JPS@岩手大学 (2017/9/21)

量子トライマー模型のRVB状態と $\mathbf{Z}_3$ トポロジカル秩序

桂 法称 (東大院理)

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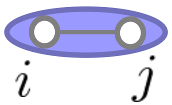
➢ *Phys. Rev. B*, **95**, 060413(R) (2017) [arXiv:1612.06899]

Resonating valence bond (RVB) state

■ What are dimers?



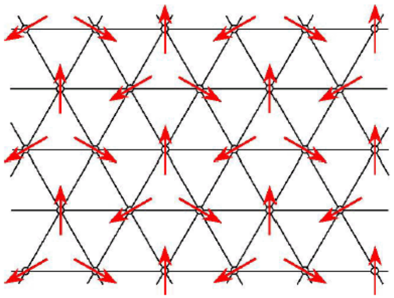
$$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Dimer = spin singlet
= valence bond

■ What is RVB?

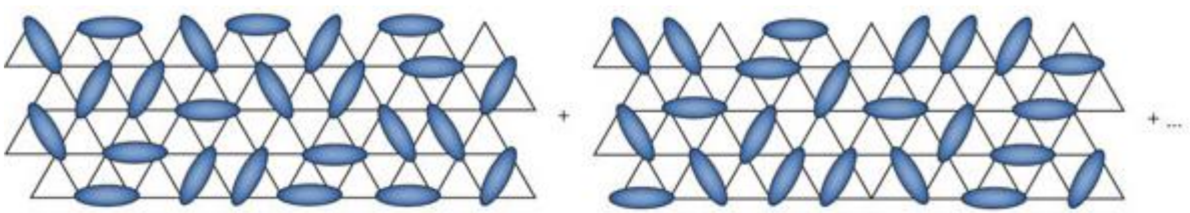
- *S*=1/2 Heisenberg AFM model on △ lattice

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$



Classically, the g.s. exhibits 120° order. What about quantum?

RVB = Equal-weight superposition of all dimer coverings
P.W. Anderson, *Mat. Res. Bull*. **8**, 153 (1973).



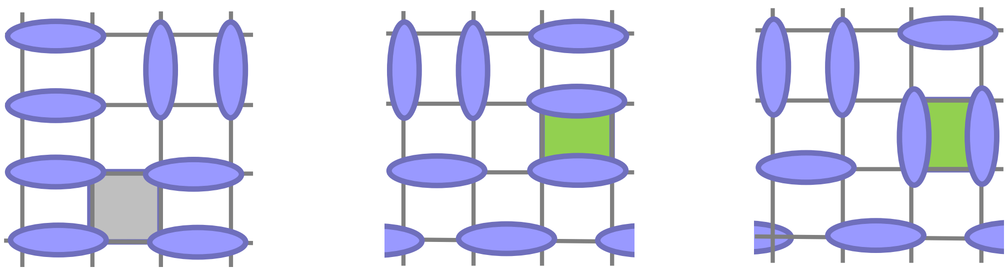
Common belief: This is unlikely. The g.s. has 120° order.

Quantum dimer model

■ Model

Rokhsar-Kivelson, *PRL* **61**, 2376 (1988).

• Basis states: dimer coverings on a square lattice



Dimers don't touch/overlap.
Different configurations are orthogonal.

$$\langle \mathcal{C} | \mathcal{C}' \rangle = \delta_{\mathcal{C},\mathcal{C}'}$$

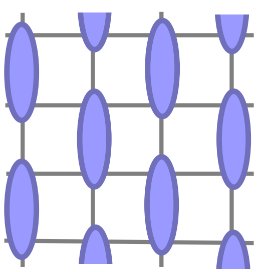
• Hamiltonian

kinetic potential

$$\mathcal{H}_{\rm dimer} = \sum_{\rm plaquettes} \left[-J\left(| \Vert \rangle\langle = | + \mathrm{H.c.} \right) + V\left(| = \rangle\langle = | + | \Vert \rangle\langle \Vert | \right) \right]$$

