Z_2 invariant for topological magnon insulators

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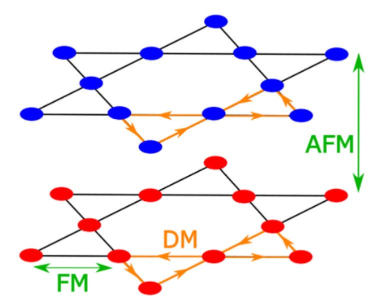


- H. Kondo, Y. Akagi & HK, arXiv:1808.09149

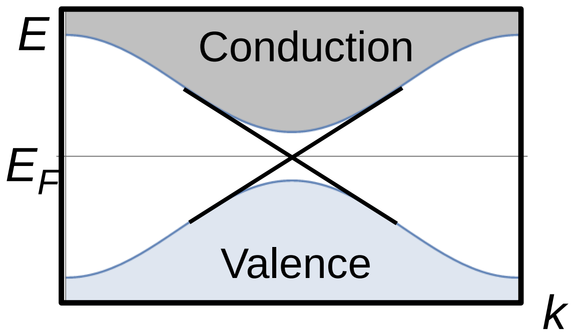
Outline

1. **Introduction & motivation**
   - What are topological insulators (TI)?
   - Disordered TI
   - What about boson?

2. **Magnon Hall effect**
3. **Magnetic analog of 2D class AII Topo. Ins.**
4. **Summary**

What are topological insulators (TI)?

- Band insulators (free-fermions)
- Characterized by **topological invariants**
  TKNN invariant = Chern #
- Robust gapless **edge/surface states**



Reviews and textbooks:
M.Z. Hasan and C.L. Kane, *RMP*. **82**, 3045 (2010).
X-L. Qi and S-C. Zhang, *RMP*. **83**, 1057 (2010), …

■ Examples

- Integer quantum Hall effect ('80s) Magnetic field
  von Klitzing *et al*., TKNN
- **2D** Quantum spin Hall effect Spin-orbit
  Kane-Mele ('05), Molenkamp's group, …
- **3D** TI ($Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3)
  Fu-Kane-Mele, Hasan's group, …

Periodic Table -- Classification of TI & SC --

Symmetries: Time-reversal(T), particle-hole(C) & Chiral(S).

Topo. Numbers: $\mathbf{Z}_2$ = {0,1}, **Z**={0, ±1, ±2, …}, 2**Z**={0, ±2, ±4, …}

Schnyder *et al*., *PRB* **78** (2008); Kitaev, AIP Conf. Proc. **1134** (2008).

Symmetry				Spatial dimension d							
CAZ	TRS	PHS	CHS	1	2	3	4	5	6	7	8
A	0	0	0		$\mathbb{Z}$		$\mathbb{Z}$		$\mathbb{Z}$		$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$				$\mathbb{Z}$		$\mathbb{Z}$	
AI	+1	0	0				$2\,\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	+1	+1	1	$\mathbb{Z}$				$2\,\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$				$2\,\mathbb{Z}$		$\mathbb{Z}_2$
DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$				$2\,\mathbb{Z}$	
AII	−1	0	0		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$				$2\,\mathbb{Z}$
CII	−1	−1	1			$\mathbb{Z}$		$\mathbb{Z}$			
C	0	−1	0		$2\,\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$		
CI	+1	−1	1			$2\,\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	

IQHE

p+ip SC

QSHE

3D TI

Bott Periodicity

Can be refined with additional symmetry, e.g., inversion.

What about bosons?

- Eigenvalue problem
  - Fermionic quadratic form can be diagonalized by **unitary** tr.
  - Bosonic quadratic form can be diagonalized by **para-unitary** tr.
- Fermion v.s. boson in 2D

Fermion (electron)	Boson (magnon)
Chern insulator (A) Hall effect (quantized) Chern (TKNN) number	**Topo. magnon insulator** Thermal Hall effect (not quantized) Chern (TKNN) number
Quantum spin Hall ins. (AII) Spin Hall effect Z2 (Kane-Mele) invariant	*What's the counterpart?* *What's topo. invariant?* *(What's TR symmetry?)*

Outline

1. Introduction & motivation

2. **Magnon Hall effect**

- Chern insulators
- Thermal Hall effect
- Magnon Chern insulators
- Examples

3. Magnetic analog of 2D class AII Topo. Ins.

4. Summary

Chern insulators

■ TKNN formula   Thouless *et al., PRL*, **49** (1982)

- Bloch wave function $\psi_n(\boldsymbol{k})$
- Berry connection $\boldsymbol{A}_n(\boldsymbol{k}) = i\langle\psi_n(\boldsymbol{k}), \nabla_{\boldsymbol{k}}\psi_n(\boldsymbol{k})\rangle$
- Berry curvature $\Omega_n(\boldsymbol{k}) = [\nabla_{\boldsymbol{k}} \times \boldsymbol{A}_n(\boldsymbol{k})]_z$

