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Restraint Conditions in Estimating Endmembers

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Abstract

The linear spectral unmixing method is a commonly accepted approach to estimation of coverages of some categories in hyperspectral imagery. If, however, the values of coverages are concentrated on roughly 50%, for example in agricultural land, it is difficult for this method by itself to estimate coverages because the observed spectra obtained in hyperspectral imagery are quite similar. Therefore a solution of this problem needs to be discovered.

In this paper, a new method is suggested to solve that problem. It is found that it can be applied to agricultural land by the singular value decompdsing method adding a new conditional restraint. This focuses on the low correlation between the endmember of soil and that of crop. Even if coverages of all areas are 40%-60%, Maximum error is shown to be less than 0.018, sufficiently small. Additionally, accurate estimation of coverages is proven to be possible.

Key words: Endmember, Mixed pixel (mixels), Remote Sensing, Restraint condition, Singular value decomposing method

1. Introduction

Remote sensing is an effective technology because it can measure a wide area in a moment, observe the same area in chronological order and obtain data in digital fashion. For example, in respect to agricultural management, remote sensing makes it possible to understand the situation of crop growth and easier to improve the application of fertilizer and irrigation by using remote sensing for a large agricultural area. Moreover, production volume becomes predictable by getting information of crop coverage. For these reasons, the use of the remote sensing technology of agricultural land is a trend.

Recently, remote sensing technology is used to classify land-use. However, if the subject is agricultural land, it is difficult. Almost all pixels have the crop and soil areas combined in agricultural land. That mixed pixel problem needs to be solved.

To solve the problem, unmixing the mixed pixels and obtaining information such as endmember and coverage is necessary. It has been reported that some researchers estimate coverage in the cases that endmembers are known or when observed spectra are quite similar to endmembers. Both cases are ideal, nevertheless, they are far from representing real agricultural land. Generally, the pixels that are homogenous are elusive. In this paper, a new method to estimate endmembers from only observed spectra in agricultural land is presented. It uses the singular value decomposing method and some restraints are suggested in this paper assuming that endmembers are unknown. Specifically, it is assumed that observed spectra $\boldsymbol{X}$ is expressed by a linear combination between endmember $\boldsymbol{S}$ and coverage $\boldsymbol{C}$ and only observed spectra $\boldsymbol{X}$ are acquired. Estimation of $\boldsymbol{S}$ and $\boldsymbol{C}$ from only $\boldsymbol{X}$, and comparison between values of estimation and true values is carried out.

2. Method

2.1 Singular value decomposing method

The method to estimate endmembers and coverage includes a singular value decomposing method. First, the number of pixels selected from a remote sensing image is defined as K. An observed matrix $\boldsymbol{X}$ is defined as

$$X = \begin{Bmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,K} \\ & \cdots & \cdots & \\ X_{N,1} & X_{N,2} & \cdots & X_{N,K} \end{Bmatrix} \qquad (1)$$

where $X_{i,j}$ is a radiance of band i from area j and N is the number of total bands. Additionally, an endmembers matrix $\boldsymbol{S}$ and a coverage matrix $\boldsymbol{C}$ is defined as

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$$S = \begin{Bmatrix} S_{1,1} & S_{1,2} & \cdots & S_{1,L} \\ & \cdots & \cdots & \\ S_{N,1} & S_{N,2} & \cdots & S_{N,L} \end{Bmatrix} \quad (2)$$

$$C = \begin{Bmatrix} C_{1,1} & C_{1,2} & \cdots & C_{1,K} \\ & \cdots & \cdots & \\ C_{L,1} & C_{L,2} & \cdots & C_{L,K} \end{Bmatrix} \quad (3)$$

where $S_{i,j}$ is an endmember of band i of category j, L is the number of total categories and $C_{i,j}$ is a coverage of category i from area j. X is assumed to have a linear relation between S and C, and X can be expressed as

$$X = S \cdot C^{t} \quad (4)$$

Next, X can be divided into 3 matrices that are expressed as a function of only X. They are defined as U, P, and V. They consist of eigenvalues and eigenvectors of X, and can be calculated easily. U is defined as the matrix that is made of column-eigenvectors of $X \cdot X^{t}$, V is defined as the matrix that is made of line-eigenvectors of $X^{t} \cdot X$ and P is defined as the diagonal matrix that is made of square root of eigenvalues of $X^{t} \cdot X$. A following equality is formed:

$$X = U \cdot P \cdot V \quad (5)$$

M is defined as the number of eigenvalues, eigenvectors, and U is a N-M matrix, P is a M-M matrix and V is a M-K matrix. M is Rank of matrix X that is the number of categories. Consequently, S is a N-M matrix, and C is a M-K matrix. From equality (4) and (5), with adopting a square matrix T (a M-M matrix), the following equations are formed:

$$S = U \cdot T \quad (6)$$
$$C = T^{-1} \cdot P \cdot V \quad (7)$$

Because endmembers and coverage are never negative, the following restraint is valid:
Restraint 1 non-negative restraint condition :

$$S \geqq 0$$
$$C \geqq 0$$

From this restraint, some inequalities as much as the number of factors of S and C confine factors of an unknown matrix T effectively. So, it becomes possible to estimate endmembers S and coverage C.

2.2 Estimation of endmembers

It is generally thought that one pixel in remote sensing data of agricultural land includes various proportions of crop and soil. When the coverage of crop is near 0% or 100%, it is comparatively easier to estimate endmember S from observed spectra X because X becomes very similar to S. Here, estimation of endmembers is carried out with changing the situation of coverage in mixed pixels. In this research, 79bands are used. Figure 1 shows two spectra measured indoors and Figure 2 shows the flowchart that is used in this procedure.

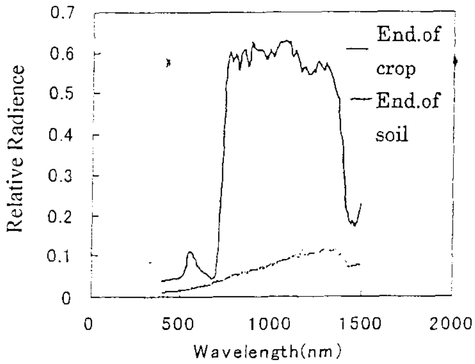

Fig.1. Endmembers of crop and soil.

(Case) Coverage 40%-60%
As Table 1 shows, coverage is changed from 40% to 60%. It is difficult to estimate endmembers because the observed spectra are very similar. After X is created from S and this C, unknown matrix T is moved. Concretely, 4 factors are changed from −1000 to 1000 by 0.1 so that S and C should not be negative as shown in 2.1.

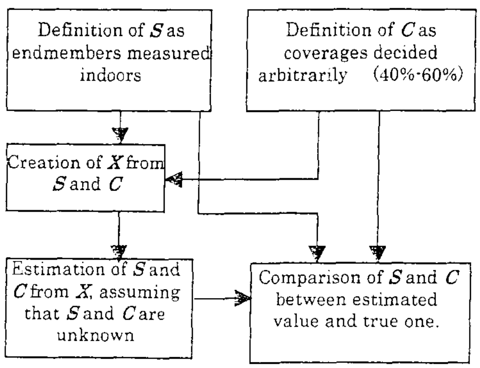

Fig.2. Flowchart of this method.

Table 1. Coverage of each area (40%-60%)

	Area1	Area2	Area3	Area4	Area5
Crop	0.41	0.52	0.45	0.6	0.55
Soil	0.59	0.48	0.55	0.4	0.45

However, T is not yet fully restricted, so some additional restraints are needed. Hence, the coverage restraint is adopted. Briefly, the sum of coverages in one pixel is about 1.$Csum_i$ is defined as $C_{i,1}+C_{i,2}$, and T are restrained as $Csum_i$ are about 1(i=1,2,3,4,5).

Restraint 2 Csum restraint conditions :

$$Csum_i \fallingdotseq 1$$

In this paper, only one answer is estimated by calculating the average of maximum endmember and minimum endmember showing equation (8) when a few answers are estimated.

$$S_{Estimate} = \left(S_{Max} + S_{Min}\right)/2 \qquad (8)$$

Figure 3 shows the estimation results from the singular value decomposing method with Restraint 1 and Restraint 2. As it shows, the estimation results of both crop and soil have large deviations and it was impossible to restrain them sufficiently. 28 sets of [T] are estimated and the maximum errors between estimated and true is 0.015 for the crop and 0.110 for the soil. It is found that it is very high, especially in the soil. Additionally, a sudden gradient is found in wavelengths from 700nm to 800nm in soil. Mentioned above, in the case of coverage 40%-60%, it is difficult to estimate endmembers using the singular value decomposing method with Restraint 1 and Restraint2 and some improvement is needed.