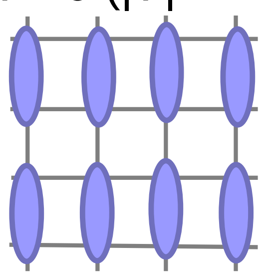
■ Ground state

• $V >> J > 0$



staggered

• $V < 0$ ($|V| >> J$)



columnar

• $V = J$ (RK point) **RVB state!**

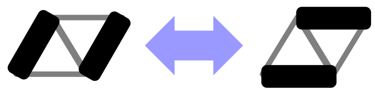
Exact g.s.: $|\Psi_0\rangle = \sum_{\mathcal{C}:\ \text{flippable}} |\mathcal{C}\rangle$

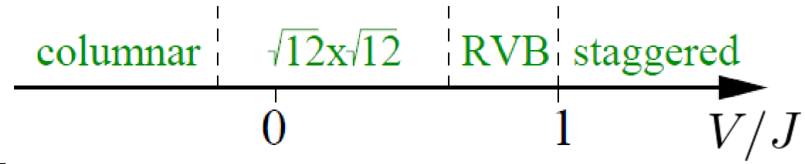
Critical dimer-dimer correlation at RK → Critical (gapless) point

Topological order

The model on a triangular lattice exhibits *topological order!*

Moessner-Sondhi, *PRL* **86** (2001), Ivanov, *PRB* **70** (2004).





■ What is topological order?

- Spectral gap above the g.s.

  gap in RVB phase, vison excitations

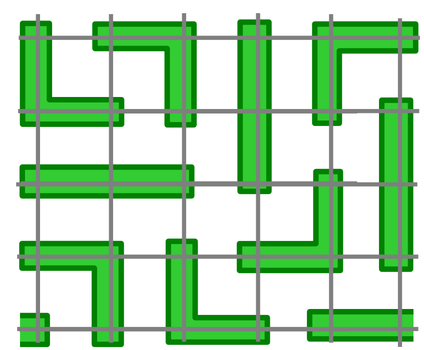
- G.S. degeneracy depends on topology

  4-fold deg. on a torus

- All g.s. are indistinguishable locally

■ Beyond dimers?

- RVB states consisting of trimers
- RK type Hamiltonian and exact ground states
- Topological degeneracy, $\mathbf{Z}_3$ vortex excitations



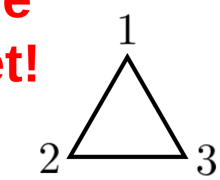
Trimer covering

What are trimers?

Trimer = SU(2) singlet made up of three S=1

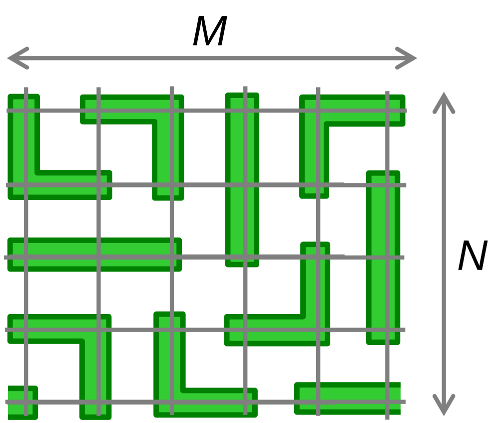
$$\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{3} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{0}$$

Unique singlet!

