物理学会@関西大学（2015/9/16）

16aCQ-12

平坦バンドのあるBose-Hubbard模型におけるペア朝永-Luttinger液体相

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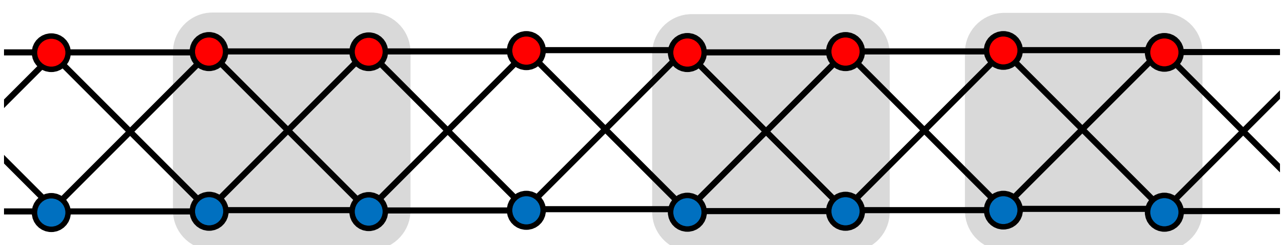


- S. Takayoshi, H.K., N. Watanabe, and H. Aoki, *Phys. Rev. A* **88**, 063613 (2013).

Creutz ladder

- Some of my favorite subjects:

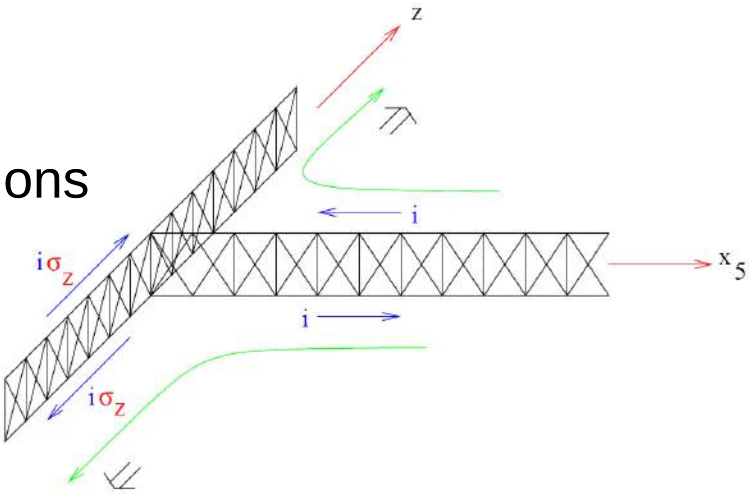
1. Bethe ansatz
2. Flat bands
3. Topological insulators

All of them lead to ***non-perturbative*** results.
Marriage of these 3 in the Creutz ladder → ***Pair TL liquids!***

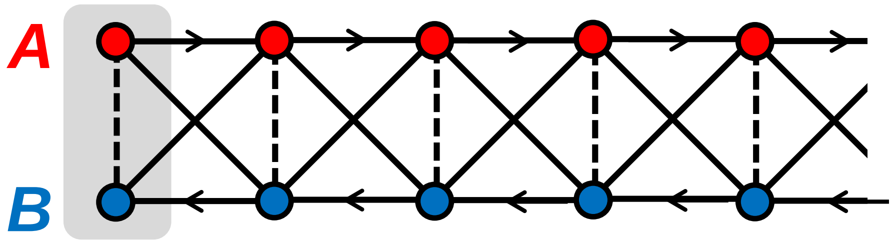
- Creutz ladder (free-fermion model)

M. Creutz, *PRL* **83**, 2636 (1999).

- 1d toy model for domain-wall fermions
- Edge states
- Spin separater?



What about Bose-Hubbard on the Creutz ladder?



Bosons a_j live on chain A, while bosons b_j on chain B.

