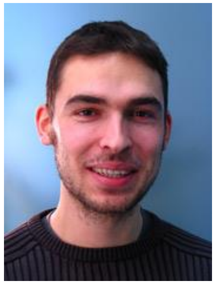
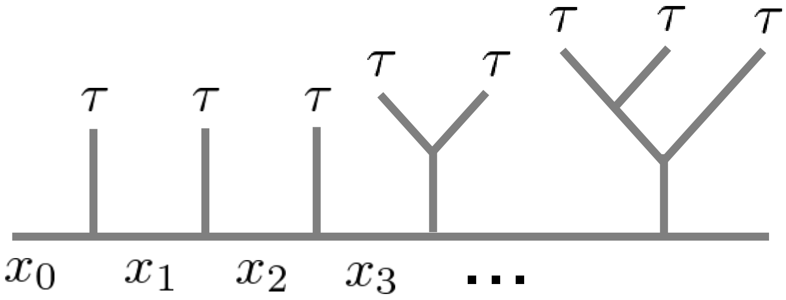
18aAL-12

Rydberg blockadeを用いたFibonacciエニオンの実現

桂 法称（学習院大理）

Collaborator:
Igor Lesanovsky（Nottingham Univ.）



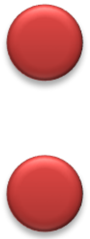


- I. Lesanovsky and Hosho Katsura., [arXiv:1204.0903]

What are Fibonacci anyons?

Anyons: Quasiparticles with fractional charges
FQHE, 2d magnetic models (Kitaev model), …

1. Abelian anyons: acquire a phase factor $e^{i\phi}$ under braiding
2. Non-abelian anyons: unitary rotation in the ground space



Fibonacci anyons: Read-Rezayi state, Levin-Wen model

➢ ***quantum dimension:*** $\boldsymbol{d_\tau = (1+\sqrt{5})/2}$

Fusion rule: $\tau \times \tau = \mathbf{1} + \tau \qquad \mathbf{1} \times \tau = \tau$

Precise meaning: There are two possible quantum states for two Fibonacci anyons.
1 has trivial statistics, τ has the braiding properties of a single Fibonacci anyon.

1 $\mathbf{1}$

1 τ

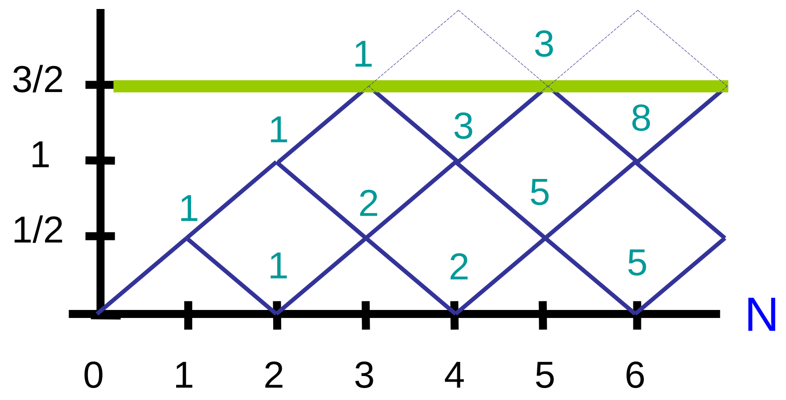
2 $\tau \times \tau = \mathbf{1} + \tau$

3 $(\tau \times \tau) \times \tau = \tau + \tau \times \tau = \mathbf{1} + 2\tau$

5 $((\tau \times \tau) \times \tau) \times \tau = 2\,\mathbf{1} + 3\tau$

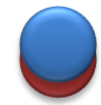
8 $(((\tau \times \tau) \times \tau) \times \tau) \times \tau = 3\,\mathbf{1} + 5\tau$

Fibonacci #: $F_N \sim (d_\tau)^N$

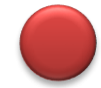


What are Fibonacci anyons?

Anyons: Quasiparticles with fractional charges
FQHE, 2d magnetic models (Kitaev model), …



1. Abelian anyons: acquire a phase factor $e^{i\phi}$ under braiding
2. Non-abelian anyons: unitary rotation in the ground space



Fibonacci anyons: Read-Rezayi state, Levin-Wen model

- ***quantum dimension:*** $\boldsymbol{d_\tau = (1+\sqrt{5})/2}$

Fusion rule: $\tau \times \tau = \mathbf{1} + \tau \qquad \mathbf{1} \times \tau = \tau$

Precise meaning: There are two possible quantum states for two Fibonacci anyons.
1 has trivial statistics, τ has the braiding properties of a single Fibonacci anyon.

