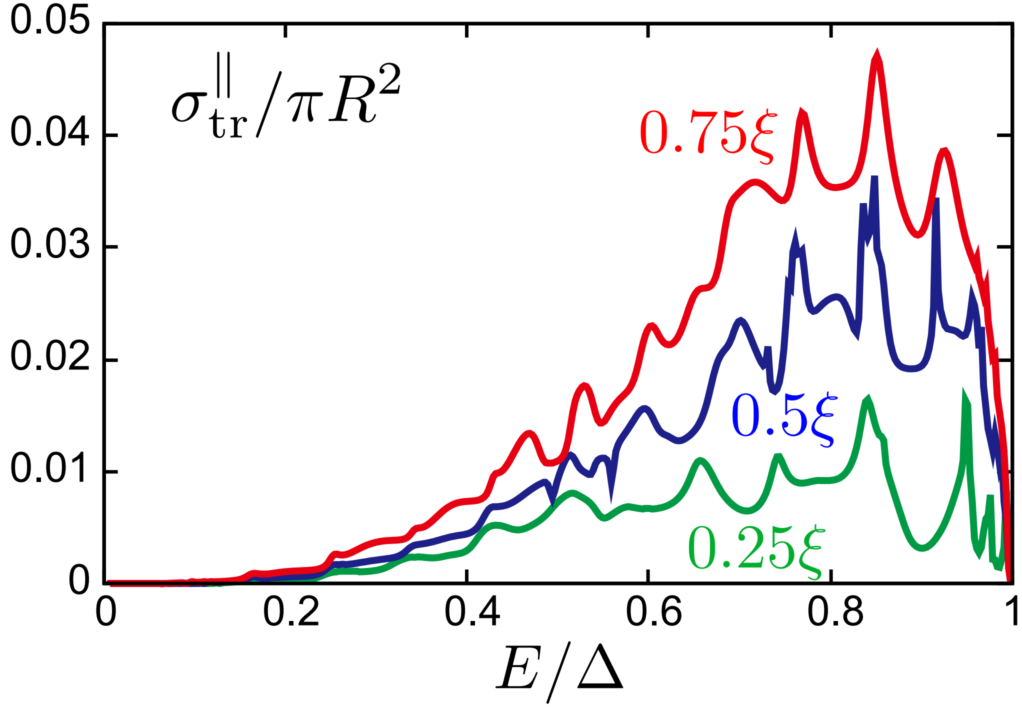
$$\sigma_{\rm tr}^{\perp}(E,z) = \left(\frac{\pi N_{\rm F}}{k_{\rm F}}\right)^2 \frac{(\Delta^2 - E^2)E^2}{\Delta^4} 3 \int_0^{2\pi} d\varphi \sum_{s=\pm 1} \sum_{\sigma,\sigma'} |t(\boldsymbol{k}_\parallel, sk_\perp, \sigma \to \boldsymbol{k}'_\parallel, -sk_\perp, \sigma')|^2$$

$[k_\perp(E)N(E)]^2$

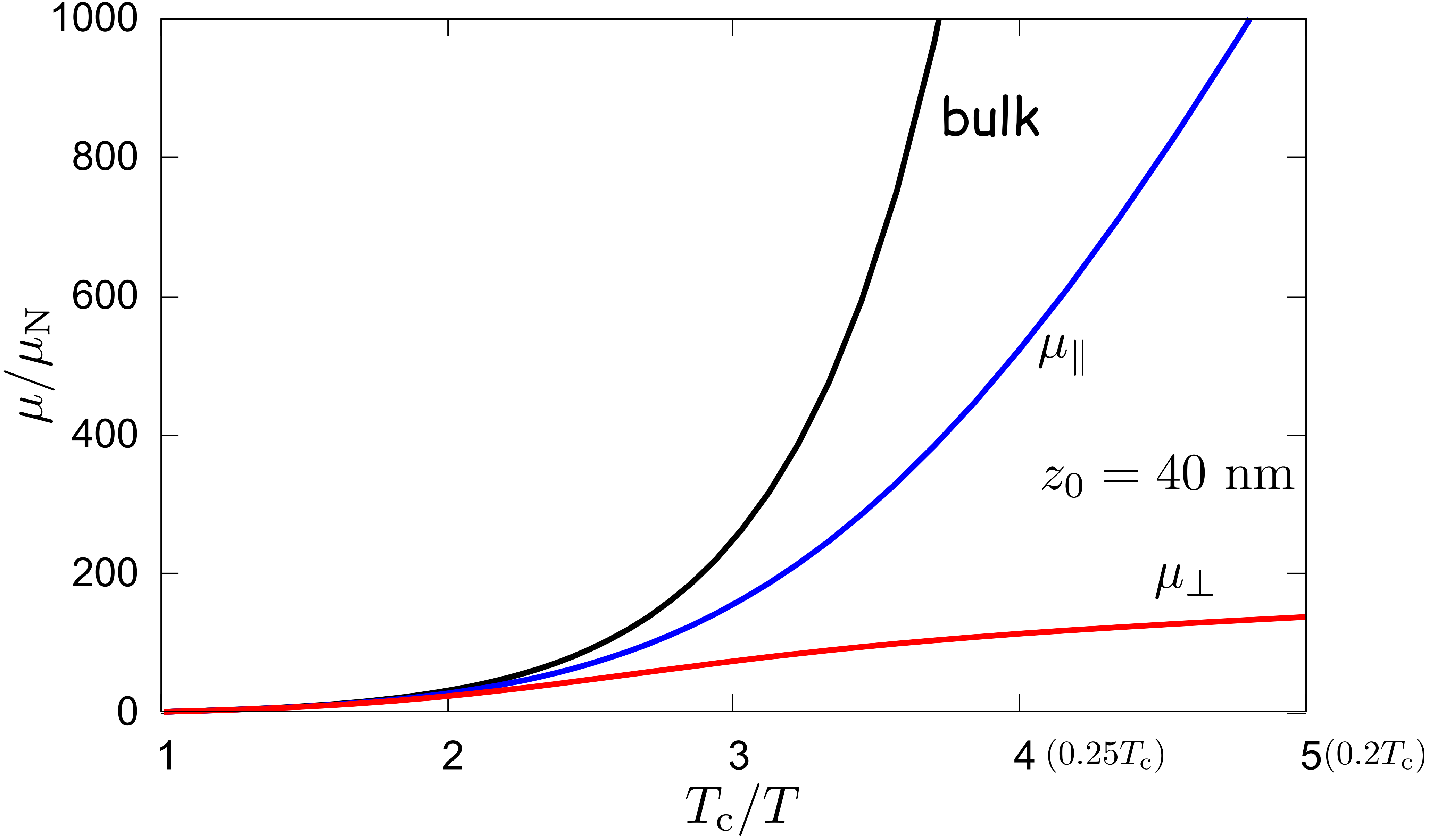








Mobility perpendicular to surface



Summary

We can quantitatively reproduce observed mobility
by formulating impurity scattering of surface Majorana fermions.

Surface Majorana fermions have been detected.

^{3}He-B is the first system hosting Majorana fermions
where experiment and theory show quantitative agreement.

Depth dependence of mobility is observed
if electron bubbles are trapped over $4\xi \sim 500$ nm.

Perpendicular mobility is strongly suppressed in low temperatures.