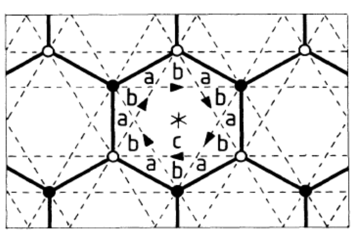
Chern number

$$N = -\sum_{n\in\mathrm{occ.}} \int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}\,\Omega_n(\boldsymbol{k})$$

$$\sigma_{xy} = N\frac{e^2}{h}$$

■ Examples

- Haldane's model (*PRL* **61**, 2015 (1988), Nobel prize 2016)

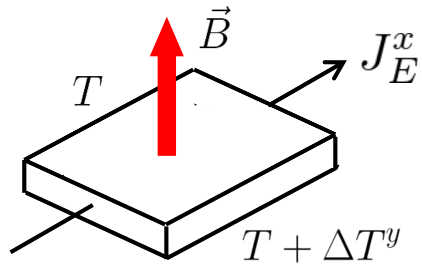


Zero net magnetic field.
But local field breaks TRS!
n.n. real and n.n.n. complex hopping → Integer QHE

- Ohgushi-Murakami-Nagaosa (*PRB* **62** (2000))
  A similar model on kagome derived from double-exchange model

Thermal Hall effect

■ Righi-Leduc effect



Transverse temperature gradient is produced in response to heat current

In itinerant electron systems from Wiedemann-Franz $\kappa_{xy} = LT\sigma_{xy}$

■ TKNN-like formula for bosons

- Bloch w.f. $\psi_n(\boldsymbol{k})$, Berry curvature $\Omega_n(\boldsymbol{k})$
- Earlier work had some flaw…

  H.K., Nagaosa & Lee, *PRL* **104**; Onose *et al*., *Science* **329**, (2010)

- Modified linear-response theory

  Matsumoto & Murakami, *PRL* **106**, 197202; *PRB* **84**, 184406 (2011)

$$\kappa_{xy} = -\frac{k_{\mathrm{B}}^2 T}{\hbar}\sum_n \int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}\, c_2(\rho_n(\boldsymbol{k}))\,\Omega_n(\boldsymbol{k})$$

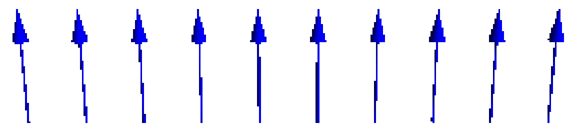
$$c_2(\rho) = (1+\rho)\left(\ln\frac{1+\rho}{\rho}\right)^2 - (\ln\rho)^2 - 2\mathrm{Li}_2(-\rho)$$

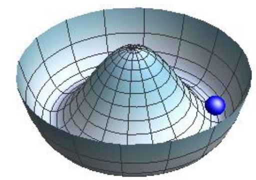
NOTE) No quantization.

What are magnons?

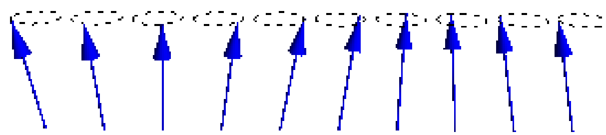
■ Intuitive picture

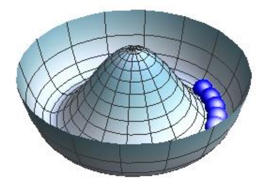
FM GS





Excitation
=NG mode





$E(k) \propto k^2$

■ Holstein-Primakoff transformation

- Bose operators

$$[b_i, b_j^\dagger] = \delta_{ij}, \quad [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0$$

Number op.: $n_i = b_i^\dagger b_i$

- Spins in terms of *b*

$$S_j^+ = \sqrt{2S - n_j}\, b_j, \quad S_j^- = b_j^\dagger \sqrt{2S - n_j}, \quad S_j^z = S - n_j$$

Obey the commutation relations of spins

Often neglect nonlinear terms. $S_j^+ \sim \sqrt{2S}\, b_j, \quad S_j^- \sim \sqrt{2S}\, b_j^\dagger$

- Magnetic ground state = vacuum of bosons $\quad b_j|\mathrm{vac}\rangle = 0$

Magnon Chern insulators

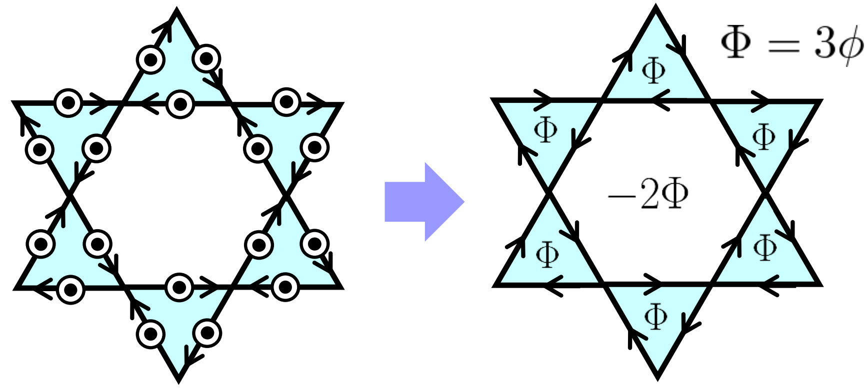
■ Kagome model

$$H = \sum_{\langle i,j \rangle} [-J \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \boldsymbol{D}_{ij} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j)]$$

