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Knot-related Patterns in Folk Arts

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Folk patterns often include examples of knots or line/string patterns. The latter patterns are very similar in graphical structures and we were able to analyze and synthesize them as knot patterns. These string and knot patterns have some unsolved problems, different from those of Eulerian cycles, which are now becoming topics in knot/link theory in mathematics.

1: Folk patterns as cycle or knot patterns

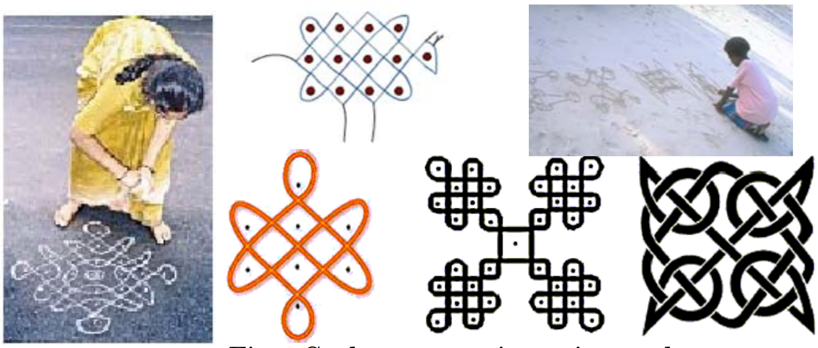


Fig.1 Cycle patterns in various cultures

In folk art patterns there are several kinds of real knots including Asian knots e.g. Chinese knot, Maedeub or Korean knots, Mizuhiki in Japan, and several kinds of string patterns e.g. Celtic knot design in Europe (right pattern in Fig. 1), Kolam ground-painting by Tamil women in South India (left photo, and red pattern: equivalent to the Olzii-Hee knot in Mongolia, Takara-musubi1宝結び in Japan, crest/knot or Panchang-jie盤長 in Chinese knots, the black recursive pattern), Sona sand-painting by Chokwe in Central Africa(top-center pattern), Nitus sand-painting in Vanuatu, South Pacific Oceania (right photo) or some Arabic designs etc. The latter patterns are very similar in graphical structures and we found we were able to analyze and synthesize them as knot patterns.

2: Rules for Kolam or other cycle/knot patterns

1: Create a dot array (an arbitrary array in principle, but usually a regular grid or interlaced grid, in Fig.2) in a pattern of the designer's choice.

2: Make (un-visual) lines (called Navigating/N-lines, shown in black in Fig.2) between connecting dots, or set tiles around each dot as a space-filling polygon (usually square). Tamil women do this process unconsciously in practical drawing. We make it explicit here.

3: Make a crossing (1, in Fig.2) at a middle point on each N-line (or edge of the tiles), or if you choose the third status condition, set an open turning (2) avoiding the N-line around the dot. The second status is used for ***Sikku*** (in Tamil: entangled/linked) Kolam and the third status for ***Neli*** (snaky, squiggle) Kolam or Celtic knots.

4: How to draw the line: Start from a crossing at any edge (usually at a corner), and go strait (smoothly without a sharp angle) at the crossing, and then turn around the dot (0) next to the adjacent crossing (1) or open turning (2). Each turning direction is alternated clockwise/anti-clockwise after the crossing, (and also the side of each up-down crossing is changed alternately for knot patterns). A sample (Fig. 2 left) is drawn on a chain code of "1-0-0-0-1-2-2-0-0-0-2-0-0-2-1-0-0-1-0-0-0-1-1-0-1" from the left edge of the red-dot-tile. Another sample (right) shows that the ivy of the line twines round the tree-branch of the N-line and twists at the crossings, and then untwists and untwines to be minimized.

