The percolation of cooperation: The evolution of collective strategies in communities

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Abstract

The emergence of cooperation in systems of interacting agents is a nontrivial phenomenon that has long intrigued biologists, sociologists and mathematicians alike. Many previous investigations into this phenomenon have utilized the paradigm of the spatial Prisoner's Dilemma game to explore the roles played by network topology and update rule in the evolution of collective strategies. In this context, it has been shown that the size of the largest cooperator cluster in the asymptotic state approaches zero below a critical value of initial fraction of cooperators. In this study, we examine how this percolation threshold is mediated by the choice of update rule. Furthermore, we consider the role of modularity, or community structure, in the sustenance of cooperation, and connect our results with known observations for the behaviour of an Ising spin model over a range of temperatures.

Keyword: Cooperation, Game theory, Prisoner's dilemma, Complex networks, Percolation, Modularity, Ising spin model

Cooperation is one of the fundamental mechanisms underlying the organization of systems as diverse as genomes, multicellular organisms and human societies [1]. The theoretical framework of game theory has been employed to investigate the dynamical evolution of cooperation in social dilemmas [2], microbial populations [3] and, more generally, networks of interactions [4]. Perhaps the best-known paradigm for the emergence of cooperation is the Prisoner's Dilemma (PD) game [5], in which players, or "agents", can either cooperate or defect, and receive payoffs in accordance with the ensuing set of choices. While defection is known to be the dominant strategy in the traditional two-player PD game, it was shown that network reciprocity can lead to the sustenance of cooperation of PD agents in an iterated game [6]. That is, when agents that are spatially arranged on a lattice (or network) play the PD game with all their neighbours, and update their strategy after each iteration through a set of (deterministic or stochastic) rules, it can be seen that the population can sustain some level of cooperation.

A wide variety of update rules have been utilized to study the possible emergence of cooperation in iterated PD games. These include deterministic rules such as *unconditional imitation* [6], in which each agent copies the strategy of the agent in its neighbourhood (including itself) that has received the highest payoff. Several stochastic rules have also been employed, including (i) *proportional imitation* [7] and (ii) the *Fermi rule* [8], in which each agent i randomly picks a neighbour j and copies its strategy with a probability proportional to (i) the difference between their respective payoffs, $(\pi_j - \pi_i)/(T \max(k_i, k_j))$, where k_i and k_j are the respective degrees and T is the temptation of an agent to defect, and (ii) the Fermi distribution function $1/(1 + \exp(-\beta(\pi_j - \pi_i)))$, where β can be thought of as the inverse of temperature. When the iterated PD is played on a network, the game is characterized by T, as well as the initial fraction of cooperators, ρ_0, and its outcome can depend on the connectivity structure of the network, as well as the update rule [9].

When considering a square lattice using an unconditional imitation rule, it was shown that there exists a critical value of ρ_0, corresponding to a percolation threshold, below which the size of the biggest cluster of cooperators in the lattice approaches zero [10]. In this study, we investigate how the choice of update scheme affects the critical initial fraction of cooperators required to produce a giant cooperator cluster in the steady state. We have explicitly verified that a proportional update rule can significantly reduce the minimum initial number of cooperators needed to sustain cooperation. Furthermore, we examine how the evolving dynamical state can be impacted by aspects of the network topology, such as modularity, which characterizes the community structure of the network. The effect of modular structure on the outcome of the PD game has thus far been investigated in the context of unconditional imitation [11], but the role of temperature has not been examined. The implementation of the PD game on a modular network with a Fermi rule has a close analogy to an investigation into the dynamics of the Ising spin model [12]. In such a system, it was observed that the system will be in a state of global order at a sufficiently low value of temperature, while for larger temperatures there exists a critical value of the modularity parameter r below which the system moves into a state of "modular order", and for even larger temperatures the system moves into a state of disorder, regardless of r. We discuss our results in the context of these observations

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