- Hamiltonian: $\mathcal{H} = \mathcal{H}_{\text{hop}} + \mathcal{H}_U$

  Hopping term:

$$\mathcal{H}_{\text{hop}} = -t\sum_j (e^{\mathbf{i}\theta} a_j^\dagger a_{j+1} + e^{-\mathbf{i}\theta} b_j^\dagger b_{j+1} + \text{h.c.}) - t_\times \sum_j (a_j^\dagger b_{j+1} + b_j^\dagger a_{j+1} + \text{h.c.}) - t_\perp \sum_j (a_j^\dagger b_j + h.c.)$$

Class AIII & BDI ***topological insulator*** (Mazza *et al*, *NJP* **14** ('12))
Edge states for $\theta = \pi/2$ and small $t_\perp$.

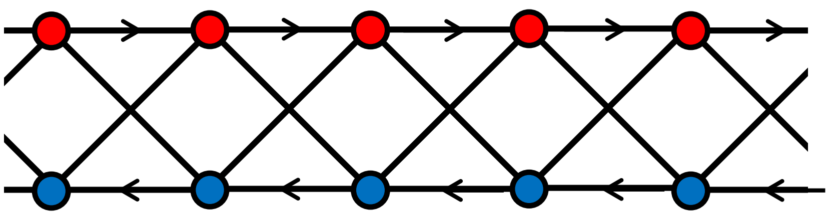
On-site repulsion (U >0):

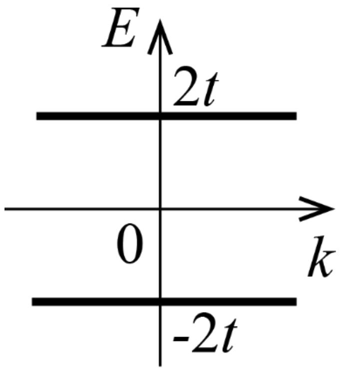
$$\mathcal{H}_U = \frac{U}{2}\sum_j a_j^\dagger a_j^\dagger a_j a_j + \frac{U}{2}\sum_j b_j^\dagger b_j^\dagger b_j b_j$$

How the interaction affects 1d topo. ins.?
Fermions v.s. bosons?

Flat band and Wannier basis

- Flat-band case





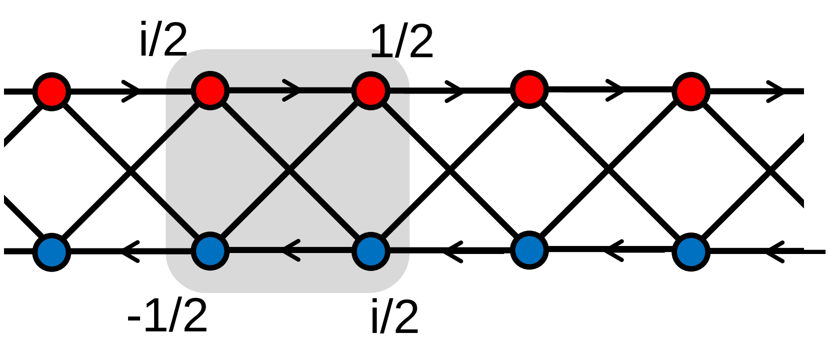
Completely flat bands!

$$t = t_\times, \quad t_\perp = 0, \quad \theta = \pi/2$$

$$E(k) = \pm 2t$$

- Wannier states

Usually, flat bands are spanned by non-orthogonal basis states. Here, we can construct a ***Wannier basis*** for both upper and lower flat bands. For the lower band, we have



$$w_j = \frac{1}{2}(\mathbf{i}a_j - b_j - a_{j+1} + \mathbf{i}b_{j+1})$$

$$[w_i, w_j^\dagger] = \delta_{i,j}$$ Orthonormal!