$$-\tilde{J} S (e^{i\phi} b_i^\dagger b_j + e^{-i\phi} b_j^\dagger b_i) + \cdots$$

$$\tilde{J} = \sqrt{J^2 + D^2}, \quad \tan\phi = D/J$$

DM vectors



Bosonic ver. of Ohgushi's model!

Scalar chirality order there $\boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$ ←→ DM.

Nonzero Berry curvature! κ_{xy} is expected to be nonzero.

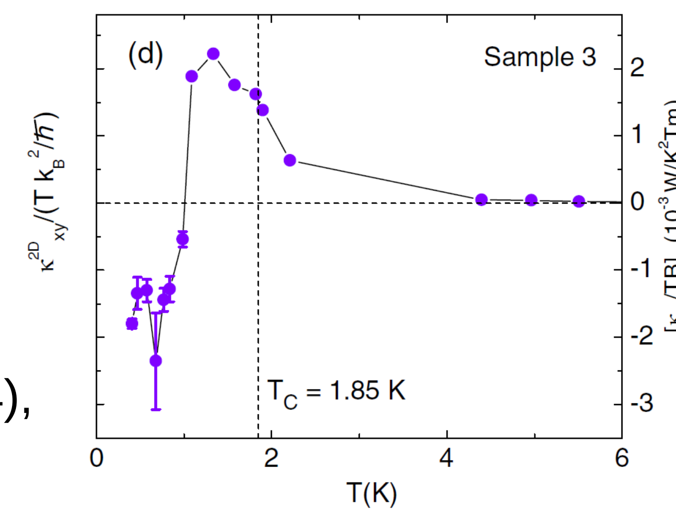
■ MOF material Cu(1-3, bdc)

FM exchange int. b/w Cu^{2+} moments

- Hirschberger *et al.*, *PRL* **115**, 106603 (2015)
- Chisnell et al., *PRL* **115**, 147201 (2015)

Nonzero THE response.

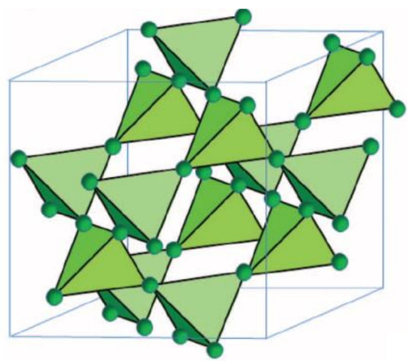
Sign change consistent with theories:
Mook, Heng & Mertig *PRB* **89**, 134409 (2014),
Lee, Han & Lee, *PRB* **91**, 125413 (2015).



Magnon Hall effect in 3D

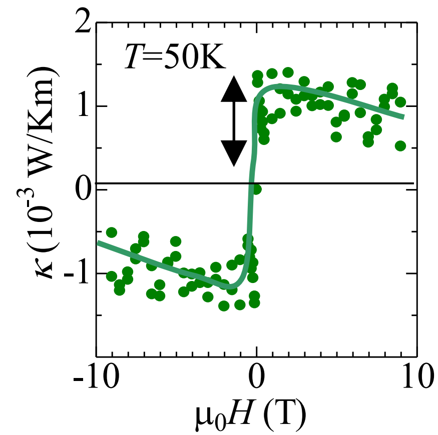
■ Pyrochlore ferromanget $Lu_2V_2O_7$

Onose *et al.*, *Science* **329**, 297 (2010)



● V^{4+}: $(t_{2g})^1$, S=1/2

- Insulator
- Isotropic ferromagnet (T_c=70K)
- $\kappa_{xy} \neq 0$ below T_c
- "Spontaneous" component

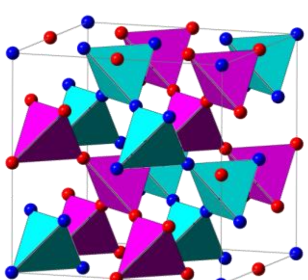


Theoretical model (Heisenberg + DM) well explains the magnon Hall effect!

