Universality of Tsallis q-exponential of interoccurrence times within the microscopic model of cunning agents

Mateusz Denys[1], Tomasz Gubiec[1], and Ryszard Kutner[1]

[1]Faculty of Physics, University of Warsaw, Hoża 69, PL-00681 Warsaw, Poland
E-mail: mateusz.denys@fuw.edu.pl (correspondence author); tomasz.gubiec@fuw.edu.pl; ryszard.kutner@fuw.edu.pl

Abstract

We proposed the agent-based model of financial markets, where agents are represented by three-state spins located on the plane lattice or social network. The spin variable in this model represents only the advice that each agent gives to his nearest neighbors. In the model the agents can be considered as cunning. That is, although agent, having currently a maximal value of the spin, advises his nearest neighbors to buy some stocks he, perfidiously, will sell some stocks in the next Monte Carlo step or will occupy a neutral position. For the minimal value of the spin situation is analogous (or reverse). The main stylized facts coming from real-life markets, are well reproduced by our model in particular, the very recent ones concerning a universal distribution of interoccurrence times in the form of the Tsallis q-exponential.

Keyword: Agent-based model, Cunning agent, Return, Loss, Interocurrence time, Simulation, Tsallis q-exponential

1 Model of cunning agents

Agent-based modeling of financial markets is a modern trend and challenge. In this work we merge two microscopic, agent-based approaches: (i) the threshold model of the Sieczka-Hołyst type [1] with (ii) the concept of the negotiation round inspired by the Iori model [2]. We consider $N = 1024$ interacting agents on a square lattice of linear size $n = 32$ (where $N = n \times n$) represented by three-state spin variable $s_i = 0, \pm 1$, $i = 1, \ldots, N$. The value of s_i represents only an advice, which i-th agent gives to his four nearest neighbors by the social rule (1). Value $s_i = +1$ means the advice to buy stocks, while $s_i = -1$ to sell them. Value $s_i = 0$ simply means no advice or a neutral advice of the agent.

The single time step t, called a round, consists of N drawings of spin values[1]. After each drawing, the chosen spin is updated according to the social rule,

$$s_i(t) = \mathrm{sgn}_{\lambda|M(t-1)|}\left[\sum_{j=1}^{N} J_{ij} s_j(t) + \epsilon_i(t)\right], \quad (1)$$

where threshold characteristic

$$\mathrm{sgn}_Y(x) = \begin{cases} +1 & \text{if} \quad x \geq Y, \\ 0 & \text{if} \quad -Y \leq x < Y, \\ -1 & \text{if} \quad x < -Y, \end{cases} \quad (2)$$

[1]The single round resembles 1MCS/spin used in the dynamical Monte Carlo methods, while a single drawing is equivalent to 1MCS.

and the constant λ is a positive number. The strength of pair interaction $J_{ij} > 0$ if agent j is one of the four nearest neighbors of the agent i, otherwise $J_{ij} = 0$. The nonvanishing value of J_{ij} is drawn from the range $[J(1-\gamma), J(1+\gamma)]$, where $0 < \gamma < 1$ is an additional parameter of the model. In this way we somehow take into account the varying mutual cogency of agents, extending our recent model [3]. The (local) additive noise term, $\epsilon_i(t)$, represents the temporal, intrinsic random opinion of agent i. In our simulation we used the additive noise distribution in the form of the Weierstrass-Mandelbrot probability density function (cf. Eq. (7) in Ref. [3]). Apparently, we consider both multiplicative and additive noises. It should be added that the change of any spin can affect its neighbors immediately, i.e., within the same round.

The activity of the agent requires two subsequent rounds leading to the change of the spin's value, $d_i(t) = s_i(t) - s_i(t-1)$, i.e. to the change of the agent's opinion during subsequent rounds. For $d_i > 0$ we deal with the agent's demand, while for $d_i < 0$ with the agent's supply. The agent buy stocks if his spin's value increases in the current round in comparison with its value in the previous round. The agent sell stocks if the value of his spin variable decreases.