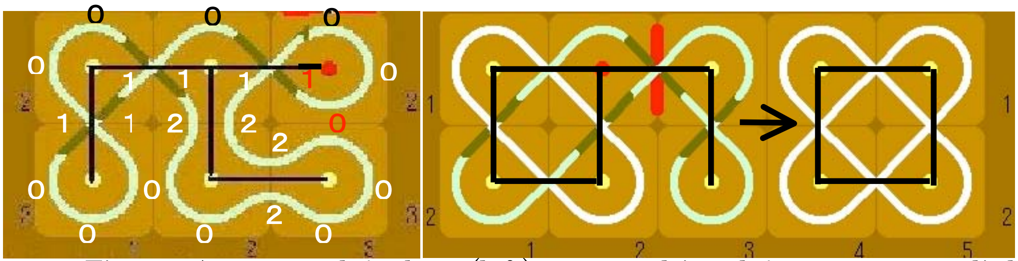


Fig.2 A mono-cycle/unknot (left), a multi-cycle/two component link (right)

3: Characteristics of the patterns

The line goes back to the beginning line as a unicursal cycle and then the pattern consists of a single or some combined cycles), encircling all dot once (in a case except 2-status). In the original Kolam and other sand-painting patterns, a crossing is not represented as an up-down crossing, as they overwrite it in time course, so they are not knot/link patterns. Our drawing rules, however, make an alternating crossing knot/link pattern. Sample Kolam patterns corresponding to Knot designs are shown in Fig.3.

4: Problems: What conditions required for making a single cycle/knot pattern, and how to find the number of cycles/components of a given cycle pattern/links.

Answer 1: a pattern, the N-line of which is only an open/tree structure, consists of a single cycle/knot, and un-twining eventually reduces it to a cycle/unknot (SC, Fig.2-left).

2: a pattern, the N-line of which is closed in a circuit and has odd crossings, consists of a single cycle/knot (SC, Fig.3-1,-3), but having even crossings, it consists of two-cycle/component links (Fig.2-right, Fig.3-10,-9,-12).

3: a pattern, the N-line structure of which is coupled with some closed circuits is not defined simply. In the case of a rectangle dot array of NxM, where all dots are connected with a crossing on the N-line, the pattern consists of cycle(s)/components of the number GCD(N,M), e.g. GCD(2,3) (Fig.3-5)= GCD(3,5)=1, GCD(3,3)=3, etc.

5: Conclusions

We discussed folk art designs consisting of single or multiple cycle patterns and showed how to analyze and synthesize them as knot patterns. To the question of how we find the number of cycles in a given pattern, or how to draw such patterns consisting of a single cycle pattern in a given dot array, we have some answers. We don't yet, however, have general answers to the full range of possible/open problems for folk art designs.

References

FORMA Vol.22-1, Commemorative Issue of the Conference ISKAF06: The Beauty, Dynamics and

Design of String Patterns in Folk Arts, edited by S. Nagata, pp. 1-140, 2007

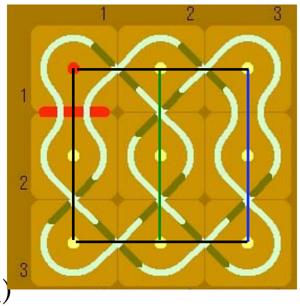
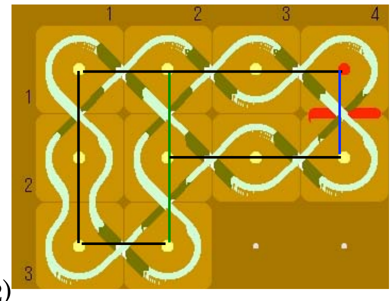
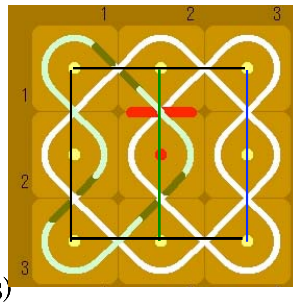
How many components in the case of the combined two N-line closed patterns

A part of them is overlapped

odd crossing x odd crossing, odd overlapped (1)->single, even->multi

odd crossing x even crossing, odd overlapped ->multi, even->single (2)

even crossing x even crossing, odd overlapped (3) ->multi, even->single

(1)  (2)  (3) 

N of All crossings (1,2 of a knot, and 3) are even

Crossings of each closed N-line are odd except of overlapped.