Fibonacci #: $F_N \sim (d_\tau)^N$

1 $\mathbf{1}$

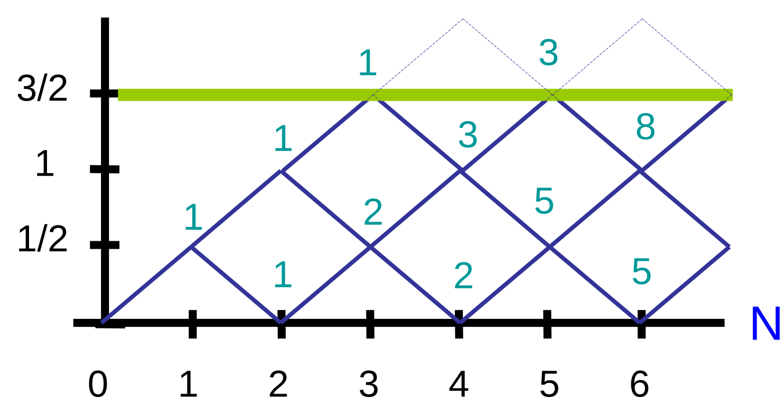
1 τ

2 $\tau \times \tau = \mathbf{1} + \tau$

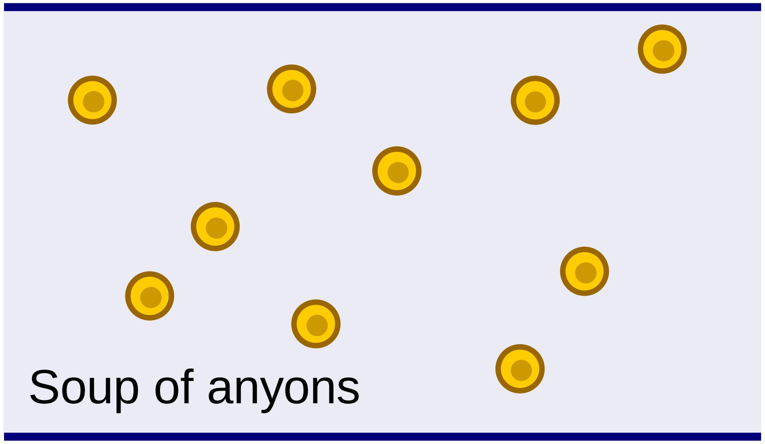
3 $(\tau \times \tau) \times \tau = \tau + \tau \times \tau = \mathbf{1} + 2\tau$

5 $((\tau \times \tau) \times \tau) \times \tau = 2\,\mathbf{1} + 3\tau$

8 $(((\tau \times \tau) \times \tau) \times \tau) \times \tau = 3\,\mathbf{1} + 5\tau$



Golden chain of Fibonacci anyons



$a \gg \xi_m$

The ground state has a macroscopic degeneracy $\sim \varphi^N$ $(\varphi : \text{ golden ratio})$

$a \sim \xi_m$

Anyons approach each other and interact. The interactions will lift the degeneracy.

- **1d array of anyons (pinned)** A. Feiguin *et al*., *PRL.* **98**, 160409 (2007).

Heisenberg int.

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

triplet

splitting / J

singlet

Anyonic Heisenberg int.

$$\tau \times \tau = 1 + \tau$$

$\tau \times \tau \to \tau$

splitting / J

$\tau \times \tau \to 1$

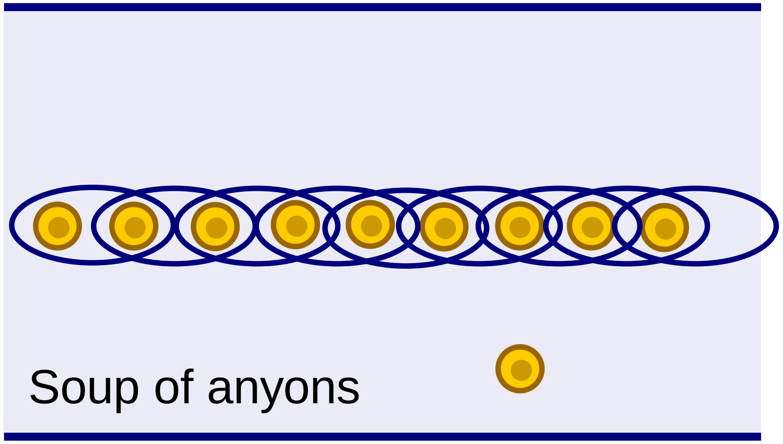
Penalize fusion outcome 1 or τ.

Numerical results:
g.s. → unique, critical
Low-energy physics
→ minimal CFT (c<1)

- **Mapping & exact solution**

Anyonic spin chain = Integrable RSOS model (Andrews-Baxter-Forrester)

Golden chain of Fibonacci anyons



$a \gg \xi_m$

The ground state has a macroscopic degeneracy $\sim \varphi^N$ $(\varphi:\ \text{golden ratio})$

$a \sim \xi_m$

Anyons approach each other and interact.
The interactions will lift the degeneracy.

- **1d array of anyons (pinned)**

A. Feiguin *et al*., *PRL.* **98**, 160409 (2007).

Heisenberg int.