Wigner solid

- Boson density

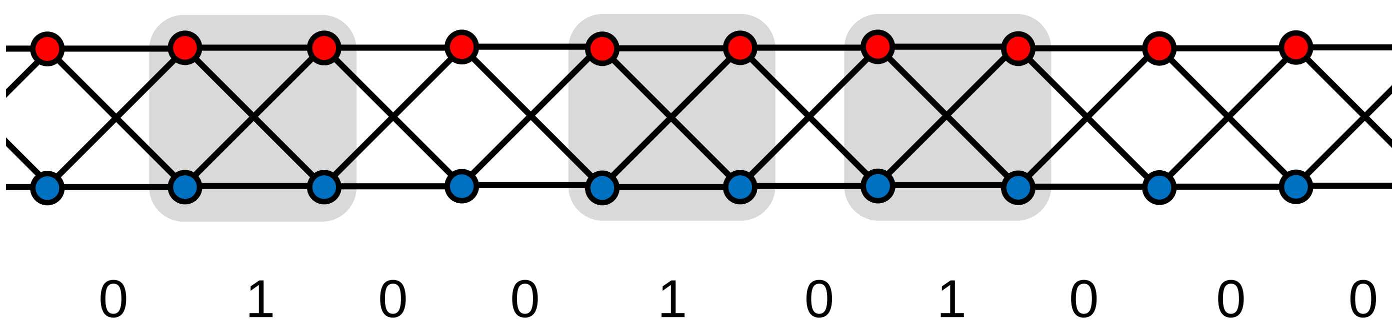
$$\rho := \frac{N}{2L}$$

L: Number of sites on one chain
N: Total number of bosons

- Ground states for low densities ($\rho \leqq 1/4$)

Since H_U is ***positive semi-definite***, any zero-energy state of H_U spanned by $w^\dagger$ is an exact ground state of $H = H_{\mathrm{hop}} + H_U$.

→ *1-to-1 correspondence with non-overlapping configurations!*

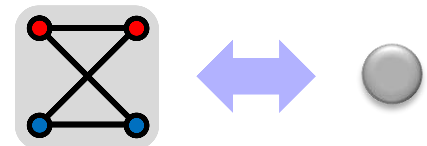


They do not touch each other as long as $N \leqq L/2$.
→ ***Wigner solid*** phase for $\rho \leqq 1/4$.

Low-energy effective Hamiltonian ($U<t$)

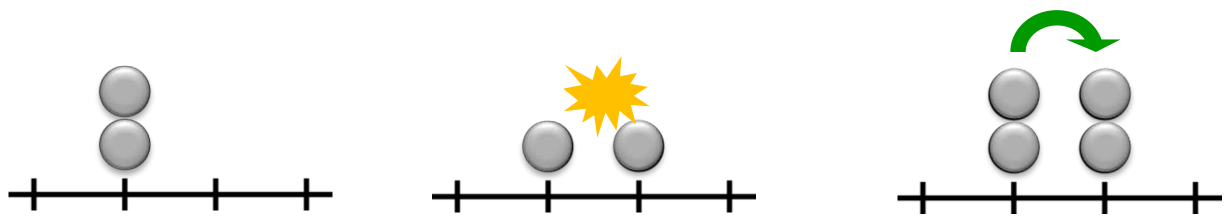
- Projected interaction

Consider the case $U \ll 4t$ (band gap).
H_U projected onto the lower band reads



$$\mathcal{H}^{\mathrm{int}}_{\mathrm{proj}} = \sum_{j=1}^{L} \left[\frac{U}{8} w_j^\dagger w_j^\dagger w_j w_j + \frac{U}{4} w_j^\dagger w_{j+1}^\dagger w_j w_{j+1} - \frac{U}{16} \left(w_j^\dagger w_j^\dagger w_{j+1} w_{j+1} + \mathrm{H.c.} \right) \right]$$