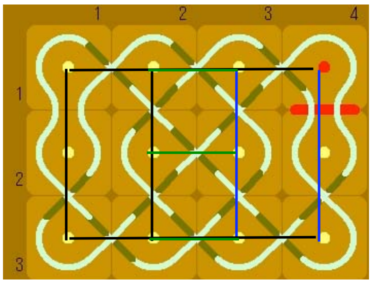
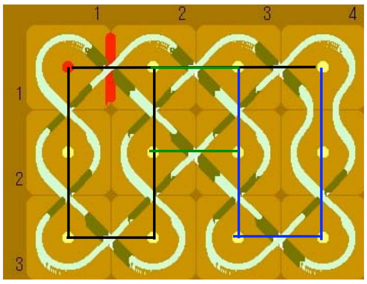
A part of them is connected with crossings

odd crossing + odd crossing, with odd crossing (1) ->single, even->multi

odd crossing + even crossing, with odd crossing ->multi,

even continued(2) ->single , separated->multi

even crossing + even crossing, with odd crossing ->multi, even->multi

(1)  (2) 

N of All crossings (1,2 of a knot,) are odd

Crossings of each closed N-line are odd except of the connected.

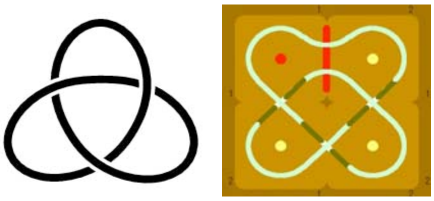
necessary and sufficient condition (NC, SC)

The persons interested in this topic!

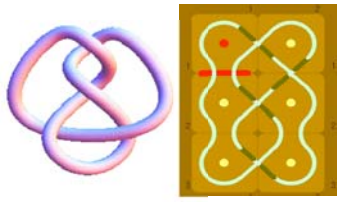
Read this journal for more detail, please.

FORMA, Commemorative Issue of the Conference ISKFA06: TheBeauty, Dynamics and Design of String Patterns in Folk Arts, edited by S. Nagata

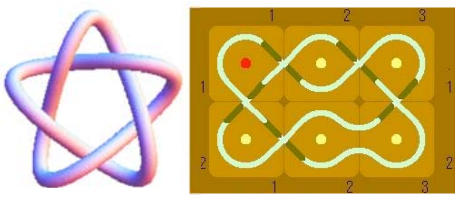
and contact with S. Nagata for using his Kolam/Knot Designer software on Windows as well.