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

triplet

splitting / J

singlet

Anyonic Heisenberg int.

$$\tau \times \tau = 1 + \tau$$

$\tau \times \tau \to \tau$

splitting / J

$\tau \times \tau \to 1$

Penalize fusion outcome 1 or τ.

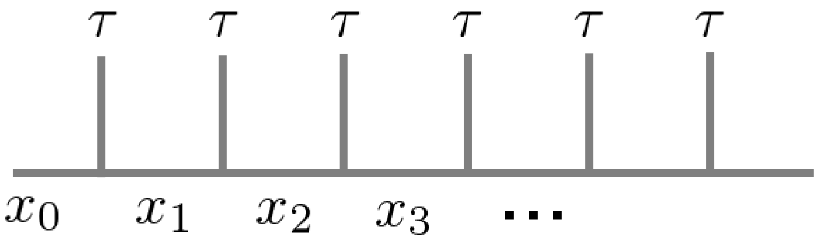
Numerical results:
g.s. → unique, critical
Low-energy physics
→ minimal CFT (c<1)

- **Mapping & exact solution**

Anyonic spin chain = Integrable RSOS model (Andrews-Baxter-Forrester)

Anyonic Hilbert space

- **Fusion tree basis**

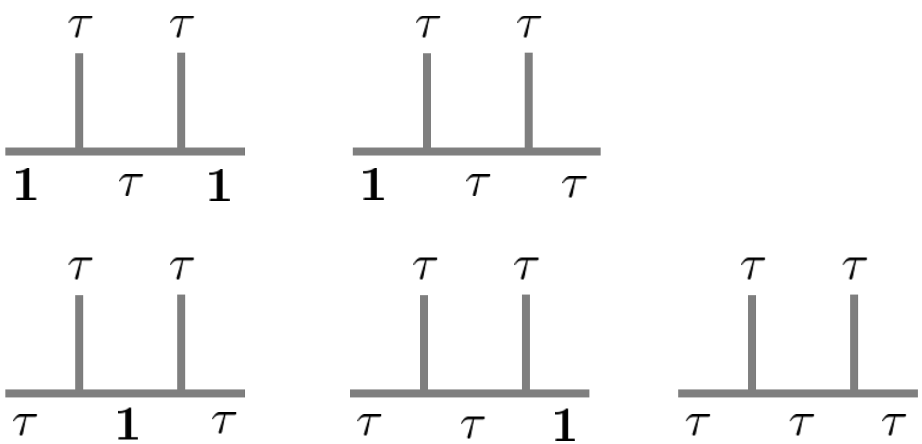


Orthonormal basis

$$|x_0, x_1, x_2, x_3, \ldots\rangle \quad (x_i = 1 \text{ or } \tau)$$

Hilbert space is spanned by all possible fusion paths.

➢ Example:



$$|\mathbf{1}\tau\mathbf{1}\rangle,\ |\mathbf{1}\tau\tau\rangle,\ |\tau\tau\mathbf{1}\rangle,\ |\tau\mathbf{1}\tau\rangle,\ |\tau\tau\tau\rangle$$

Fusion rules

$$\begin{aligned} 1 \times 1 &= 1 \\ \tau \times 1 &= \tau \\ 1 \times \tau &= \tau \\ \tau \times \tau &= 1 + \tau \end{aligned}$$

Constraint:

$$\cancel{x_i = x_{i+1} = \mathbf{1}}$$

$$\dim \mathcal{H}_N = \mathrm{Fib}_{N+1} \sim \varphi^N$$

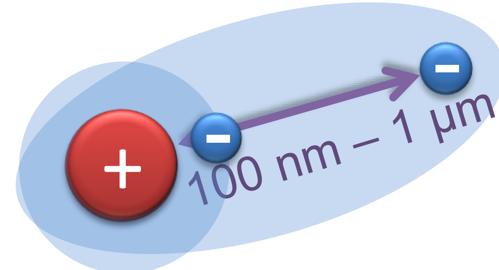
φ : golden ratio

Exclusion constraint → Rydberg blockade!

Rydberg原子によるFibonacci anyonの実現

What is special about Rydberg atoms?

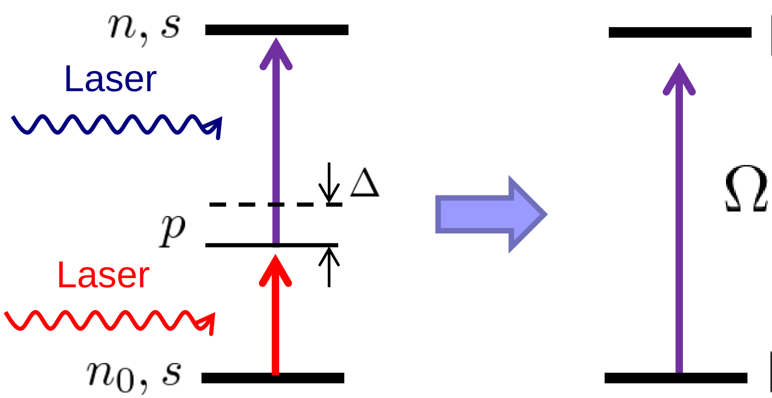
Rydberg atoms are atoms in which one of the electrons is in the excited state with a very *high principal quantum number (n)*.