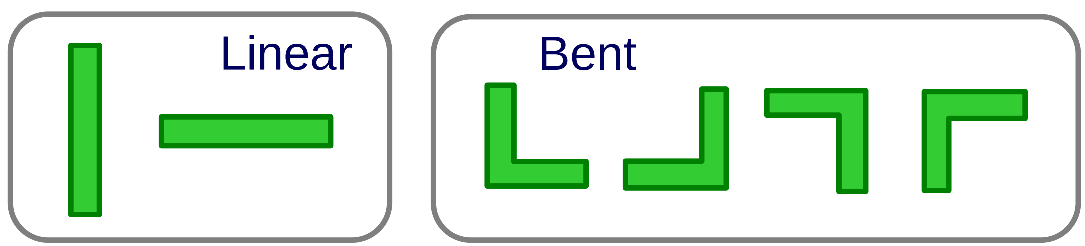


0 is the g.s. of 3-site AFM Heisenberg chain

Trimers on square lattice



- Allowed trimers



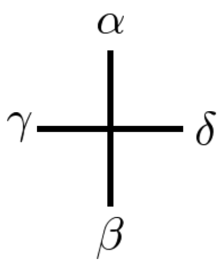
- Rules
  - -Place trimers without making holes
  - -Trimers should not touch or overlap

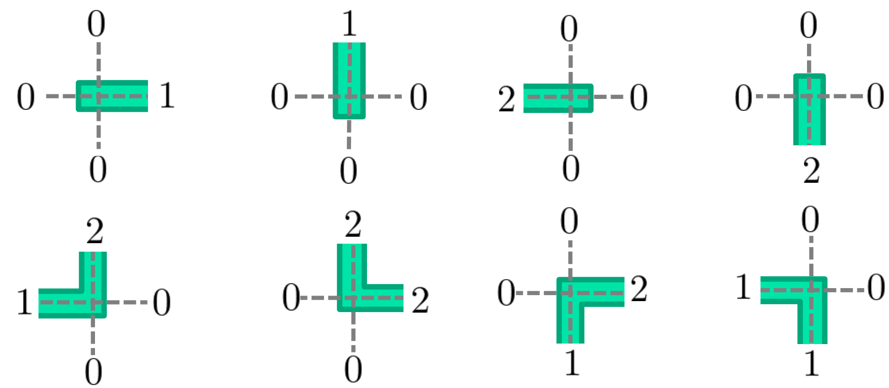
Q. How many ways to arrange trimers?

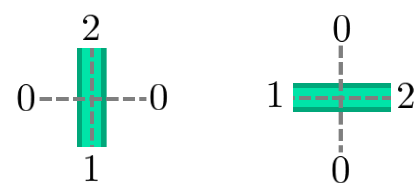
Tensor network approach

■ Local tensor

$$A^{\alpha,\beta}_{\gamma,\delta} =$$





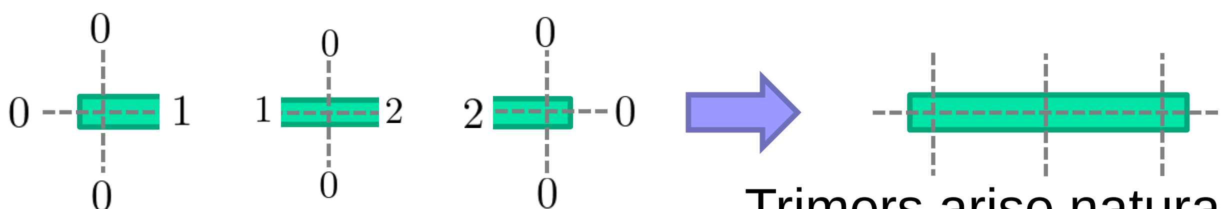


$$A^{0,0}_{0,1} = \cdots = 1$$

α, *β*, *γ*, *δ* = 0, 1, or 2 labels
a state on each ***edge***.
Only 10 nonzero elements.

■ How it works

Dimers never appear!



Trimers arise naturally.

Results

■ Counting

Local tensor → transfer matrix T_M

#(configs.) = $Z = \mathrm{Tr}\,(T_M)^N$

Z	N=1	2	3	4	5	6
M=3	3	33	174	585	2,598	11,550
6	3	297	11,550	54,417	705,708	9,027,000
9	3	2,913	1,094,943	7,111,413	325,897,458	15,280,181,589

Z grows exponentially with system size!