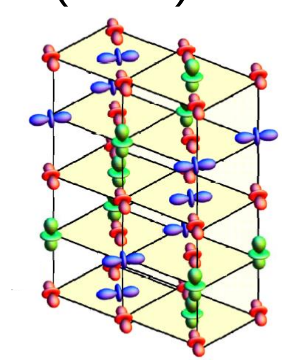
Estimate of FM int.
$|D/J|$ ~ 0.38

■ Other materials

Ideue *et al.*, *PRB* **85**, 134411 (2012)



$Ho_2V_2O_7$
$In_2Mn_2O_7$



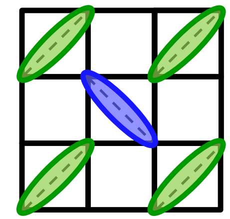
$BiMnO_3$

$\kappa_{xy} = 0$ in $YTiO_3$ and La_2NiMnO_6
consistent with the DM scenario
(Zero Berry curvature)

Topological boson systems

■ Thermal Hall effects of bosonic particles

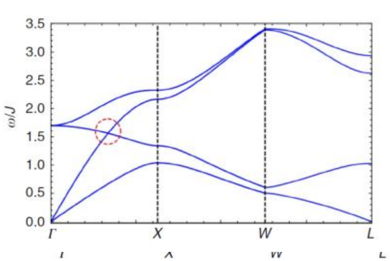
- Phonon:
  (Exp.) Strohm, Rikken, Wyder, *PRL* **95**, 155901 (2005)
  (Theory) Sheng, Sheng, Ting, *PRL* **96**, 155901 (2006)
  Kagan, Maksimov, *PRL* **100**, 145902 (2008)



- Triplon: (Theory) Romhányi, Penc, Ganesh, *Nat. Comm*. **6**, 6805 (2015)
- Photon: (Theory) Ben-Abdallah, *PRL* **116**, 084301 (2016)

■ Topological magnon physics

- Dirac magnon
  Honeycomb: Fransson, Black-Schaffer, Balatsky, *PRB* **94**, 075401 (2016)
- Weyl magnon
  Pyrochlore AFM: F-Y. Li *et al*., *Nat. Comm.* **7**, 12691 (2016)
  Pyrochlore FM: Mook, Henk, Mertig, *PRL* **117**, 157204 (2016)



- Topological insulators
  Nakata, Kim, Klinovaja, Loss, *PRB* **96**, 224414 (2017)

Outline

1. Introduction & motivation
2. Magnon Hall effect
3. **Magnetic analog of 2D class AII Topo. Ins.**

- Kramers pairs and Z2 invariant (electrons)
- Magnon spin Nernst effect
- Kramers pairs and Z2 invariant (magnons)
- Examples, helical edge states

4. Summary

Kramers pairs

■ Time-reversal symmetry (TRS)

- Quadratic form

$H(\boldsymbol{k})$ (in *k*-space)

$$\mathcal{H} = \sum_{\boldsymbol{k}} (\boldsymbol{c}_\uparrow^\dagger, \boldsymbol{c}_\downarrow^\dagger) \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{k}) & h_{\uparrow\downarrow}(\boldsymbol{k}) \\ h_{\uparrow\downarrow}(\boldsymbol{k}) & h_{\downarrow\downarrow}(\boldsymbol{k}) \end{pmatrix} \begin{pmatrix} \boldsymbol{c}_\uparrow \\ \boldsymbol{c}_\downarrow \end{pmatrix}$$

Annihilation op.

$$\boldsymbol{c}_\sigma = [c_{1\sigma}(\boldsymbol{k}), \cdots, c_{N\sigma}(\boldsymbol{k})]^{\mathrm{T}}$$

- (Odd) Time reversal

$$\Theta = UK$$

$$H(-\boldsymbol{k}) = \Theta H(\boldsymbol{k}) \Theta^{-1}$$

Antiunitary! Standard choice: $\Theta = i\sigma_y \otimes 1_N$ with $\Theta^2 = -1$.

- Important property $\langle \Theta\psi, \Theta\varphi \rangle = \langle \varphi, \psi \rangle$

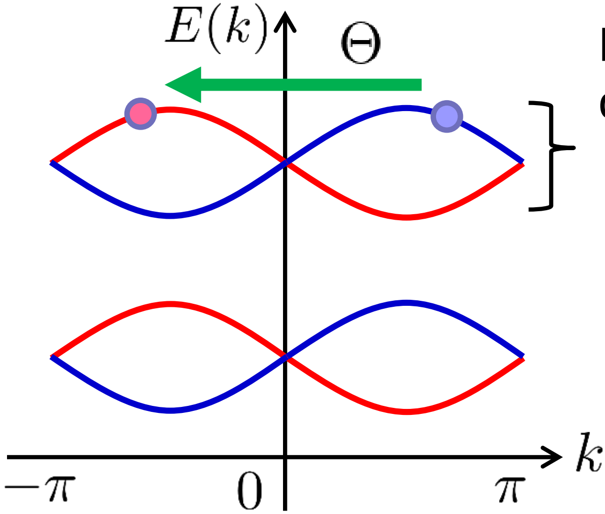
■ Kramers degeneracy

Suppose $\psi(\boldsymbol{k})$ is an eigenstate of $H(\boldsymbol{k})$ with energy $E(\boldsymbol{k})$.
Then, $\varphi(-\boldsymbol{k}) := \Theta\psi(\boldsymbol{k})$ is an eigenstate of $H(-\boldsymbol{k})$ with the same energy.
They form a **Kramers pair**.