3. A new restraint added to method of estimating endmembers.

In Fig.3, all estimation results of crop by the singular value decomposing method with Restraints 1 and 2 in the wavelength of 700nm-800nm have a sudden gradient, while only some estimation results of soil have such a gradient. Generally it is considered that there is no sudden gradient in the wavelength of 700nm-800nm in soil. Hence, estimation results that have a sudden gradient need to be removed. A gradient of the spectra shape is focused on in order to

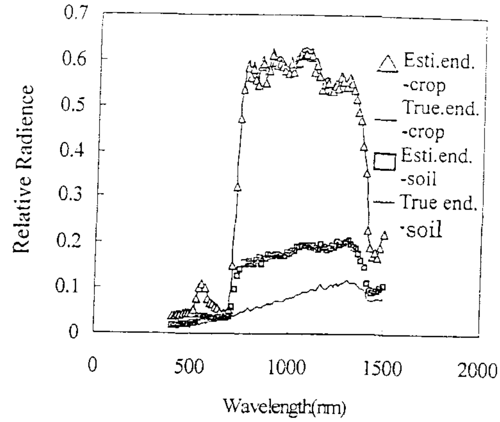

Fig.3. Result of estimation of endmembers.

remove them. Correlation of soil and crop is regarded as an index of similarity of shape. Concretely, $S_{j,1}$-$S_{j-1,1}$ is defined as $dif_{1,j}$ and $S_{j,2}$-$S_{j+1,2}$ is defined as $dif_{2,j}$ and Cor, correlation coefficient of $dif_{1,j}$ and $dif_{2,j}$,is defined as equation(9). ($\overline{dif_1}$ means average of $dif_{1,k}$. $\overline{dif_2}$ means average of $dif_{2,k}$).

$$Cor = \frac{\sum_{k=1}^{78}\left(dif_{1,k} - \overline{dif_1}\right)\left(dif_{2,k} - \overline{dif_2}\right)}{\sqrt{\left(\sum_{k=1}^{78}\left(dif_{1,k} - \overline{dif_1}\right)^2\right)}\sqrt{\left(\sum_{k=1}^{78}\left(dif_{2,k} - \overline{dif_2}\right)^2\right)}} \qquad (9)$$

Because a sudden gradient leads to high correlation, estimation results of high Cor need to be removed. They are removed if they are higher than 0.2.

Restraint 3 correlation coefficient restraint condition :

$$Cor < 0.2 \qquad (10)$$

Figure 4 shows endmembers estimated by the singular value decomposing method with Restraints 1, 2 and 3 in the situation of Table 1. Comparing them with estimation results of Fig.3, a deviation of results is sufficiently constrained. In this figure, the true endmember of soil and the estimated one were very similar, so it was found that the accuracy is high and results are estimated and maximum errors from true endmembers are 0.023 in crop and 0.0031 in soil. A significant improvement can be seen. For this reason, with coverage of 40%-60%, the endmember [S] could be estimated accurately.

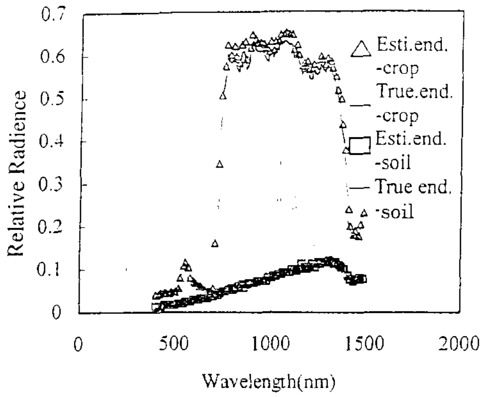

Fig.4. Result of estimation of endmembers by using a new restraint.

4. Estimation of coverages

It is indicated that the more important task is to estimate endmembers and, if it is possible to estimate them, it is not hard to estimate coverages. Table 2 shows the result of estimation of coverages. It proves that accurate estimation of endmembers leads to accurate estimation of coverages.

Table 2. Comparison of coverage between true and estimated.

	Area1	Area2	Area3	Area4	Area5
True cov-crop	0.410	0.520	0.450	0.600	0.550
True cov-soil	0.590	0.480	0.550	0.400	0.450
Esti.cov-crop	0.402	0.507	0.440	0.583	0.540
Esti.cov-soil	0.580	0.493	0.560	0.417	0.465
Error	0.010	0.013	0.010	0.017	0.015

5. Conclusion

Even if the coverages are 40%-60% and observed spectra are quite similar, this result shows that it is possible to estimate endmembers by adopting a new restraint. Accurate estimating of endmembers makes it easy to estimate coverages.

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