$$\text{Radius} \sim n^2 a_0$$

$$\text{Life time} \propto n^3 \text{ or } n^5 \qquad \mathbf{80\mu s} \qquad n = 40$$

- **Spin-1/2 representation**



$$|r\rangle := |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|g\rangle := |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Single-atom Hamiltonian

(Rotating-wave approximation)

$$H = \Omega \sigma_x + \Delta n$$

Rabi frequency **Detuning**

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad n = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

- **Strongly exaggerated interaction**

Rydberg atoms in ***s-states*** interact via van-der-Waals type interaction

$$V(\boldsymbol{R}_1 - \boldsymbol{R}_2) = \frac{C_6}{|\boldsymbol{R}_1 - \boldsymbol{R}_2|^6}$$

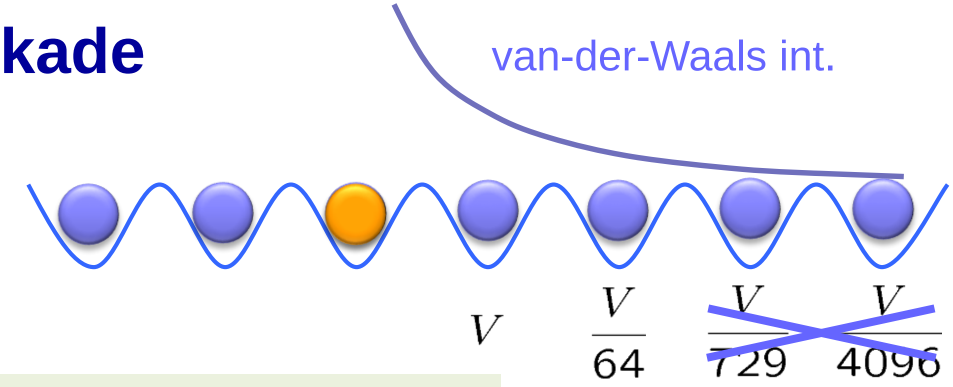
tens of MHz (|***R***|~μm)

$$\boldsymbol{C_6 \propto n^{11}}$$

For typical *n*-values (*n=40-80*), the interaction is **10 orders of magnitude stronger** than that of ground-state atoms.

Rydberg lattice gas & blockade

- Atoms trapped in 1d optical lattice
- Interaction decays quickly (~ r^{-6})
  → Consider only up to n.n.n. interaction



- Hamiltonian

$$H_{\mathrm{Ryd}} = \Omega \sum_{i=1}^{N} \sigma_i^x + \Delta \sum_{i=1}^{N} n_i + V \sum_{i=1}^{N} n_i n_{i+1} + \frac{V}{64} \sum_{i=1}^{N} n_i n_{i+2}$$

Connection to Fibonacci anyons

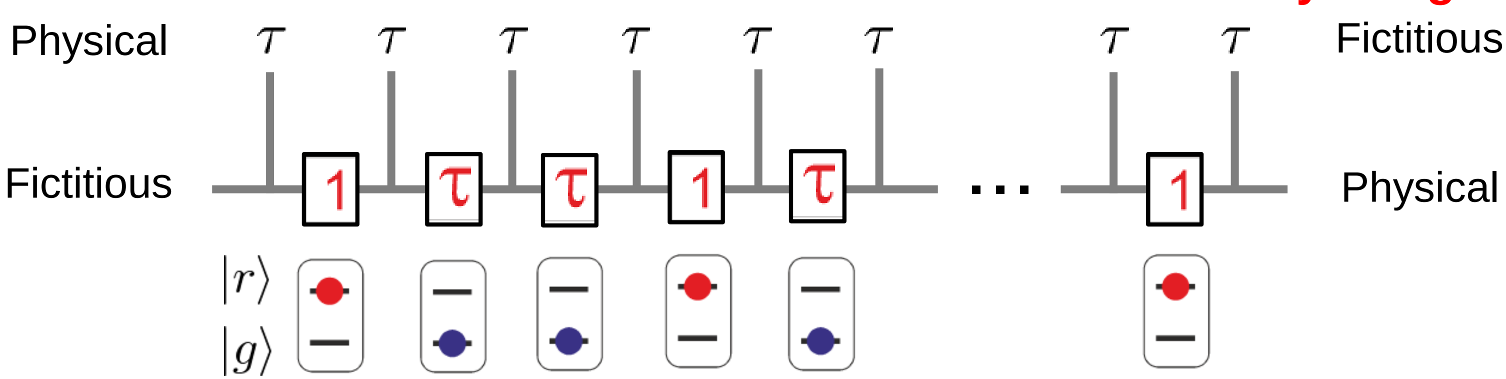
- Hilbert space (with exclusion constraint)
  Fusion rule (reminder): $\tau \times \tau = \mathbf{1} + \tau$