Positive semi-definite

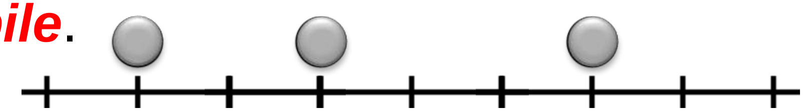


Onsite interaction

NN interaction

Pair hopping

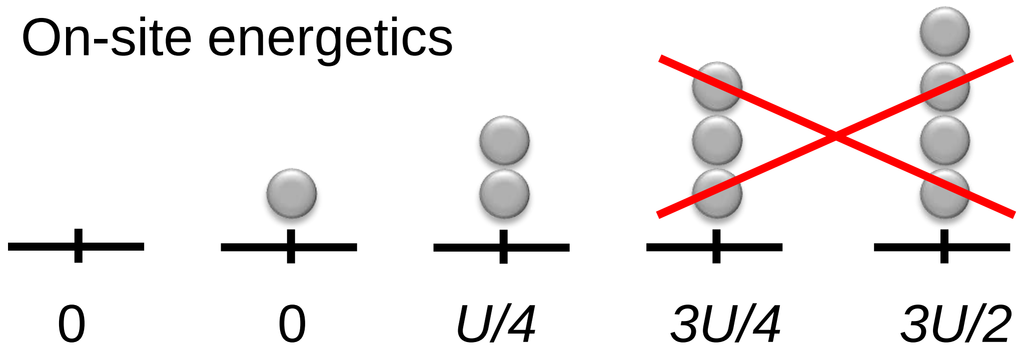
Isolated single bosons are ***immobile***.
Wigner solids correspond to



Pairs of bosons can lower the “kinetic” energy.
(Similar to 2-magnon bound states in ***nematic phases***.)

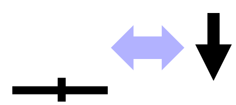
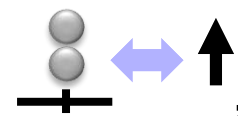
Effective spin model for high densities

- On-site energetics

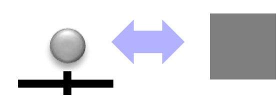


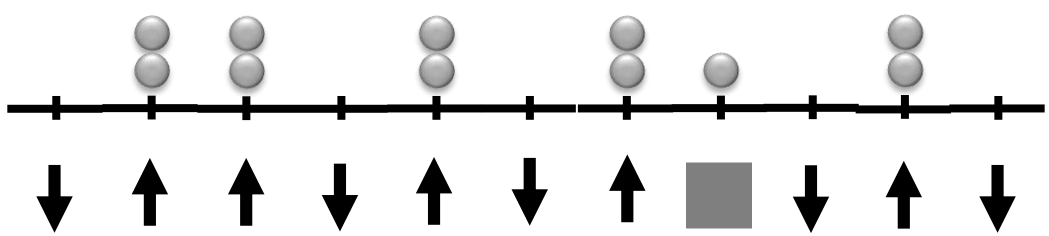
- *Neglect states with more than 2 bosons at one site.*
- *On-site int < NN int.*

- Spin Hamiltonian

With the identification  and ,
effective spin model = Spin-1/2 XXZ chain in a magnetic field

$$\mathcal{H}_{\mathrm{XXZ}} = \frac{U}{4}\sum_j \left(-S_j^x S_{j+1}^x - S_j^y S_{j+1}^y + 4 S_j^z S_{j+1}^z\right) + \frac{5}{4}U\sum_j S_j^z$$

Single-boson sites can be thought of as “walls”. 



Each configuration is an ensemble of open XXZ chains separated by walls.

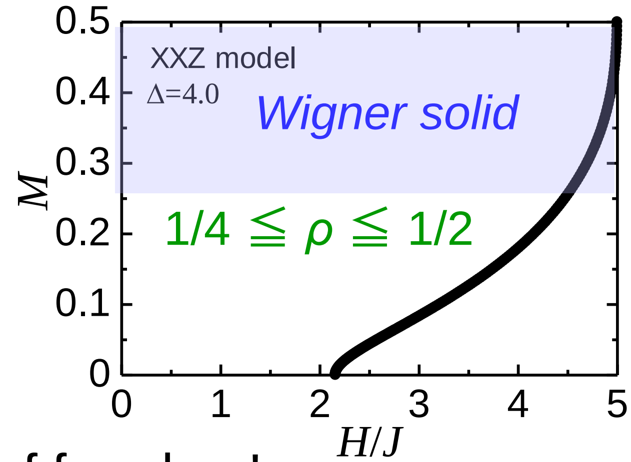
Pair Tomonaga-Luttinger (TL) liquid phase

- Dictionary

  Boson-pair number ⇔ $2S_j^z + 1$

  Boson density   Magnetization

$$\rho = \frac{1}{2} - M \quad \Leftrightarrow \quad M = -\frac{1}{L}\sum_j S_j^z$$



Pair-TL liquid ⇔ Field-induced TL liquid

Pairs of bosons are fundamental degrees of freedom!

