Network of Seifert surgeries

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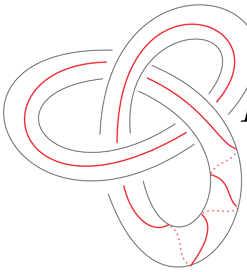
ABSTRACT In a preliminary work with K. Miyazaki and K. Motegi, we introduced a Network of Seifert surgeries in order to find a global explanation to the production of Seifert fiber spaces by Dehn surgeries on knots, called Seifert surgeries for short; for instance, Seifert surgeries on **Torus knots** are well-understood as their exterior (in S^3) is "already" Seifert fibered. This Network is usefull to understand Seifert surgeries on Hyperbolic knots as "descendants" of Seifert surgeries on Torus knots. We studied many known examples as **Berge** or **Dean** Seifert surgeries, and recently, worked on the case of the **Covering knots** that are obtained by the Montesinos trick; in particular, the examples introduced by M. Eudave-Muñoz.

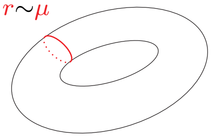
Motivation – Introduction

Dehn surgery



$E(K) = S^3 - intN(K)$, $\partial E(K) \hookleftarrow r \in \mathbb{Q} \cup \{\infty\}$

$E(K) \bigcup_{r\sim\mu} S^1 \times D^2 = K(r) =$ Seifert Fiber Space

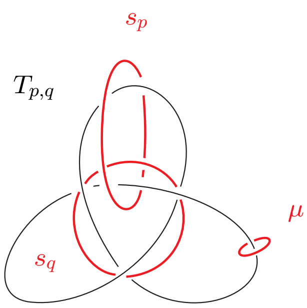


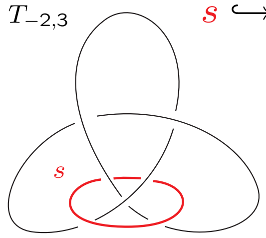
$\Rightarrow (K, r)$ called a Seifert surgery

VIEWPOINT

Global picture of Seifert surgeries with a NETWORK using Seiferters to connect **Torus knots**

Vertices are Seifert surgeries (K, m)'s





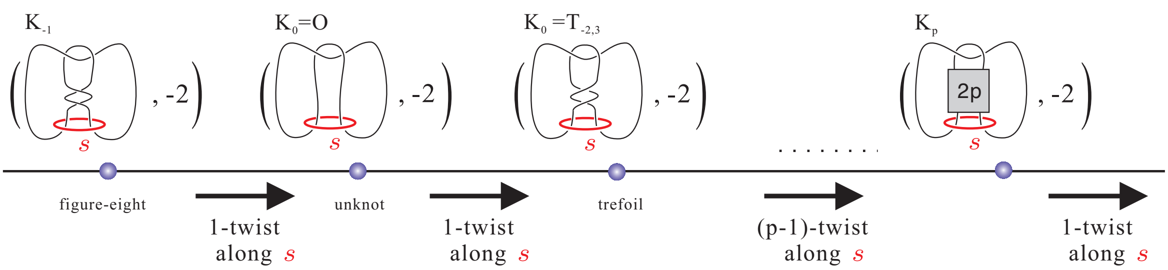
$s \hookrightarrow E(K)$ is a Seiferter for (K, m):