At time-reversal invariant momentum (TRIM), $\langle \psi(\boldsymbol{k}), \varphi(\boldsymbol{k}) \rangle = 0$

Z2 topological invariant

- Band structure



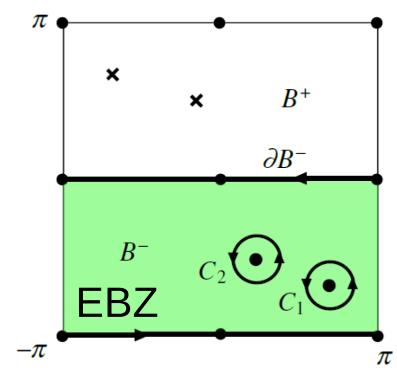
Kramers pair of bands $\psi_{n,\alpha}(\boldsymbol{k})$ *n*: band index
α: label for doublet

- Berry connection

$$\boldsymbol{A}_{n,\alpha}(\boldsymbol{k}) = i\,\langle \psi_{n,\alpha}(\boldsymbol{k}), \nabla_{\boldsymbol{k}} \psi_{n,\alpha}(\boldsymbol{k}) \rangle$$

- Berry curvature

$$\Omega_n(\boldsymbol{k}) = [\nabla_{\boldsymbol{k}} \times \boldsymbol{A}_{n,\alpha}(\boldsymbol{k})]_z$$



■ Fu-Kane formula of Z2

- Z2 index
  Fu & Kane, *PRB* **74**, 195312 (2006).

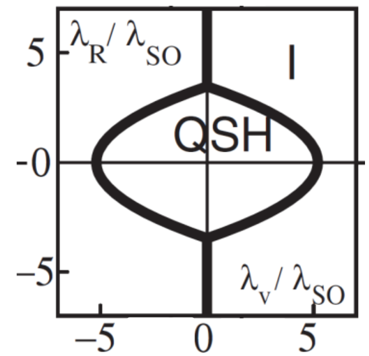
$$D_n = \frac{1}{2\pi} \sum_{\alpha=I,II} \left(\oint_{\partial \mathrm{EBZ}} d\boldsymbol{k} \cdot \boldsymbol{A}_{n,\alpha}(\boldsymbol{k}) - \int_{\mathrm{EBZ}} d^2k\, \Omega_{n,\alpha}(\boldsymbol{k}) \right) \mod 2$$

Kane-Mele model

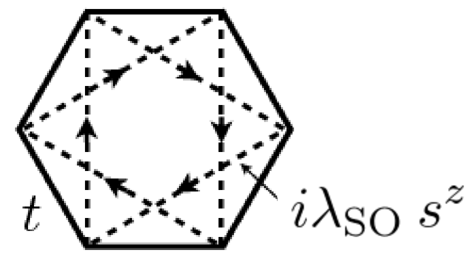
■ Hamiltonian

Kane & Mele, *PRL* **95**, 146802; 226801 (2005)

$$H = t\sum_{\langle ij\rangle} c_i^\dagger c_j + i\lambda_{SO}\sum_{\langle\langle ij\rangle\rangle} \nu_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{\langle ij\rangle} c_i^\dagger (\mathbf{s}\times\hat{\mathbf{d}}_{ij})_z c_j + \lambda_v \sum_i \xi_i c_i^\dagger c_i.$$



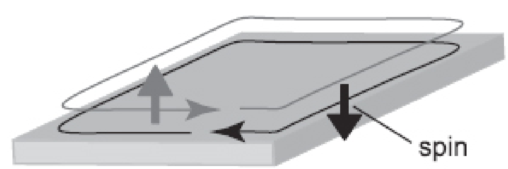
- Two coupled Haldane models
  When λ_R=0, up and down spins are decoupled.
  Two edge states propagate in opposite directions.

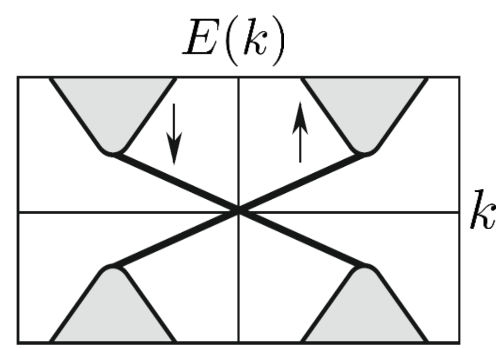


■ Z2 invariant

Each Kramers pair of bands is Z2 nontrivial
→ Helical edge states!