NOTE: Nematic TL liquid, Doucot-Vidal, *PRL* **88** 227005 (‘02)

- Bosonization analysis

  TL parameter (*K*) and velocity can be computed via *Bethe ansatz!*

  S. Qin *et al*., *PRB* **56**, 9766 (’97), Cabra *et al*., *PRB* **58**, 6241 (‘98).

- Density-density correlation ($n_r := w_r^\dagger w_r$) $\langle S_r^z S_0^z \rangle$

$$\langle n_r n_0 \rangle \simeq 4\rho^2 - 2K(\pi r)^{-2} + c_1 \cos(2\pi\rho r) r^{-2K}$$

- Off-diagonal correlation $\langle S_r^+ S_0^- \rangle$

$$\langle w_r^\dagger w_r^\dagger w_0 w_0 \rangle \simeq (-)^r c_2 r^{-\frac{1}{2K}} + (-)^r c_3 \cos(2\pi\rho r) r^{-2K-\frac{1}{2K}}$$

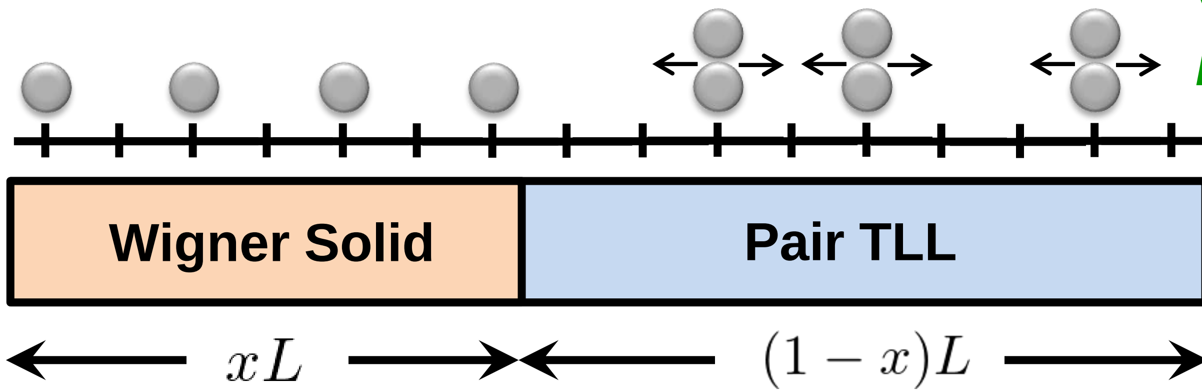
1/4 ≦ *K* ≦ 1/2

→ The latter decays faster.

→ No supersolid?

Phase separation

Coexistence phase?

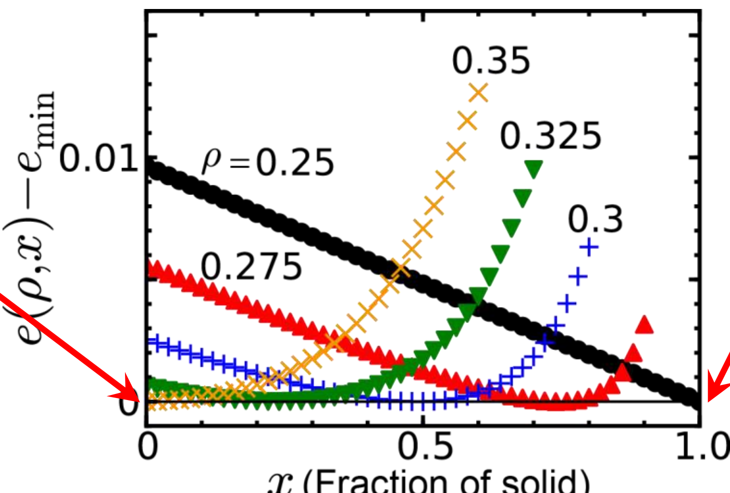


Density: $\rho_{\mathrm{WS}} = 1/2$ $\qquad \rho_{\mathrm{TLL}} = \dfrac{\rho - x/4}{1-x}$

Energy: 0 $\qquad (1-x)L\, e_{\mathrm{TLL}}(M(x))$

Can be computed by solving Bethe-ansatz integral eq.