No adjacent 1s

No adjacent Rydberg states

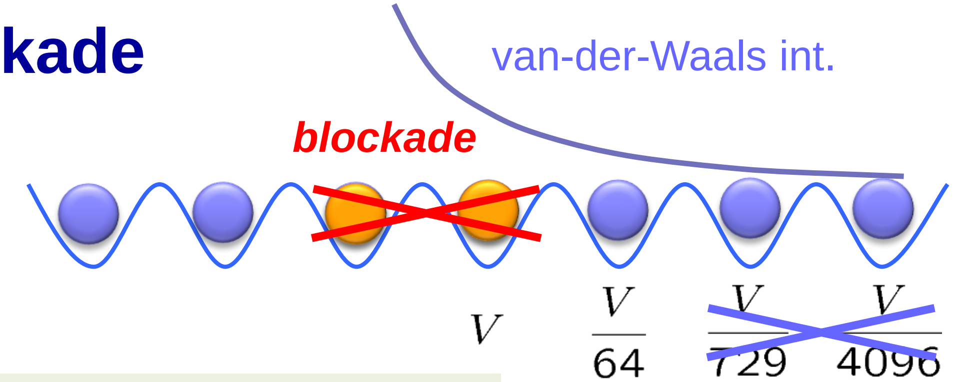
Fibonacci chain

Rydberg atom



Rydberg lattice gas & blockade

- Atoms trapped in 1d optical lattice
- Interaction decays quickly (~ r^{-6})
  → Consider only up to n.n.n. interaction



• Hamiltonian

$$H_{\mathrm{Ryd}} = \Omega \sum_{i=1}^{N} \sigma_i^x + \Delta \sum_{i=1}^{N} n_i + V \sum_{i=1}^{N} n_i n_{i+1} + \frac{V}{64} \sum_{i=1}^{N} n_i n_{i+2}$$

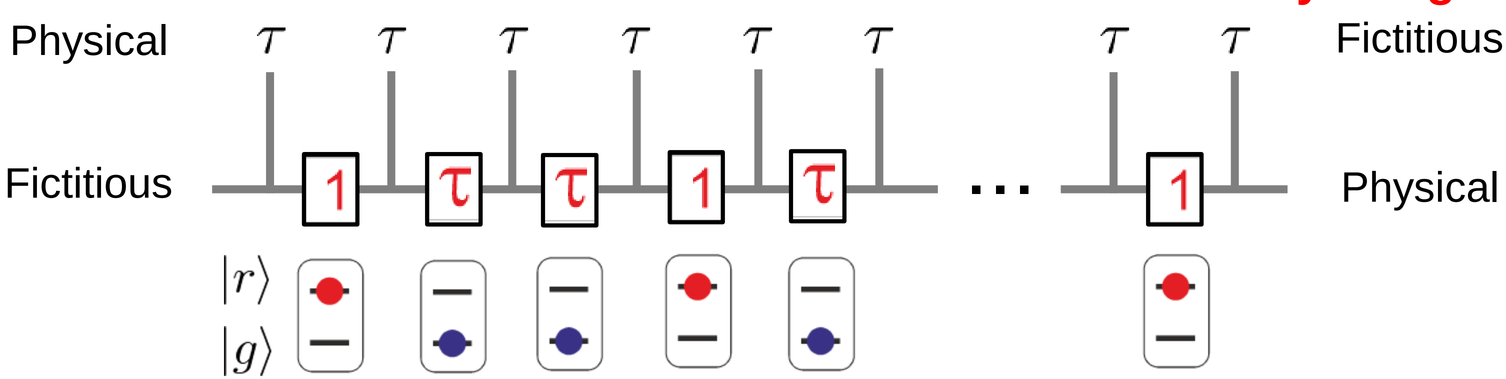
Connection to Fibonacci anyons

• Hilbert space (with exclusion constraint)
Fusion rule (reminder): $\tau \times \tau = \mathbf{1} + \tau$

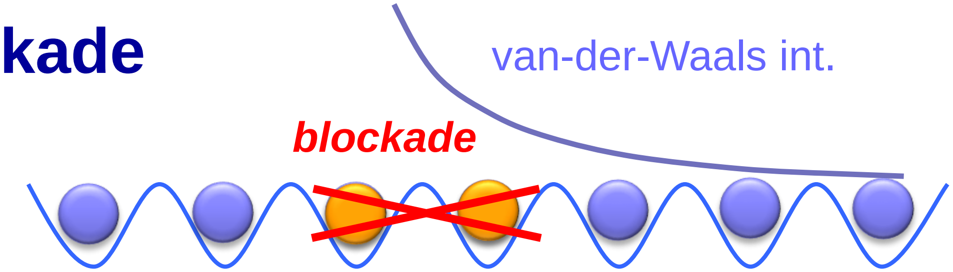
No adjacent 1s
No adjacent Rydberg states

