Spatial graph diagrams realizing prescribed subdiagrams partitions

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(OCAMI)

Γ: graph

$f : \Gamma \to \mathbb{R}^3$: embedding

$f(G)$: spatial graph

Especially

$f(G)$: knot if $G \overset{\text{homeo.}}{\sim} S^1$

$f(G)$: link if $G \overset{\text{homeo.}}{\sim} S^1 \sqcup \cdots \sqcup S^1$

G, G' : spatial graphs

$G \overset{\text{equiv.}}{\sim} G' \overset{\text{def.}}{\Leftrightarrow} \exists h : \mathbb{R}^3 \to \mathbb{R}^3$: ori. pre. homeo

s.t. $h(G) = G'$

$H \subset G$: spatial subgraph

D : diagram of G

$D(H)$: the restriction of D to H

$\overset{\text{def.}}{\Longleftrightarrow}$ $D(H)$ is a diagram of H obtained from D by removing edges and vertices not belonging to H

Theorem [Lee - Jin]

L : link

L is partitioned into sublinks $L_1, L_2, \cdots, L_n$

admitting diagrams $D_1, D_2, \cdots, D_n$, resp.

$\Rightarrow {}^{\exists}D$: diagram of L

s.t. $D(L_i) \overset{\text{equiv.}}{\sim} D_i$

$(i = 1, 2, \cdots, n)$

G: spatial graph

$V(G)$: spatial graph consisting of all vertices of G

G_1, G_2: spatial graphs

G_1 and G_2 are <u>edge disjoint</u>

$$\overset{\text{def.}}{\Longleftrightarrow} \quad G_1 \cap G_2 \subset V(G_1) \cap V(G_2)$$

Theorem [S]

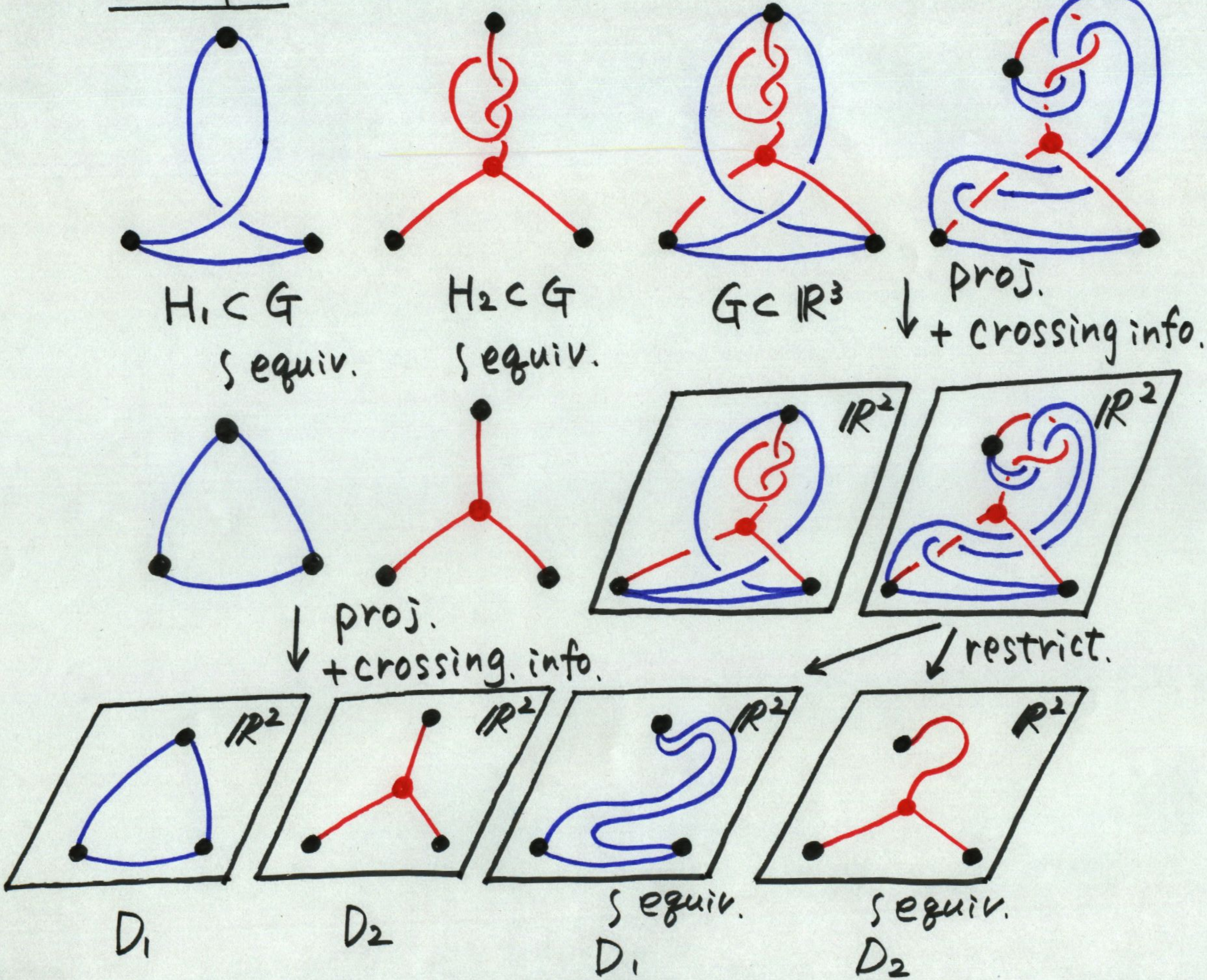
G : spatial graph

G is partitioned into mutually edge disjoint spatial subgraphs $H_1, H_2, \cdots, H_n$ admitting diagrams $D_1, D_2, \cdots, D_n$, resp.

$\Rightarrow {}^{\exists}D$: diagram of G

s.t. $D(H_i) \overset{\text{equiv.}}{\sim} D_i$

Example


$H_1 \subset G$
$H_2 \subset G$
$G \subset \mathbb{R}^3$
proj.
+ crossing info.
equiv.
equiv.
$\mathbb{R}^2$
$\mathbb{R}^2$
proj.
+crossing. info.
restrict.
$\mathbb{R}^2$
$\mathbb{R}^2$
$\mathbb{R}^2$
$\mathbb{R}^2$
D_1
D_2
equiv.
D_1
equiv.
D_2