Pair-TL liquid

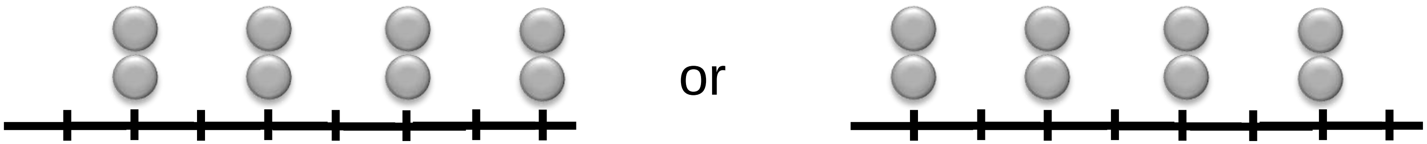


Wigner Solid

Phase separation when 0.25 < ρ ≦0.35.

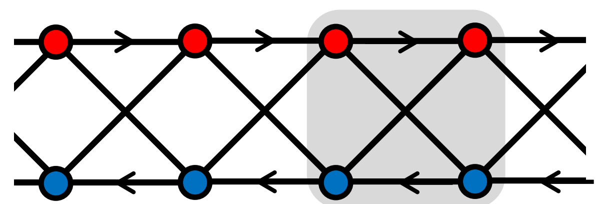
Charge-density wave at ρ=1/2

- Ising-like XXZ → Neel ground states

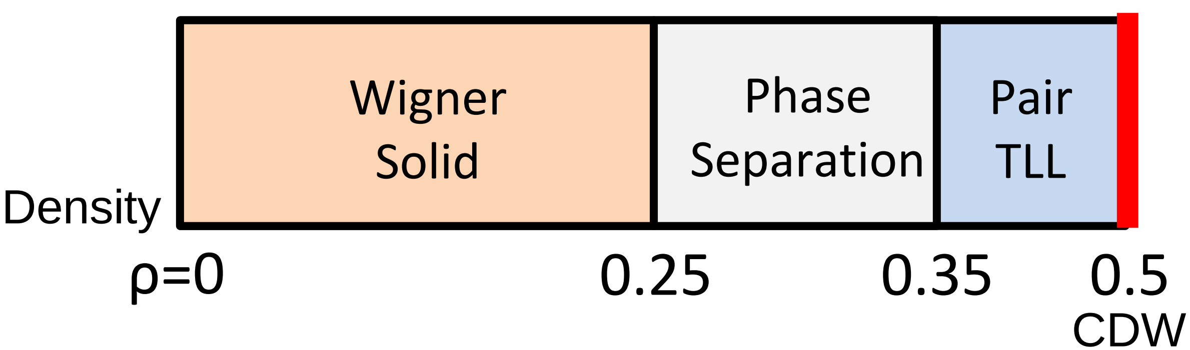


The 1st and 2nd excited energies can be computed ***exactly***!
Batchelor-Hamer, *JPA* **23** (‘90), Kapustin-Skorik, *JPA* **29** (‘96).

Summary



- Studied Bose-Hubbard on Creutz ladder
- Flat bands and **Wigner solid** ($\rho \leqq 1/4$)
- Phase diagram for $U \ll 4t$ (band gap) → **Pair-TL liquid!**



The validity of the XXZ description was checked (ED).