Fibonacci chain

Rydberg atom



Rydberg lattice gas & blockade



- Atoms trapped in 1d optical lattice
- Interaction decays quickly (~ r^{-6})
  → Consider only up to n.n.n. interaction

• Effective Hamiltonian($|V|>>|\Omega|,|\Delta|$): Lesanovsky, *PRL* ('11, '12)

$$H_{\rm eff} = \Omega \sum_{i=1}^{N} P_{i-1}\sigma_i^x P_{i+1} + \Delta \sum_{i=1}^{N} n_i + \frac{V}{64}\sum_{i=1}^{N} n_i n_{i+2}$$

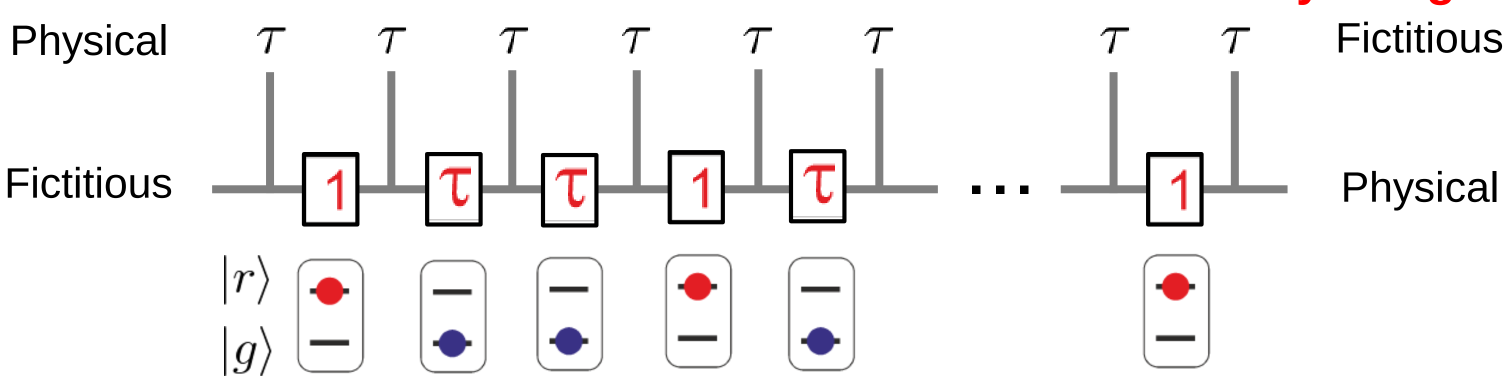
$$P_i = 1 - n_i$$
$$P_i|r\rangle_i = 0,\ \ P_i|g\rangle_i = |g\rangle_i$$

Connection to Fibonacci anyons

• Hilbert space (with exclusion constraint)
Fusion rule (reminder): $\tau \times \tau = \mathbf{1} + \tau$

No adjacent 1s
No adjacent Rydberg states

Fibonacci chain **Rydberg atom**



Practical implementation

$H_{\rm any} = H_{\rm Ryd}$ when $\boxed{\Omega = -J\,\varphi^{-3/2},\ \Delta = -J\,(\varphi^{-2} - 3\varphi^{-1}),\ V = -64 \times J\,\varphi}$

$$\varphi = (1+\sqrt{5})/2$$

- Parameter region

  $|\Omega|, |\Delta| \ll |V|$ $\quad \left|\frac{\Omega}{V}\right| \sim 0.013 \quad \left|\frac{\Delta}{V}\right| \sim 0.023$ ***The blockade can work!***

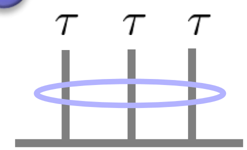
- Sign of Rydberg-Rydberg interaction

  Both positive and negative coeficient ***V*** are available.
  Negative V: R. Low *et al*., *PRL* **106**, 170401 (2011).

- Effect of imperfections

  - Non-perfect blockade 

$$H_2 = -\frac{\Omega^2}{V}\sum_i \left[2n_i - \frac{3}{2}n_i n_{i+2} \right.$$
$$\left. \boxed{P_{i-1}(\sigma_i^+\sigma_{i+1}^- + \sigma_{i+1}^+\sigma_i^-)P_{i+2}} \right]$$

