Morse complexes for manifolds with non-empty bondary and A_∞ - structures. Applications to links in S^3.

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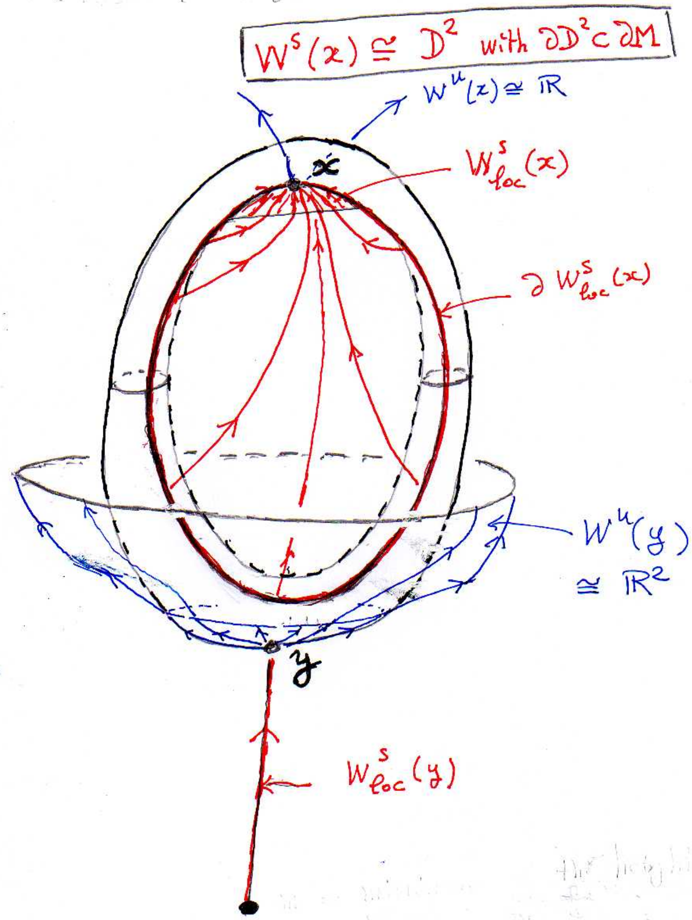
Given a generic Morse function on a manifold with non-empty boundary, two Morse complexes may be defined. The first one yields the absolute homology and the second one yields the homology relative to the boundary. Both of them are endowed with multiplicative structures, A_∞-structures indeed, similarly to a work by Fukaya&Oh for closed manifolds. When it is applied to the complement in the 3-sphere of a tubular neighborhood of a link, equipped with the standard height function, the Massey product is seen with a Morse point of view. In this way, we have a proof "à la Morse" that the Borromean link is not trivial.

Figures below show different configurations of stable manifolds for critical points of Dirichlet type; these points with an adapted pseudo-gradient generate a complex which calculates the relative homology.



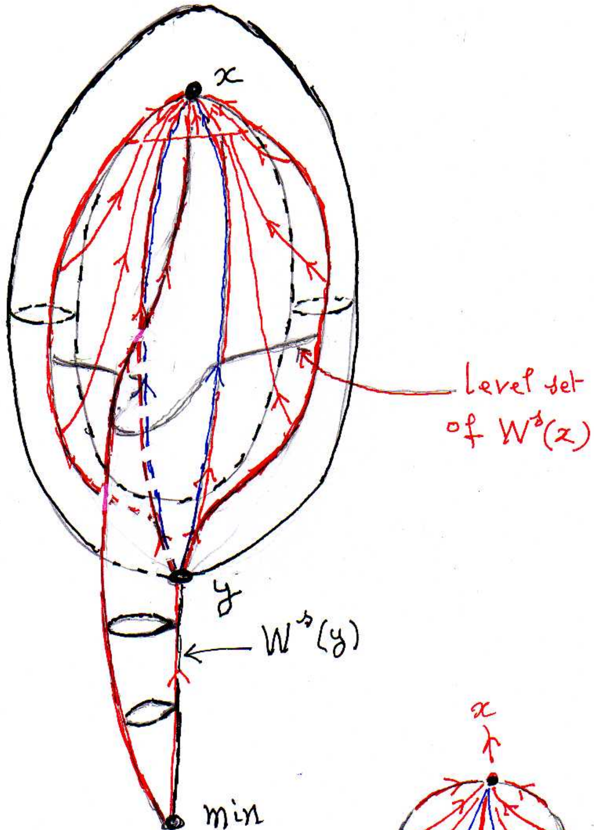
$M = S^3 \setminus$ solid torus

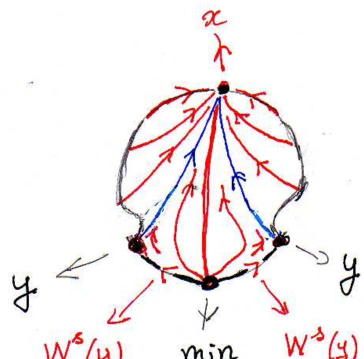
f = standard height function.