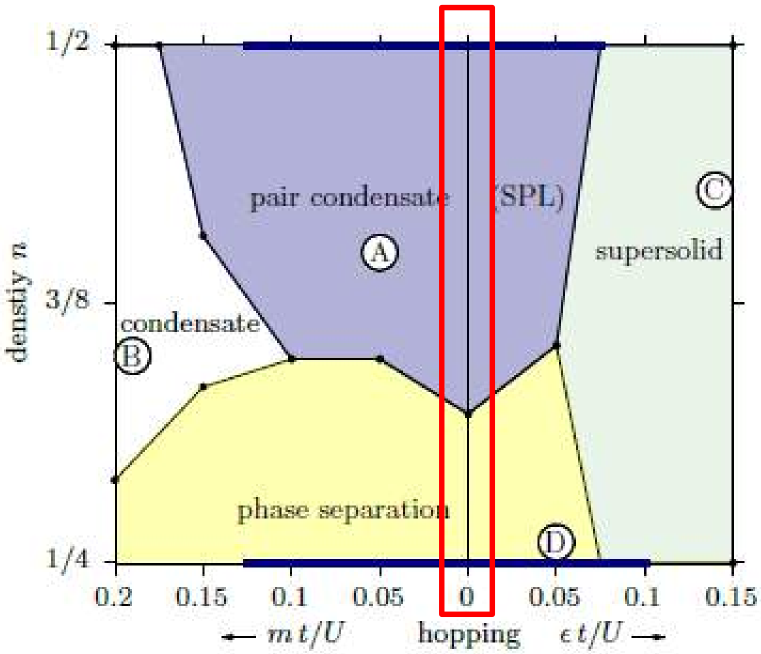
Away from the flat-band limit

- DMRG study (after our work)

Tovmasyan, Nieuwenburg & Huber, *PRB* **88**, 220510(R) (2013).

Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \sum_{j=1}^{L} \Big[\frac{U}{8} w_j^\dagger w_j^\dagger w_j w_j + \frac{U}{4} w_j^\dagger w_{j+1}^\dagger w_j w_{j+1} - \frac{U}{16}(w_j^\dagger w_j^\dagger w_{j+1} w_{j+1} + \text{H.c.}) + \frac{mt}{4} w_j^\dagger w_{j+1} + \frac{\epsilon t}{4} w_j^\dagger w_{j+2} + \text{H.c.}\Big] \qquad (mt = t_\perp, \quad \epsilon t = t_\times - t)$$



Qualitatively similar result!
Pair-TL liquid phase is stable against small perturbations.

Future directions

- Realization using synthetic gauge?

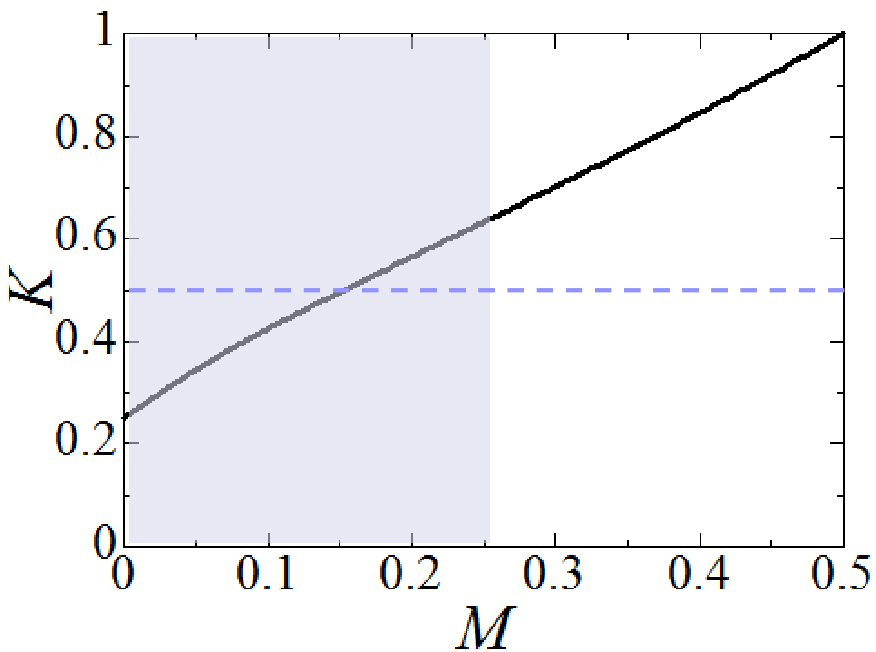
  Y.-J. Lin *et al*., *Nature* **462**, 628 (‘09).

- Fermionic models, topological phases?

  Spin-orbit interpretation is possible.

Backup slide

- TL parameter



=ρ-1/2

- TL exponents