■ Entropy/site

$$s = \frac{1}{MN} \ln Z \sim \frac{1}{M} \ln \lambda_M^{\max}$$

$\lambda_M^{\max}$: Largest eigenvalues of T_M

For large *M* and *N*, we have

$$s = \begin{cases} 0.41194 & (= \ln 1.50974) \\ 0.27693 & (= \ln 1.31907) \quad \text{bent only} \\ 0.15852 & (= \ln 1.17178) \quad \text{linear only} \end{cases}$$

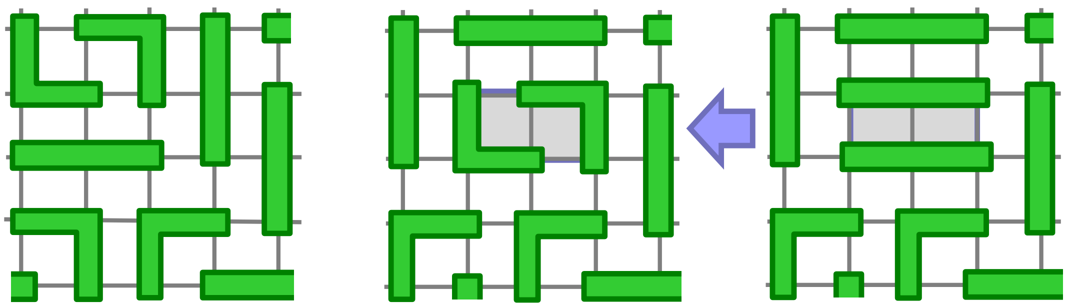
Froboese *et al*, *JPA* **29** (1996)

Ghosh *et al*, *PRE* **75** (2007)

Quantum trimer model (1)

■ Basis states

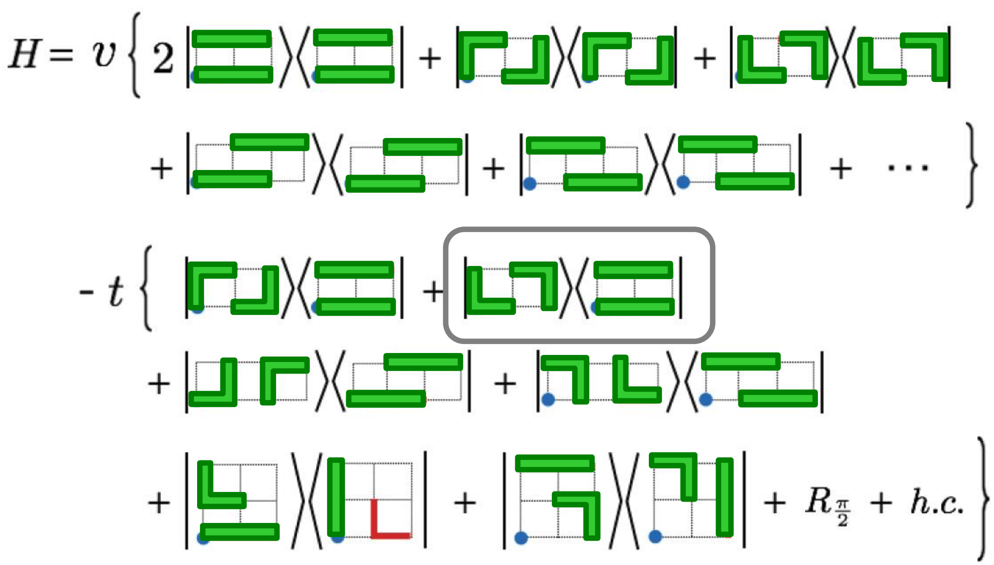
Trimer coverings on a square lattice



Trimers don’t touch/overlap.
Different configurations are orthogonal.

$$\langle \mathcal{T} | \mathcal{T}' \rangle = \delta_{\mathcal{T},\mathcal{T}'}$$