$C_3^D = \mathbb{Z}\langle Max \rangle$, $C_2^D = \mathbb{Z}\langle x \rangle$, $C_1^D = \mathbb{Z}\langle y \rangle$

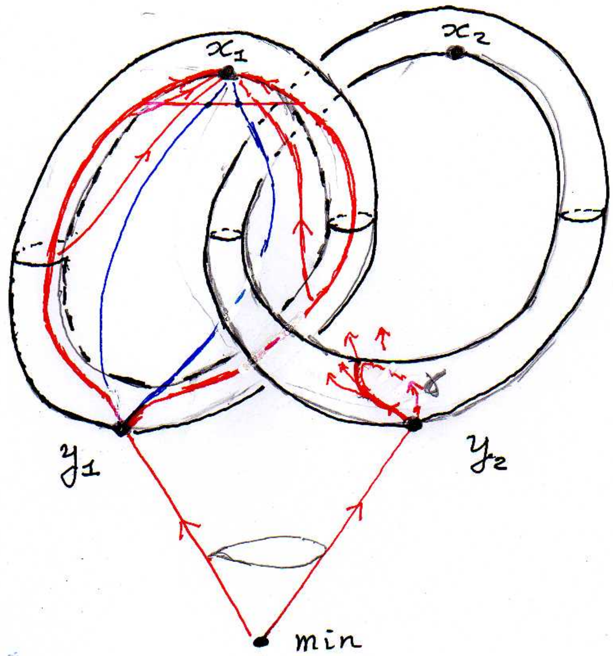
$C_0^D = \mathbb{Z}\langle min \rangle$

$W^u(y) \cap W^s(x) = 2$ lines

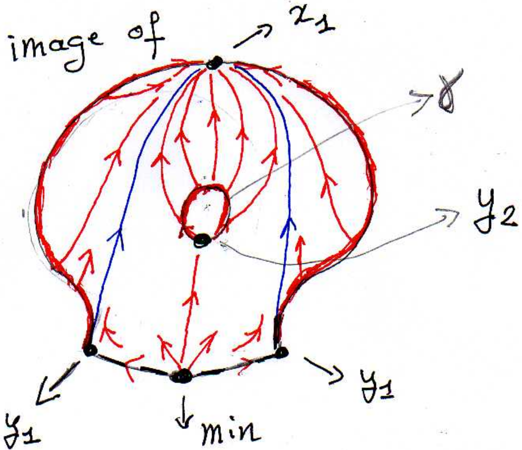


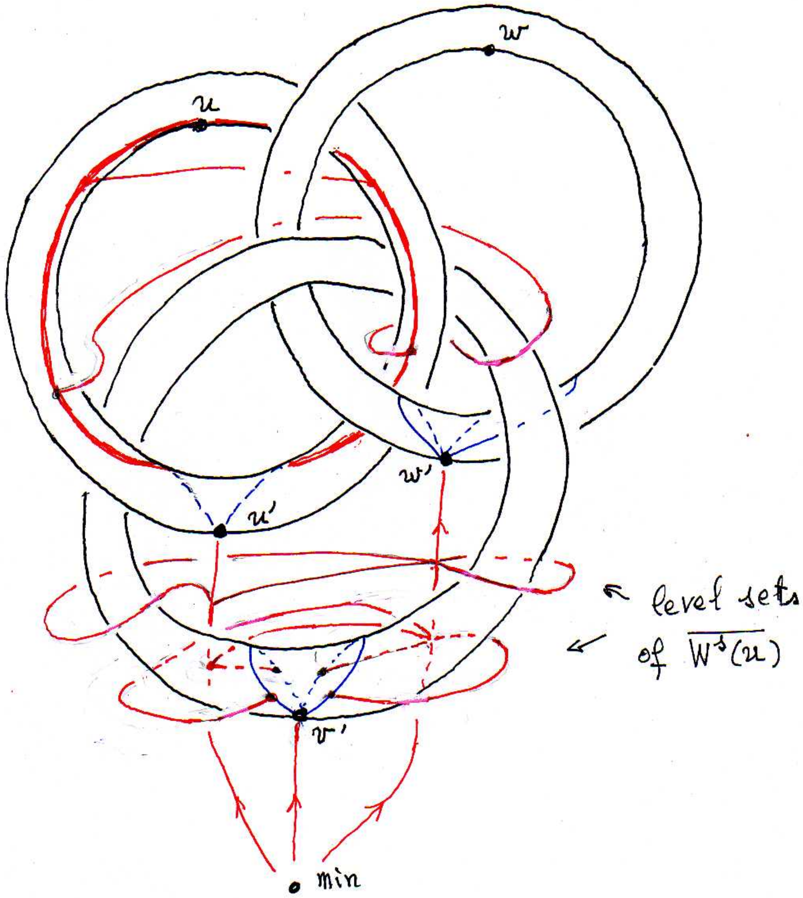


Desingularization of $\overline{W^s(x)}$ ($\cong D^2_+$)