- s is a **Trivial knot** in S^3, and
- s is a **Seifert fiber** in $K(m)$

Basic s
for $(T_{p,q}, m)$
$\forall m \in \mathbb{Z}$

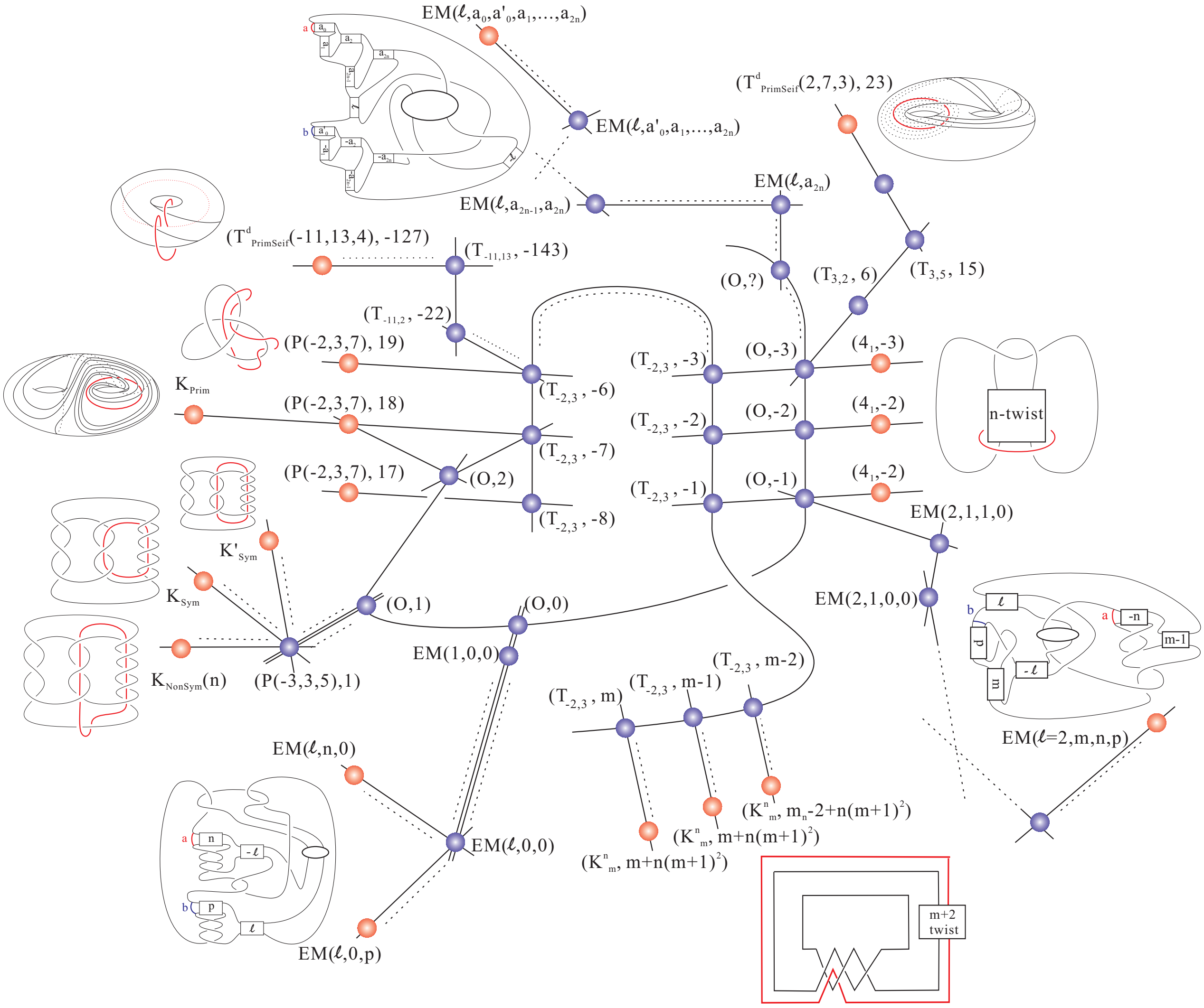
Non-basic s
for $(T_{-2,3}, i)$
$i \in \{-1, -2, -3\}$

Edges are "Seiferters"



connect two Seifert surgeries
if there is 1-**twist** along a Seiferter

Local picture of the Network



Basic Seiferters for **Torus** knots ⇒ connected Sub-network

We locate Seifert surgeries on **Twist** knots

We find ∞^{ly} 3-Successive Seifert surgeries on **hyperbolic** K_m^n

We locate Seifert surgeries on **Primitive/Primitive** knots K_{Prim}

We locate Seifert surgeries on **Primitive/Seifert** knots $T_{PrimSeif}^d$

We find Seifert surgeries on **Non-symmetric** $K_{\text{Non-Sym}}(n)$

We locate Seifert surgeries on **Covering** knots $EM(\ell, \dots)$