Leading to spin-Hall effect



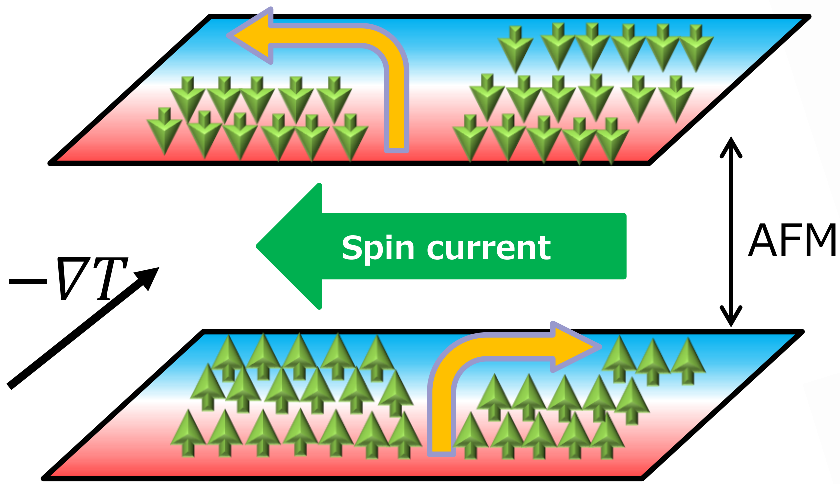


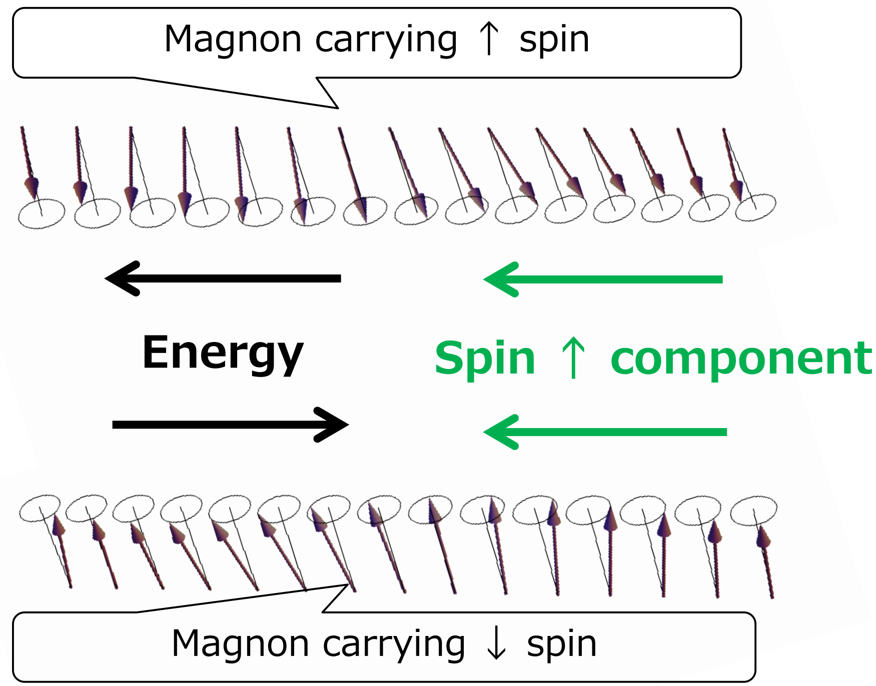
What is magnetic analog?

- Anti-ferromagnetically coupled 2D ferromagnets

Up/down electrons ⟺ magnons at layer 1/2

■ Magnon spin Nernst effect





V.A. Zyuzin *et al.*, *PRL* **117** (‘16).
K. Nakata *et al.*, *PRB* **96** (‘17).

■ Pseudo time-reversal symmetry?

System is invariant under TR PLUS the interchange of the layers.

Bosonic quadratic form

- Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}}[\boldsymbol{\beta}^\dagger(\boldsymbol{k})\boldsymbol{\beta}(-\boldsymbol{k})]H(\boldsymbol{k})\begin{bmatrix}\boldsymbol{\beta}(\boldsymbol{k})\\ \boldsymbol{\beta}^\dagger(-\boldsymbol{k})\end{bmatrix}$$

$\beta(\boldsymbol{k}) = [b_\uparrow(\boldsymbol{k}), b_\downarrow(\boldsymbol{k})]^{\mathrm{T}}$

$b_\sigma(\boldsymbol{k})$: $N-$component vec.

- Para-unitary transformation

See, e.g., Colpa, *Physica* **93A**, 327 (1978).

$$\begin{pmatrix}\gamma(\boldsymbol{k})\\ \gamma^\dagger(-\boldsymbol{k})\end{pmatrix} = \mathcal{T}\begin{pmatrix}\beta(\boldsymbol{k})\\ \beta^\dagger(-\boldsymbol{k})\end{pmatrix}$$

Should leave the boson commutations unchanged.

$$\mathcal{T}\Sigma_z\mathcal{T}^\dagger = \mathcal{T}^\dagger\Sigma_z\mathcal{T} = \Sigma_z \qquad \Sigma_z = \sigma_z\otimes 1_{2N}$$

- Diagonalization

$$\mathcal{T}^\dagger H(\boldsymbol{k})\mathcal{T} = \begin{pmatrix}E(\boldsymbol{k}) & 0\\ 0 & -E(\boldsymbol{k})\end{pmatrix} \qquad E(\boldsymbol{k}) = \mathrm{diag}[E_1(\boldsymbol{k}, \cdots, E_{2N}(\boldsymbol{k})]$$

$E_n(\boldsymbol{k})$ are positive eigenvalues of $\Sigma_z H(\boldsymbol{k})$. Non-Hermitian!