As usual, the temporal magnetization, $M(t)$, of the network, is defined as a mean of the current spin values. Accordingly, the magnetization represents the aggregated opinion of agents. Apparently, index $\lambda|M(t-1)|$ in the definition of the threshold characteristic (2) is a decisive temporal threshold.

Usually, in agent-based models one considers the formula of a price formation, where logarithmic return is proportional to the excess demand [4]. That is,

$$S_\tau(t) = \ln P(t) - \ln P(t-\tau) \propto ED_\tau(t) \equiv \sum_{i=1}^{N}[s_i(t) - s_i(t-\tau)] = N[M(t) - M(t-\tau)], \quad (3)$$

where τ is a delayed time. Apparently, the excess demand, $ED_\tau(t)$, can change only if the opinion of any agent changes. Hence, price P changes if and only if magnetization M changes, as it is required.

Now, we explain how a possible trapping (equivalent to the vanishing of market liquidity) is avoided in our simulation. By trapping we understand, herein, a fully ferromagnetic state. Then the market has a great chance of being trapped for a long time by this extreme magnetization state. To avoid this trapping effect the system was activated by an exogenous factor, which can play the role of a market maker, performing an abrupt transition of the system to the paramagnetic state[2]. Next, the evolution of the system is continued and the above given analysis of the system is repeated until subsequent abrupt transition.

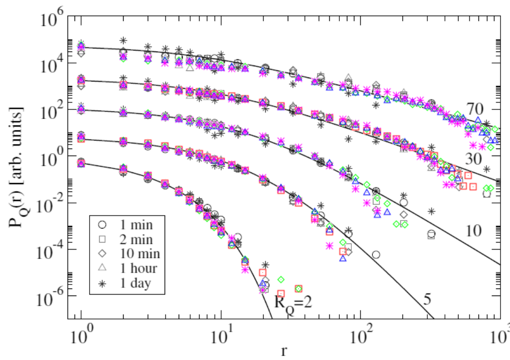

Figure1: Distribution function $P_Q(r)$ vs. interoccurrence time r taken from Ref. [5] for the volatility detrended returns of the NASDAQ from March 16, 2004 to June 5, 2006 for time resolutions (or steps, in minutes) 1, 2, 10 and 60 and additionally for daily returns between 1971 and 2010 (see legend). Data points belong to the mean interoccurrence relative times R_Q = 2; 5; 10; 30 and 70. Solid curves show the q-exponentials given by Eq. (4). Colored symbols present invariant results of our simulations for four different delayed times τ – short, medium, large, and huge values, and for $J = 3$, $\gamma = 0.33$, and $\lambda = 5$. This invariance presents additional aspect of the universality discovered in [5].

2 Empirical q-exponentials

Main stylized facts coming from financial markets are already well reproduced by our model [3]. Furthermore, we show in Fig. 1 a comparison of our model predictions with recently discovered universal distribution of interoccurrence times in the form of the Tsallis q-exponential [5]. It has been shown that for returns, irrespective of the asset class and trading period, the distribution $P_Q(r)$ of the interoccurrence times between losses greater than some fixed negative threshold $-Q$ follows the Tsallis q-exponential form:

$$P_Q(r) \propto \frac{1}{[1+(q-1)\beta r]^{1/(q-1)}}, \quad (4)$$

where parameter q increases logarithmically with mean interoccurence time R_Q as follows $q = 1 + q_0 \ln(R_Q/2)$, with $q_0 \approx 0.168$ [5]. Parameter β decreases only slightly with R_Q and above $R_Q = 6$ reaches a plateau having $\beta \approx 0.23$. One can see that our simulational results well agree with the empirical data – this is the first description of the universality discovered in [5], by the microscopic, agent-based model.

[2]The abrupt transitions observed from time to time are a characteristic feature of modern financial markets.

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