

# Generalized friendship paradox in complex networks

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## Abstract

The friendship paradox states that your friends have on average more friends than you have. Does the paradox “hold” for other individual characteristics like income or happiness? To address this question, we generalize the friendship paradox for arbitrary node attributes in complex networks. By analyzing two coauthorship networks of Physical Review journals and Google Scholar profiles, we find that the generalized friendship paradox (GFP) holds at the individual and network levels for various attributes, including the number of coauthors, the number of citations, and the number of publications. That is, your coauthors have on average more coauthors and more citations than you have. The origin of the GFP at the network level is shown to be rooted in positive correlations between degree and attribute. However, how the GFP holds for individual nodes needs to be understood in more detail. For this, we first analyze a solvable model to characterize the paradox holding probability of nodes for the uncorrelated case. Then we numerically study the correlated model of networks with tunable degree-degree and degree-attribute correlations. In contrast to the network level, we find at the individual level that the relevance of degree-attribute correlation to the paradox holding probability may depend on whether the network is assortative or disassortative. These findings help us to understand the interplay between topological structure and node attributes in complex networks. As a fruitful application of the GFP, we suggest effective and efficient sampling methods for identifying high attribute nodes in large-scale networks.

**Keyword:** complex networks, social networks, friendship paradox

## References

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