■ Rokhsar-Kivelson Hamiltonian



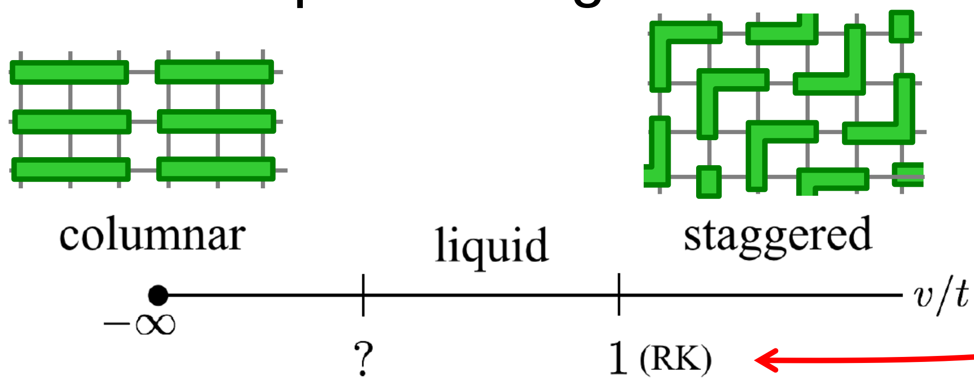
Potential term

Kinetic (resonance) term

Assume t >0.
Resonance involves only two trimers.

Quantum trimer model (2)

- Schematic phase diagram



Exact g.s. at RK point

$$|\mathrm{tRVB}\rangle = \sum_{\mathcal{T}:\,\text{flippable}} |\mathcal{T}\rangle$$

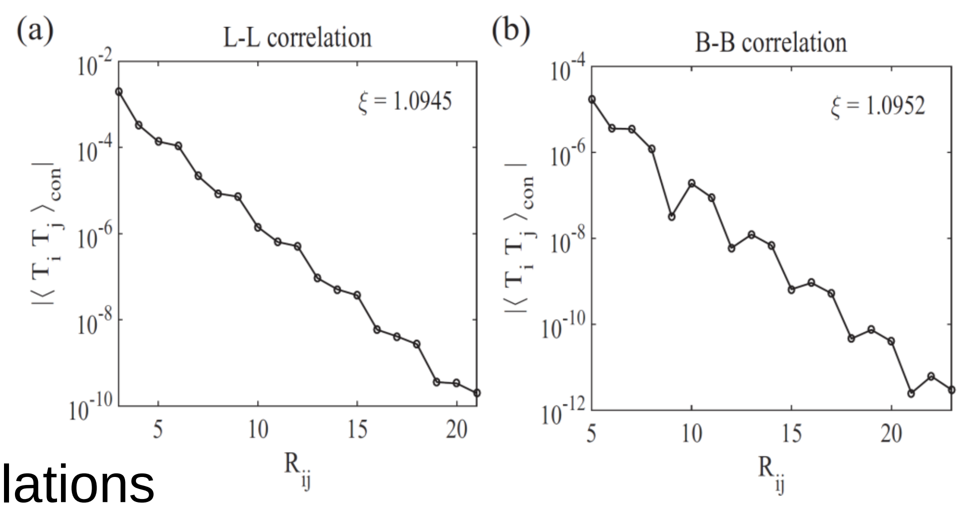
Trimer RVB state!

- Ground-state correlations in tRVB

$$\langle \mathrm{tRVB}\,|T_i T_j|\,\mathrm{tRVB}\rangle \propto \frac{Z'_{ij}}{Z}$$

Z: Total # of trimer configs.