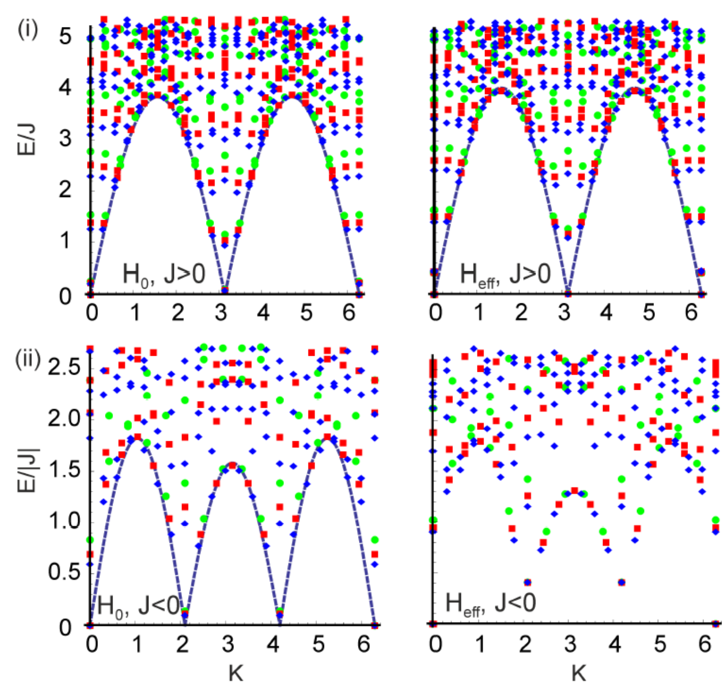


  - Longer-range interactions

$$H_{\rm LR} = V \sum_{|i-j|>2} \frac{n_i n_j}{|i-j|^6}$$

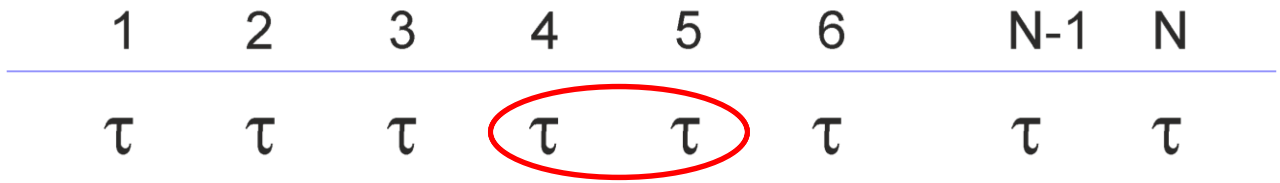
Negative V (AFM: c=7/10): critical g.s. → ***Robust!***
Positive V (FM: c=4/5): critical g.s. → ***Gapped***

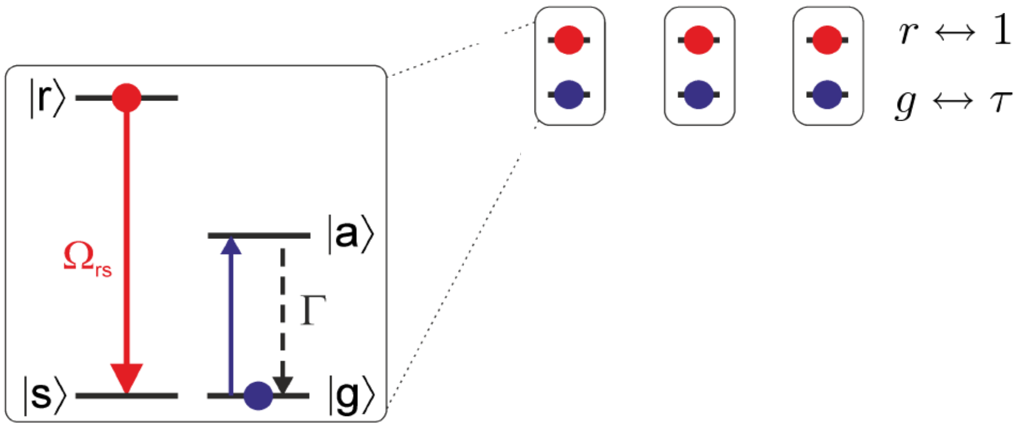
Low-energy spectra

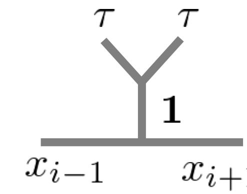


Measurement of anyonic observables

- quantity of interest is e.g. fusion outcome of neighboring anyons







$$|\mathbf{111}\rangle,\ |\mathbf{1}\tau\tau\rangle,\ |\tau\tau\mathbf{1}\rangle,\ |\tau\mathbf{1}\tau\rangle,\ |\tau\tau\tau\rangle$$

$$|\mathbf{1}\tau\mathbf{1}\rangle,\ |\mathbf{1}\tau\tau\rangle,\ |\tau\tau\mathbf{1}\rangle,\ |\tau\mathbf{1}\tau\rangle,\ |\tau\tau\tau\rangle$$

$$\Pi_k = \begin{pmatrix} 1 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & \varphi^{-2} & \varphi^{-3/2} \\ & & & \varphi^{-3/2} & \varphi^{-1} \end{pmatrix}_k$$

$$R_n^\dagger \boldsymbol{n} R_n, \boldsymbol{n} = (1+\sigma^z)/2$$

$$\langle \Pi_k \rangle = 1 \rightarrow 1$$
$$\langle \Pi_k \rangle = 0 \rightarrow \tau$$

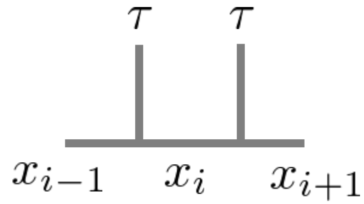
- need to perform projective measurement
- collect three independent single site measurements

Projective measurement of a single atom

E. Urban *et al*., *Nature Phys*. **5**, 110 (2009).

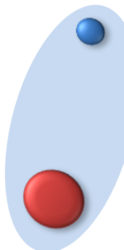
A. Gaetan *et al*., *Nature Phys*. **5**, 115 (2009).

