A Contribution to the Theory of Economic Stagnation

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Abstract

This paper provides a theoretical interpretation of the economic stagnation during the pre-industrialized era. I consider an overlapping generations economy with a labor-intensive family production structure and endogenous fertility. In the absence of financial markets and physical capital accumulation, each generation relies on the labor input of the next one for his late life consumption. I consider various degrees of knowledge about the state of the technology and levels of altruism towards future generations. Under complete knowledge about the production function, convergence to suboptimal output per capita and fertility levels is possible. Productivity improvement increases the population size in accordance with Malthus theory but has ambiguous long term effects on the average standard of living. Under incomplete knowledge, I analyze and compare how beliefs about the input-output relationship and degrees of altruism affect the long-run standards of living and fertility.

Keyword: Overlapping Generation, Family production, Altruism

1. A Family Production Structure

Let us consider an infinitely lived overlapping generations economy with a single perishable consumption commodity produced at the family level. Individuals live for three periods but become decision makers at the beginning of the second one. At every time $t \in \mathbb{Z}_+$, the economy is populated by N_t families composed of $1+n_t+n_{t+1}$ member(s): 1 parent or "old" born at time $t-2$, n_t kids or "adult" born at time $t-1$ and n_{t+1} grand-kids or "young" born at time t. The labor input is provided by adult members who inelastically allocates 1 unit of time to production. Therefore, the family output at time $t+1$ denoted by Y_{t+1} can be written as:

$$Y_{t+1} = f\left(n_{t+1}\right)$$

where f stands for the family production function. Individual lifetime preferences are described by a utility function U which depends on first and second-period consumptions: c_1, c_2, respectively.

2. A Non-Altruistic Family

The non-altruistic adult decision problem is:

$$\left(\hat{c}_{1t}, \hat{c}_{2t+1}, \hat{n}_{t+1}\right) = \underset{\left(c_{1t}, c_{2t+1}, n_{t+1}\right) \in \mathbb{R}^3_+}{ArgMax}\ U\left(c_{1t}, c_{2t+1}\right)$$

$$\text{s.t.} \qquad c_{1t} + h\left(n_{t+1}\right) = \frac{Y_t - g\left(Y_t, n_t\right)}{n_t}$$

$$c_{2t+1} = g\left(Y_{t+1}, n_{t+1}\right)$$

$$Y_{t+1} = \psi\left(n_{t+1}, Y_t, n_t\right)$$

with Y_t, n_t given

where h denotes the child bearing cost function, g represents a family allocation scheme and ψ stands an expectation function.

3. An Altruistic Family

The altruistic adult decision problem is:

$$V\left(x_t, Y_t, n_t\right) = \underset{\left(c_{1t}, c_{2t+1}, x_{t+1}, n_{t+1}\right) \in \mathbb{R}^4_+}{Max} \left\{U\left(c_{1t}, c_{2t+1}\right) + \theta V\left(x_{t+1}, Y_{t+1}, n_{t+1}\right)\right\}$$

$$\text{s.t.} \qquad c_{1t} = \frac{x_t}{n_t} - h\left(n_{t+1}\right)$$

$$c_{2t+1} = Y_{t+1} - x_{t+1}$$

$$Y_{t+1} = \psi\left(n_{t+1}, Y_t, n_t\right)$$

with x_t, Y_t, n_t given

where V represents the value function, θ stands for the degree of altruism and x represents the voluntary family bequest.

4. Population and Income Dynamics

The population and income per capita dynamics are derived and analyzed from the above two optimization problems under various degrees of knowledge about the production technology. Convergence conditions to the optimal steady-state maximizing family consumption are also discussed.

References

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