Z'_{ij}: # of configs with fixed trimers at i and j

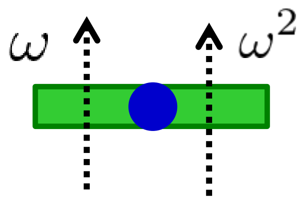


Exponentially decaying correlations
Imply **gapped** nature of the model

$\xi \sim$ lattice spacing

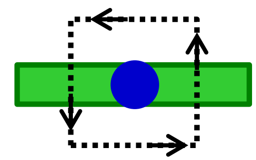
Topological sectors

Z3 flux



ω when ● is seen on the right. ω^2 when it is seen on the left

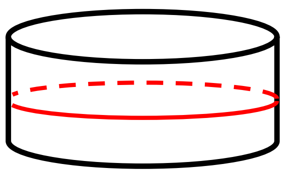
$\omega^2 \times \omega^2 = \omega$

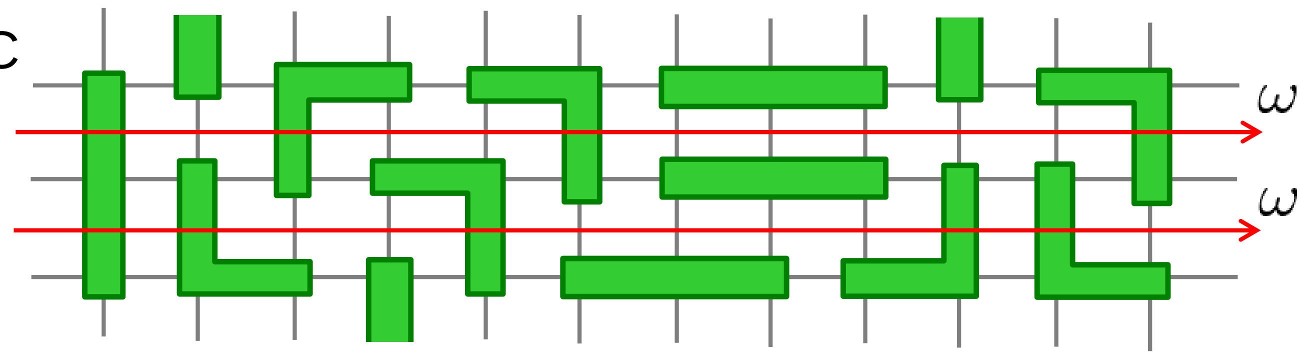


Dual plaquette carries flux ω

■ Winding numbers

Cylinder with PBC



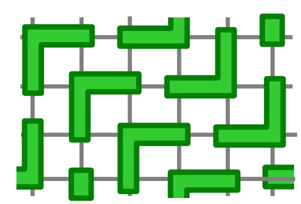


$V_\Gamma := \omega^{n_l - n_r}$ commutes with H. $V_\Gamma = 1$, ω or ω^2. → 3 sectors!

On a torus, we have 3 × 3=9 disconnected sectors.

■ Ergodicity …

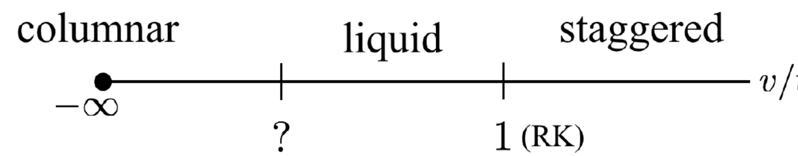
Hamiltonian H is block-diagonal w.r.t. the sectors.
Is the action of H ergodic in each sector? → NO!



Staggered states are frozen…

Z3 topological order

■ Around RK point



At RK, $|\mathrm{tRVB}\rangle = \sum_{\mathcal{T}:\ \mathrm{flippable}} |\mathcal{T}\rangle$ is the exact *E*=0 g.s. in each sector.

Perturb a little bit! $v = t - \epsilon$

Rule out staggered states but still have 9-fold degeneracy.

