Indicators and precursors of transitions in complex systems

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Abstract

We use the Tangled Nature model of co-evolving agents in a high dimensional abstract type space to develop mathematical methods that allow us to monitor and to some extend forecast approaching systems level transitions.

Keyword: Intermittent dynamics, abrupt transitions, forecasting

1 Introduction

Many complex high dimensional systems are characterised by intermittent dynamics, where relatively long quiescent periods are interrupted by sudden and quick bursts of activity during which the system suffers hectic rearrangements. These rearrangements can be seen as a transitions between metastable states. Examples of abrupt transitions have been identified in a broad range of systems [1]: in biological ecosystems transitions from a flourishing to a wild state can occur, in financial markets [2] endogenous crisis can destabilise an existing balance. Due to their widespread occurrence, these transitions have gathered a huge interest in the last decade, with research mainly focused on developing statistical methods to forecast transitions from observed time series and on the development of a general mathematical framework [3]. In present paper we contribute to both efforts by developing a mathematical analysis by use of a paradigmatic model exhibiting intermittent stochastic evolution and by identifying systemic observables that can deliver early-warning of impending transitions. We focus on the Tangled Nature (TaNa) model [4] of evolutionary ecology. The initial aim of the model was to establish a sound and simple mathematically frame work for "punctuated equilibrium", i.e. the observed intermittent mode of macro-evolution.

2 Methodology

The TaNa model is an individual based stochastic model of coevolution.The model's phenomenology is in good agreement with biological observations. At the microscopic level of individuals the dynamics is unfolding at a smooth constant pace: individuals reproduce, mutate and die at essentially constant rates. Whereas at the systemic level the generated ecological network structures jumps from one metastable configuration to another (denoted quasi-Evolutionary Stable Strategies or qESS). We investigate these macroscopic instabilities by performing a Linear Stability Analysis (LSA) of the mean field representation of the dynamics about the actual configurations produced by the full stochastic dynamics. LSA is obviously a standard procedure for analysing the nature of fixed points for deterministic autonomous equations of motion. Here we develop the method to allow applications to high dimensional *stochastic* dynamics.

This work is a collaboration with Duccio Piovani and Andrea Cairoli. For more details see [5]

References

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