- Modified inner product $\langle\!\langle\psi,\varphi\rangle\!\rangle := \langle\psi,\Sigma_z\varphi\rangle$

Bosonic Kramers pair?

■ Pseudo time-reversal symmetry

- Operation $\Theta' = PK$ $(\Theta')^2 = -1$ *P* :para-unitary
- Ex) $\Theta' = (\overbrace{\sigma_z}^{\text{p-h}} \otimes \overbrace{i\sigma_y}^{\text{up/down}} \otimes 1)\, K$
- Symmetry $\Sigma_z H(-\boldsymbol{k}) = \Theta' \, \Sigma_z H(\boldsymbol{k}) \, (\Theta')^{-1}$
- Property $\langle\langle \Theta'\psi, \Theta'\varphi \rangle\rangle = \langle\langle \varphi, \psi \rangle\rangle$

■ Kramers degeneracy

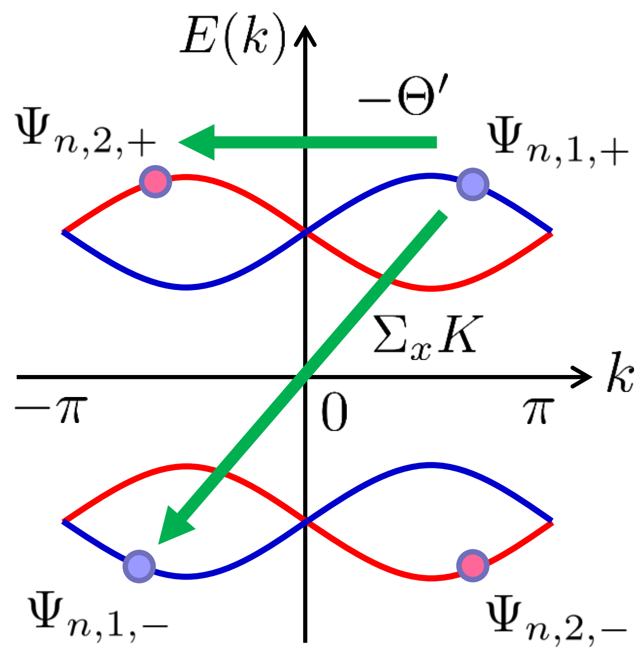
- Kramers pair $\Psi_{n,1,+}(\boldsymbol{k}),\ \Psi_{n,2,+}(-\boldsymbol{k})$

  (para) orthogonal at TRIM

- Particle-hole pair

  $\Psi_{n,1,+}(\boldsymbol{k}),\ \Psi_{n,1,-}(-\boldsymbol{k})$

Pair of particle bands and its hole conjugate have the same Berry connection.



Z2 Topological invariant

■ Fu-Kane like formula

n: band index
α: label for doublet

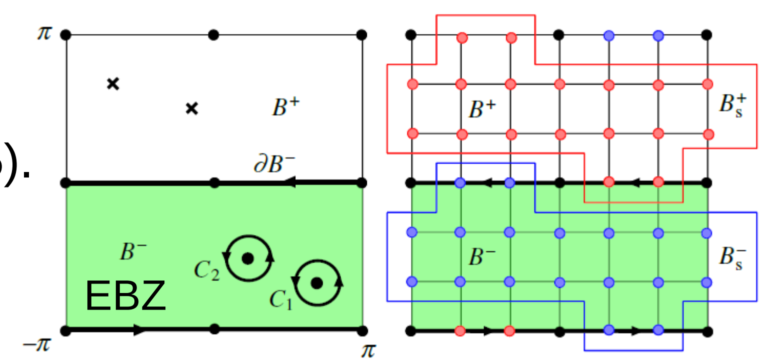
- (Particle) w.f.: $\psi_{n,\alpha}(\boldsymbol{k}) = \Psi_{n,\alpha,+}(\boldsymbol{k})$
- Berry Connection $\boldsymbol{A}_{n,\alpha}(\boldsymbol{k}) = i\,\langle\!\langle \psi_{n,\alpha}(\boldsymbol{k}), \nabla\psi_{n,\alpha}(\boldsymbol{k})\rangle\!\rangle$
- Berry curvature $\Omega_{n,\alpha}(\boldsymbol{k}) = [\nabla_{\boldsymbol{k}} \times \boldsymbol{A}_{n,\alpha}(\boldsymbol{k})]_z$
- Z2 index

$$D_n = \frac{1}{2\pi}\sum_{\alpha=I,II}\left(\oint_{\partial\mathrm{EBZ}} d\boldsymbol{k}\cdot\boldsymbol{A}_{n,\alpha}(\boldsymbol{k}) - \int_{\mathrm{EBZ}} d^2k\,\Omega_{n,\alpha}(\boldsymbol{k})\right) \mod 2$$

Essentially the same as Fu-Kane except for the inner product.