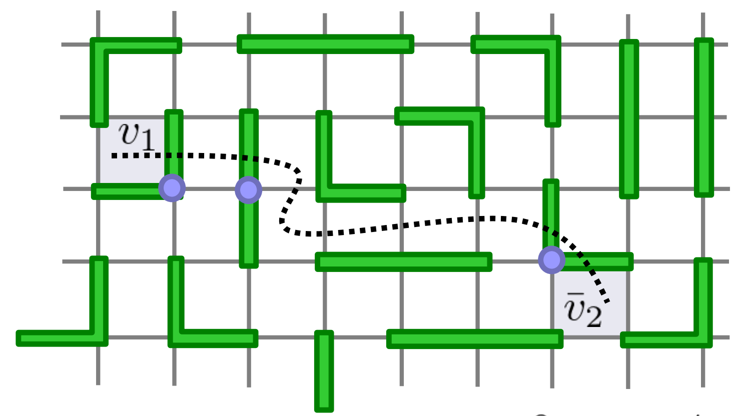
Clear sign of topological order! (NOTE: unique g.s. with OBC)

Higher genus surfaces → 9^g-fold deg.

■ Z3 vortex excitations

- Variational state

$$|v_1, \bar{v}_2\rangle = \sum_{\mathcal{T}} \omega^{n_l(\mathcal{T}) - n_r(\mathcal{T})} |\mathcal{T}\rangle$$

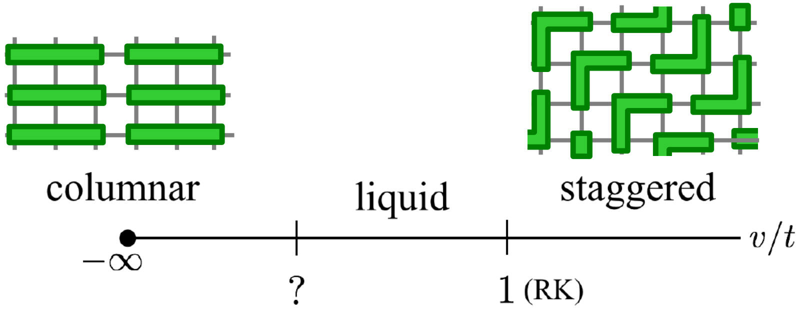


$n_l = 0, n_r = 4$

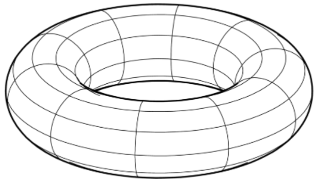
- Orthogonal to the g.s. $|\mathrm{tRVB}\rangle$
- Close to the true excited states?
- Can the pair sprit into fractional excitations?

Summary

- Trimer covering on square lattice (entropy/site s~0.41194)
- Quantum trimers on square lattice
- Exact ground states at RK point
- Short-range correlation in tRVB
- Topological order

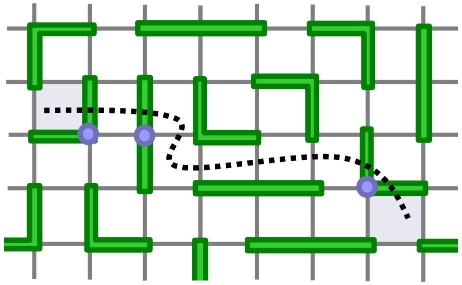


- ☐ Spectral gap above the g.s.
- ☐ 9-fold degeneracy on a torus
- ☐ Z3 vortex excitations?