Summary



- **Realization of Fibonacci anyons**

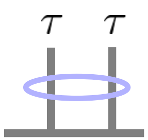
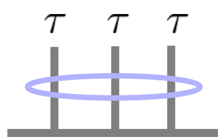
1. Rydberg blockade → restricted Hilbert space
2. Anyonic degrees of freedom are encoded in multiple atoms.
3. Experimental simulation of the anyonic Heisenberg chain.
4. Measurement scheme for fusion outcomes



Rydberg atom system → Platform for the study of interacting anyons

Future directions

- Fibonacci anyononic J1-J2 chain (2-body & 3-body interactions)
  S. Trebst *et al*., *PRL* **101**, 050401 (2008).

$H = \cos\theta$  $+ \sin\theta$ 

→ Stable critical phases

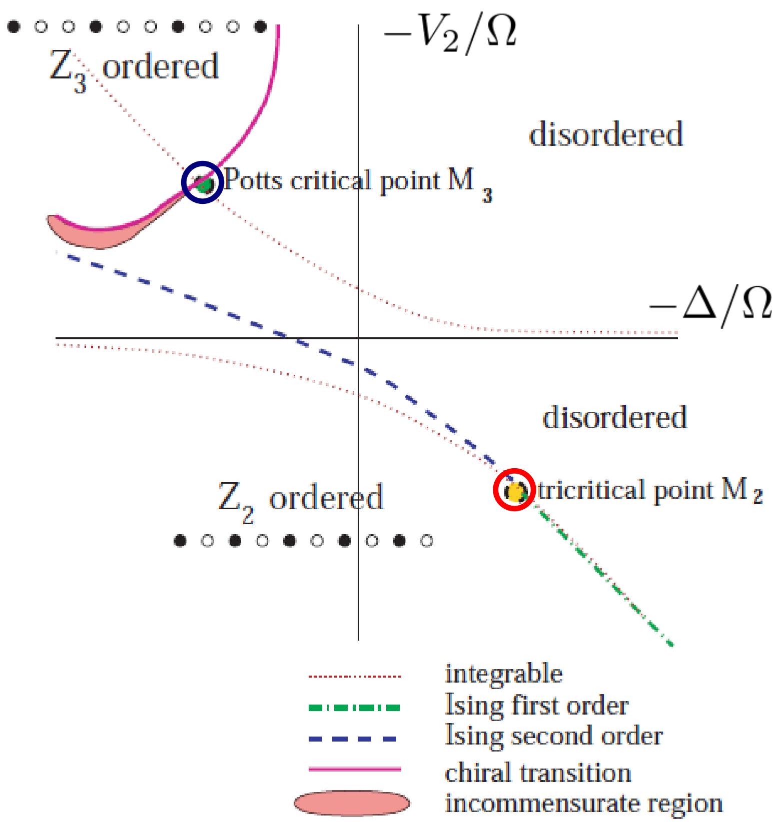
- two-leg ladders, higher-‘spin‘ models, two-dimensional models, ...

- Other class of exotic systems with Rydberg atoms
  Zamolodchikov‘s Ising field thery (E_8 symmetry and boud states)
  cf) CMT realization: $CoNb_2O_6$, Coldea *et al*., *Science* **327**, 177 (2010).

Supplement: ground state phase diagram

$$H = \Omega \sum_{i=1}^{N} P_{i-1}\sigma_i^x P_{i+1} + \Delta \sum_{i=1}^{N} n_i + V_2 \sum_{i=1}^{N} n_i n_{i+2}$$

The same model also describes trapped atoms in a tilted optical lattice.
P. Fendley, K. Sengupta & S. Sachdev, *PRB* **69**, 075106 (2004).



- Rokhsar-Kivelson line

  Igor Lesanovsky, *PRL* **106**, 025301 (2011);
  Igor Lesanovsky, *PRL* **108**, 105301 (2012).

$$\left(\frac{\Omega}{V_2}\right)^2 - \frac{\Delta}{V_2} = 3 \qquad H = \sum_k \mathbf{P}_k^\dagger \mathbf{P}_k$$

$$\mathbf{P}_k = \sqrt{\frac{\Omega}{\xi^{-1}+\xi}} P_{k-1}\left[\sigma_x^k + \xi^{-1} n_k + \xi P_k\right] P_{k+1}$$

- RK ground state

$$|\xi\rangle = \frac{1}{\sqrt{Z_\xi}} \prod_k^L \left(1 - \xi P_{k-1}\sigma_k^+ P_{k+1}\right) |\downarrow\downarrow \ldots \downarrow\rangle$$

- Exact 1st excited state is also obtained