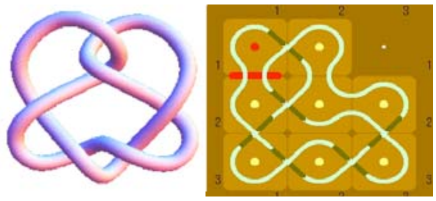
1 A knot with 3 crossings



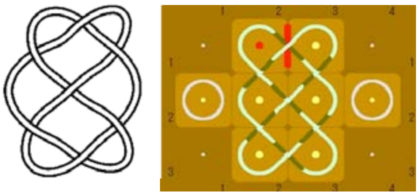
2 A knot with 4 crossings



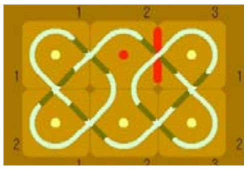
3 A knot with 5 crossings



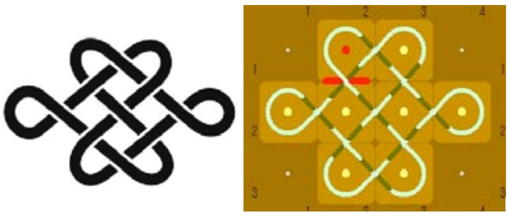
4 A knot with 6 crossings



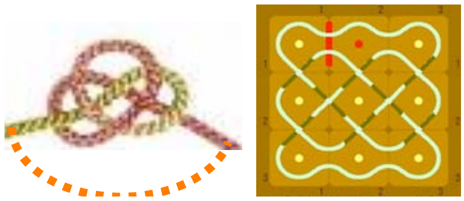
5 A knot with 7 crossings



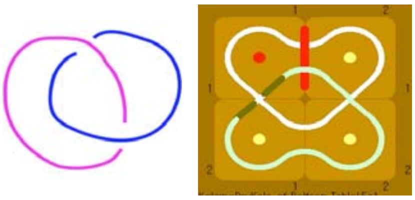
6 A Vertical knot coupled with two elemental knots



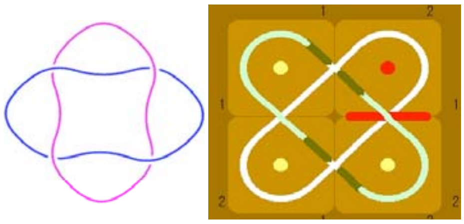
7 Takara-mon. an extended knot



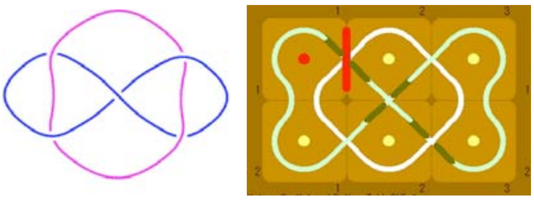
8 A knot with 8 crossings called Mizuhiki-Awaji



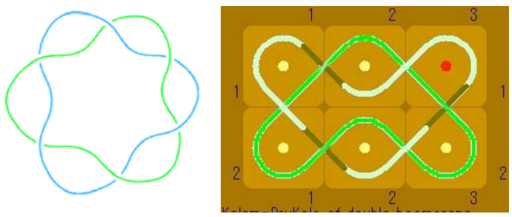
9 A 2 component link with 2 crossings



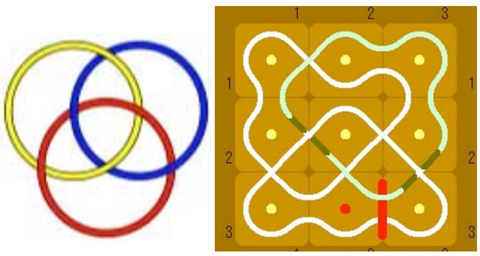
10 A 2 component link with 4 crossings



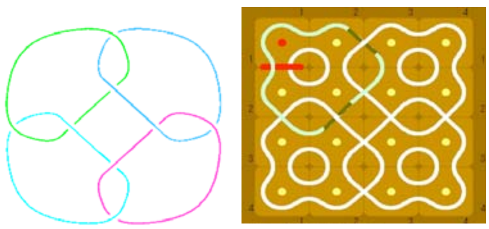
11 A 2 component link with 5 crossings



12 A 2 component link with 6 crossings

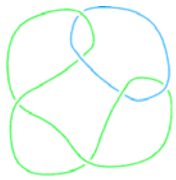


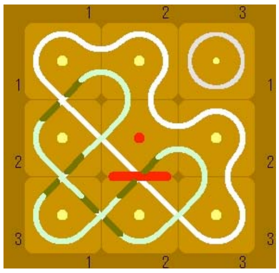
13 A 3 component link with 6 crossings



14 A 4 component link with 8 crossings

Fig.3 Kolam(right) corresponding to Knot/Link (left, from //katlas.math.utoronto.ca/wiki/) by S.Nagata





A 2 component link with 6 crossings