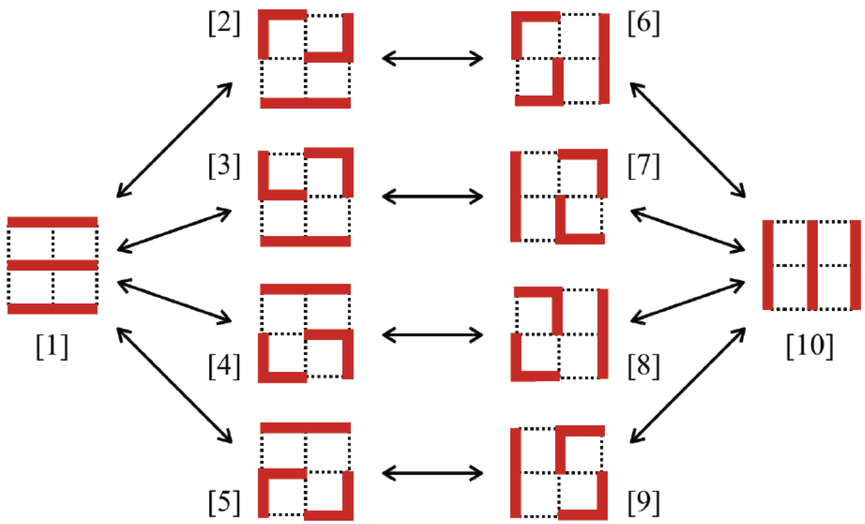


Future directions

- Rigorous proof of exponential decay in tRVB
- Work out phase diagram (ED, QMC, TN, …)
- Precise nature of the gapped excitations
- Connection to Z3 toric code? Topological QC?



Model with OBC

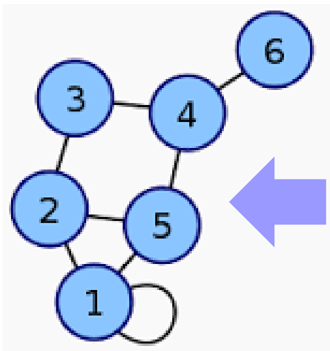


$$H = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

Unique E=0 g.s.
at least in this example

Graph analysis

■ Adjacency matrix/graph



$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

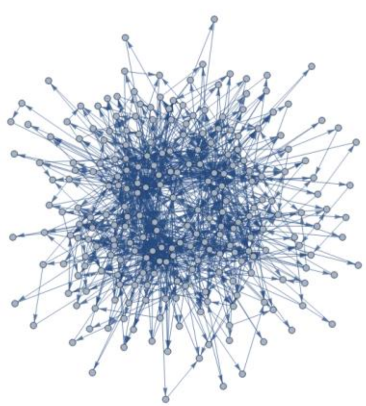
- Graph → Matrix

  $$A = (a_{ij}), \quad a_{ij} = \begin{cases} 1 & i \sim j \\ 0 & \text{otherwise} \end{cases}$$

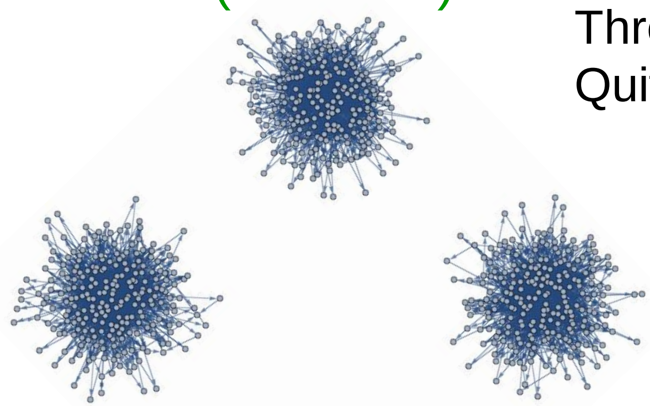
- Matrix → Graph
  Connect vertices *i* and *j* if $a_{ij} \neq 0.$

■ Adjacency graph of transfer matrix

M=5 (3^5 states)



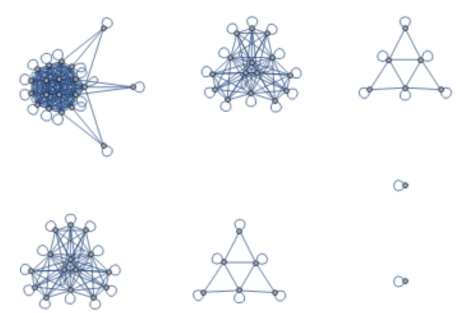
M=6 (3^6 states)



Not sparse. Unlikely to be solvable…

Single cluster for *M*=3*k*+1 or 3*k*+2.
Three disconnected clusters for *M*=3*k*.
Quite different from the dimer case.

Dimer case: *M*=6



General *M* → *M*+1 clusters.