Dragon-King dynamics vs. dynamical phase transitions: nucleation, condensation and λ-transition on financial market

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Abstract

We communicate the evidence for the existence of a dynamical phase transition on the financial market represented by the Minimal Spanning Tree (MST) network. The most striking dynamical feature observed is the emergence of a λ-peak. This is reminiscent of a non-equilibrium superstar-like superhub transition with a dragon-king-like dynamics, abruptly accelerating the evolution of the leader vertex representing a company stock in the network. Furthermore, we capture a remarkable phenomenon, where a peripheral company acquires a super-dominant role, bestriding the entire MST network over a period of time of a significant duration compared with the duration of the crash. The empirically observed complexity of the MST network is elucidated by deriving a generic 'macroscopic' dynamic nonlinear equation analytically capturing the dragon-king-like transition.

Keyword: Complex network, Financial market, Phase transition, λ-transition, Dragon-King

1 MST crash dynamics

We merge empirical and phenomenological characterisation of the dynamical phase transitions in complex networks, representing real-life financial markets [1, 2].

Specifically, we comprehensively identify, attain and interpret the empirical, numerical, and analytical evidences for the existence of three distinct dynamical phase transitions, by considering the Frankfurt Stock Exchange – a medium size market well serving as a typical example of a financial market.

By using a canonical tool of the graph theory; the Minimal Spanning Tree (MST), we observe: (i) The (initial) dynamical phase transition from the equilibrium to non-equilibrium nucleation phase of the MST network. Coalescence of edges on the FSE's transient leader (defined by its largest degree) is observed within this nucleation phase, (ii) The acceleration and transformation of the nucleation process into the condensation phase (the second dynamical phase transition). This logarithmically diverging ascending process culminates in the temporal λ-peak of the leader's degree at the second critical (temporal) point, (iii) The descending branch of the λ transition of the third dynamical phase transition. The λ peak logarithmically relaxes due to progressive fragmentation of the MST network.

2 The λ-peak

The λ-peak is a spectacular result of a non-equilibrium dynamical evolution of the complex network of interactions on the financial market. It is reminiscent of the well known λ transition of the specific heat vs. temperature formed for the equilibrium continuous phase transition from the normal fluid I ^{4}He to the superfluid II ^{4}He.

Detailed data and phenomenology driven dynamical considerations enable us to derive a generic nonlinear equation of the dragon-king dynamics [2], which describe the dynamical complexity of the MST network's core by the nonlinear preferential rule of the positive feedback type. In principle, this strategy enables us to study very different kinds of temporal peaks created by the largest node's degree, resulting from a variety of herding effects.

In Fig. 1 we depict λ-peak of the temporal vertex degree difference $k_{SZG}(t) - k_2(t)$. Here, $k_{SZG}(t)$ is the temporal degree of the richest vertex corresponding with SALZGITTER (SZG) AG-Stahl und Technologie which is the leader company during the considered span of time, while $k_2(t)$ is the degree of a vice-leader during that time. This difference plays a role of the short-range order parameter. We show that even if the vertex has initially a low degree, it can evolve towards a superhub size. We attribute this effect to the relative weakness of the correlations between the companies dominating the market activity far ahead and beyond the crash. A low cardinality of such dominating companies further adds to this effect. Hence, the correlations of these few weakly coupled dominant companies with the SZG becames relatively stronger and more significant in the nucleation phase. This effect is detected during the construction of the temporal MST network. We infer that the resulting phenomenologically rich dynamical phase transitions, can be atributed to herding effects, both on ascending – bullish and the descending – bearish, sides. Our analytical insight into the phenomenological results is in excellent agreement with data and can be considered as a significant progress towards understanding non-equilibrium dynamics of superstar-like superhub or a dragon-king [3, 4] phenomena on the complex financial networks.

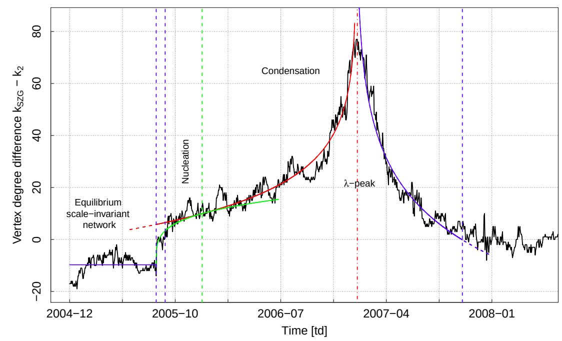

Figure1: Empirical, temporal vertex degree difference $k_{SZG}(t) - k_2(t)$ vs. time t (black jagged solid line). The plot is obtained for one day trading horizon. It forms a λ-peak transition with time playing a role of the control parameter. The red and blue solid lines follow the theoretical models, obtained from the solution of our *constitutive* equation $-A_J \ln\left((t - t_\lambda)/\tau_J\right)$, $J = L, R$. For the properly chosen values of the calibration parameters A_J and τ_J for lhs ($J = L$) and rhs ($J = R$) of the λ-peak excellent consistency with the experimental data is observed. The critical transition time, t_λ, is identical for both ascending and descending sides of the λ-peak. This confirms self-consistency of our model interpretation of the λ-peak as obtained from the empirical data. The vertical red dashed-dotted line is centered at $t_\lambda =$ 544[td]≡2007-01-25 (Thursday) where both of the above mentioned diverging ascending and descending wings of the model meet up forming the dynamic λ-transition. Additionally, the green solid line illustrates the nucleation process. This early stage critical dynamics matches with the condensation phase (red solid line). This is marked by the green dashed vertical line located at Friday 2005-12-12 (255[td]). The horizontal blue straight line at the inception of the plot has a height equal to the average value of daily data for the period from 2004-12-01 (Wednesday) to 2005-08-12 (Friday).

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