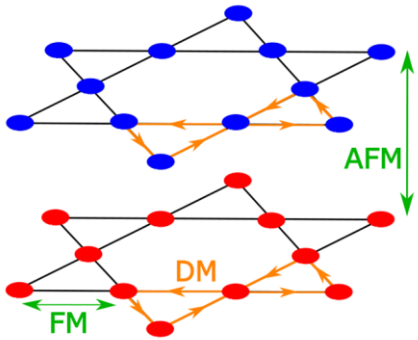
■ Numerical computation

- Lattice Brillouin zone calculation
  Fukui, Hatsugai & Suzuki, *JPSJ* **74** (2005).
  Fukui & Hatsugai, *JPSJ* **76** (2007).
- Quantized even for finite mesh



Examples

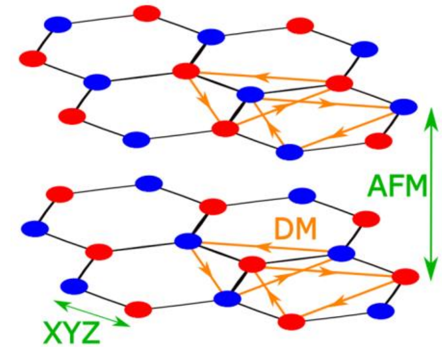
■ Kagome bilayer



●: pointing up
●: pointing down

Intra-layer: FM+DM
Inter-layer: AFM

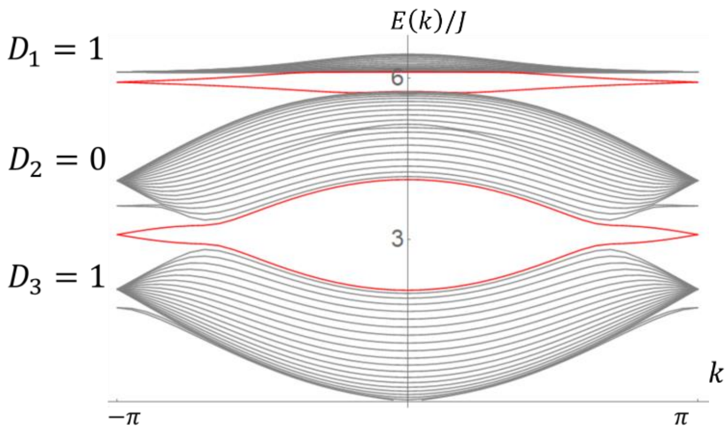
■ Honeycomb bilayer

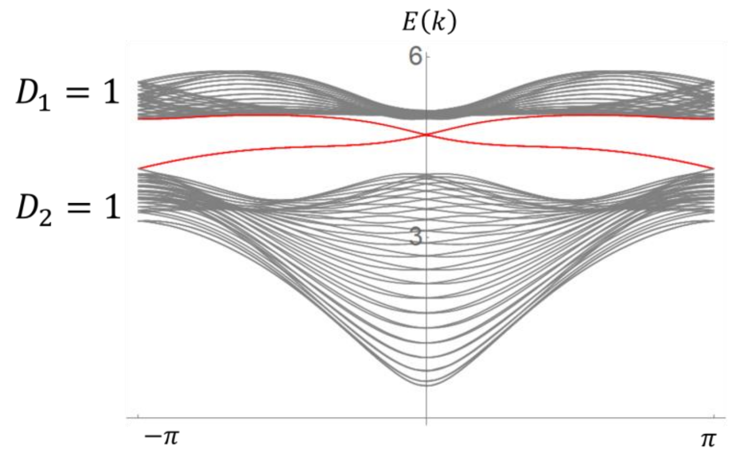


- Pseudo TR op.: $\Theta' = (\overbrace{\sigma_z}^{\text{p-h}} \otimes \overbrace{i\sigma_y}^{\text{up/down}} \otimes 1)\, K$

~ Time-reversal + exchange of layers

■ Magnon edge spectra





Presence/absence of the edge state ←→ Nontrivial/Trivial Z2

Summary

- Reviewed magnon Hall effect
- Proposed magnetic analog of Z2 topo. Ins. in 2D
- Fu-Kane like formula, helical magnon edge states

Fermion (electron) in 2D	Boson (magnon) in 2D
Chern insulator (A) Hall effect (quantized) Chern (TKNN) number	**Topo. magnon insulator** Thermal Hall effect (not quantized) Chern (TKNN) number
Quantum spin Hall ins. (AII) Spin Hall effect Z2 (Kane-Mele) invariant	**Magnon QSH insulator** Magnon spin Nernst effect Z2 invariant

Open questions: How to characterize weak TI, 3D bosonic TI, disorder…