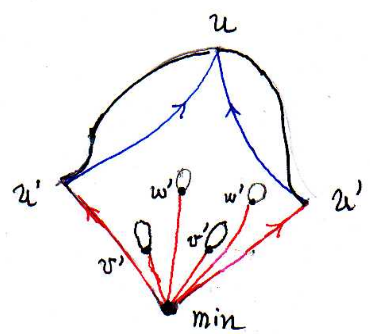


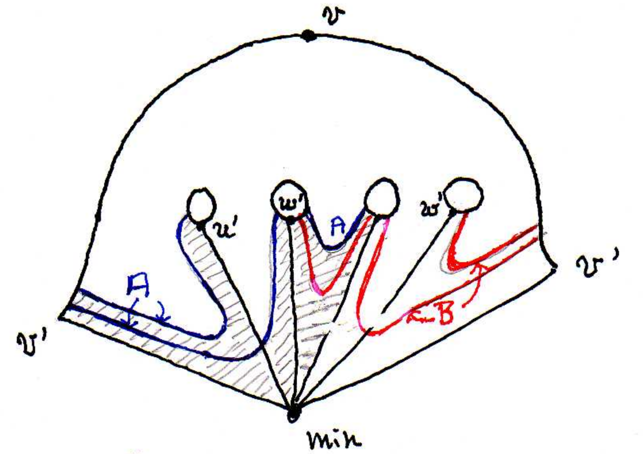
$\overline{W^s(x_1)}$ = image of





Desingularization of $\overline{W^s(u)}$





= $W^s_X(v) \cap W^s_{\tilde{X}}(\tilde{w})$

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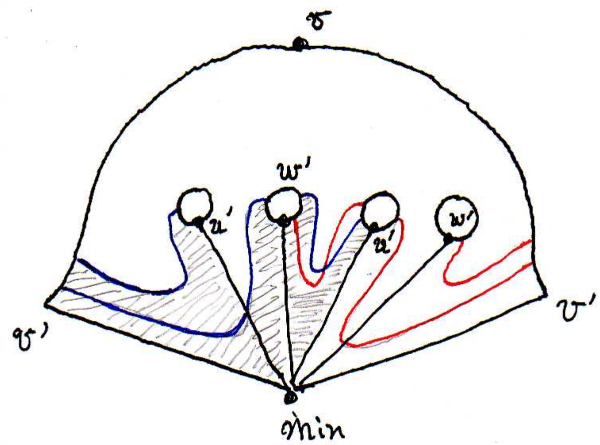
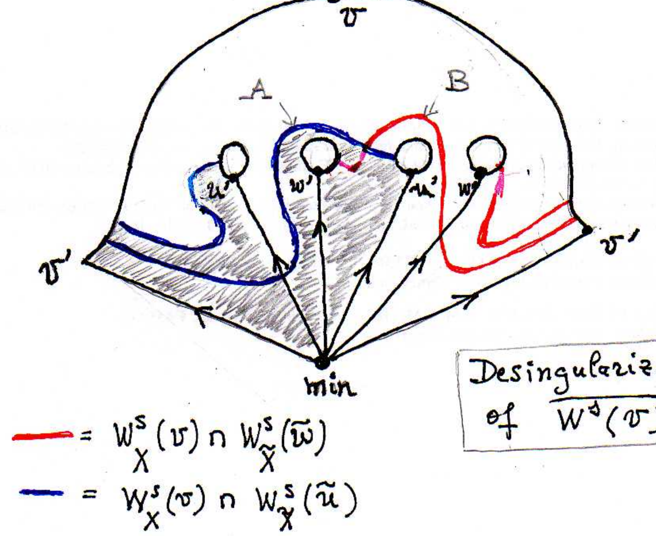
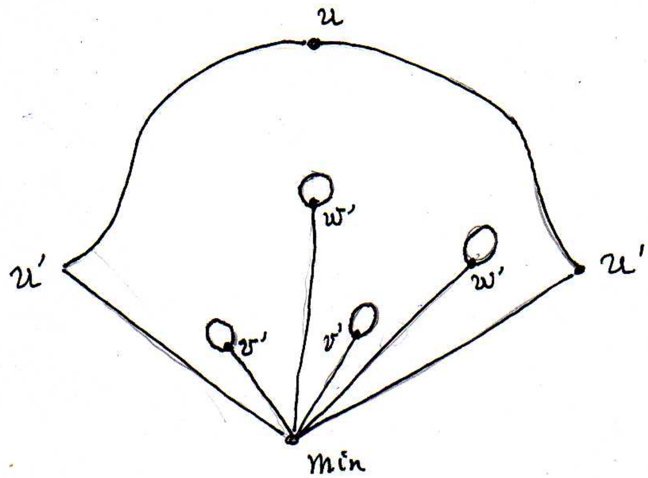


Desingularization of $\overline{W^s(v)}$



Desingularization of $\overline{W^s(v)}$



Desingularization of $\overline{W^s(u)}$