Braids, Configuration Spaces, and Quantum Topology

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Lecture Hall,
Graduate School of Mathematical Sciences, the University of Tokyo

Titles and abstracts

Kazuo Habiro (RIMS, Kyoto University)
Traces of categorified quantum groups

Categorification of quantum groups are constructed by Khovanov, Lauda and Rouquier. Roughly speaking, it is a 2-category whose split Grothendieck group is isomorphic to (a modified version of) a quantum group. The split Grothendieck group K_0 is a construction which takes an additive category and gives an abelian group (or a linear 0-category), which is generated by the isomorphism classes of objects in the category. If this construction is applied to the Hom-categories in a 2-category, one obtains a linear (1-)category. In this way, K_0 is a decategorification functor, which reduces the categorical dimension by one.

In this talk, I plan to consider another kind of decategorification, called the trace. The trace $\mathrm{Tr}(C)$ of a linear category C is the abelian group obtained as the space of the endomorphisms in C modulo the relations $fg = gf$. It is also known as the cocenter or the 0th Hochschild-Mitchell homology. Like the split Grothendieck group, the trace of a linear 2-category is a linear category.

In this talk, I plan to explain some basic facts about the trace and recent results about the traces of the categorified quantum groups. This talk is based on joint works with Beliakova, Guliyev, Lauda, Webster and Zivkovic.

Rei Inoue (Chiba University)
Cluster algebra and knot invariants

The cluster algebra was introduced by Fomin and Zelevinsky around 2000. The characteristic operation in the algebra called 'mutation' is related to various notions in mathematics and mathematical physics. In this talk I introduce an application of cluster algebra to study knot invariants, based on joint work with Kazuhiro Hikami (Kyushu University).

In three-dimensional hyperbolic geometry, a mutation is interpreted to produce an ideal tetrahedron, and cluster variables and coefficients respectively correspond to Zickert's edge parameters and the modulus of ideal tetrahedra. We define the octahedral R-operator composed of four mutations, and study the complex volume of knot complements in S^3. By q-deforming the R-operator using quantum cluster algebra a la Fock and Goncharov, we construct a braiding operator in terms of quantum dilogarithm function. In a limit that q goes to a root of unity, the braiding operator reduces to Kashaev's R-matrix.

Tetsuya Ito (RIMS, Kyoto University)

On a relation between the self-linking number and the braid index of closed braids in open books

Inspired from formulae of two-variable knot polynomial for closed 3- and 4-braids, Jones posed a conjecture that the algebraic linking number of a minimal braid representative is a knot invariant. Recently Dynnikov-Prasolov and LaFountain-Menasco proved Kawamuro's generalized version of Jones' conjecture.

In this talk we give a further generalization of the Jones-Kawamuro conjecture, for closed braids in general open books. This illustrates close connection between contact geometry and braids.

Seiichi Kamada (Osaka City University)

Generalizing braids and links in low dimensions

Every oriented link in the Euclidean 3-space can be presented by a braid. Such a braid is unique up to braid equivalence, conjugation and stabilization. This is the Alexander and Markov theorem. First we generalize it to spatial graphs in 3-space. Next we consider surfaces in the Euclidean 4-space. Every oriented surface-link in 4-space can be presented by a 2-dimensional braid. Such a 2-dimensional braid is unique up to braid ambient isotopy, conjugation and stabilization. We generalize this to singular surface-links in 4-space. In this talk we also explain some methods of describing 2-dimensional braids and their braid monodromies. This is a joint work with Victorial Lebed on spetial graphs and with Takao Matumoto on singular surface-links.

Christian Kassel (CNRS and Université de Strasbourg)

On the zeta function of the Hilbert scheme of n points in a torus

The Hilbert scheme of n points in an algebraic variety is the algebraic geometers' way to view the configuration space of n points in that variety. I will show how to compute the zeta function of this scheme over a given finite field when the variety is a two-dimensional torus. These zeta functions have a nice symmetry, which can be expressed in a simple functional equation. The computation involves a family of polynomials with nice properties: they are palindromic, their coefficients are non-negative integers and their values at 1 and at roots of unity of order 2, 3, and 4 can be expressed in terms of classical arithmetical functions. This is joint work with Christophe Reutenauer (UQAM).

Hisashi Kasuya (Tokyo Institute of Technology)
Extensions of Nomizu's Theorem

Solvmanifolds (resp. nilmanifolds) are quotients of simply connected solvable (resp. nilpotent) Lie groups by cocompact discrete subgroups. In 1950s, K. Nomizu proved that the de Rham cohomology of nilmanifolds are computed by the invariant differential forms. This theorem was generalized for "special" solvmanifolds (A. Hattori, G. D. Mostow). However, we can not obtain "simple generalization" of Nomizu's Theorem for general solvmanifolds. In this talk, we aim to obtain extensions of Nomizu's Theorem in various senses.

Takahiro Kitayama (Tokyo Institute of Technology)
Representation varieties detect essential surfaces

Extending Culler-Shalen theory, Hara and I presented a way to construct certain kinds of branched surfaces (possibly without any branch) in a 3-manifold from an ideal point of a curve in the SL_n-character variety. There exists an essential surface in some 3-manifold known to be not detected in the classical SL_2-theory. We show that every essential surface in a 3-manifold is given by the ideal point of a line in the SL_n-character variety for some n. The talk is partially based on joint works with Stefan Friedl and Matthias Nagel, and also with Takashi Hara.

Toshitake Kohno (The University of Tokyo)
Holonomy of braids and its 2-category extension

First we review basic properties of the holonomy of the KZ connection such as its relation to the homological representations and quantum group symmetry. Then we discuss a generalization of the holonomy of braid groups to higher categories. The 2-categories consist of objects, morpshims and 2-morphisms for any pair of morpshims. Using a method of formal homology connection due to K.-T. Chen, we construct a 2-functor from the path 2-groupoid of the configuration spaces. This construction gives representations of cobordism categories of braids.

Thang Le (Georgia Institute of Technology)
On the Chebyshev-Frobenius homomorphism for quantum Teichmuller spaces and skein modules

We extend the Kauffman bracket skein modules of 3-manifolds to marked 3-manifolds and show how the Chebyshev-Frobenius homomorphism appears naturally in this theory.

Ivan Marin (Université d'Amiens)
Topological applications of the BMR conjectures

During the nineties, Broue, Malle and Rouquier stated a collection of conjectures about the generalization to complex reflection groups of objects classically associated to Weyl groups : Artin groups and Iwahori-Hecke algebras. In this talk, I will describe these conjectures, recent advances towards their resolution, as well as several applications to the representations of the ordinary braid groups.

Kimihiko Motegi (Nihon University)
Genera of L-space knots

Conjecturally, there are only finitely many Heegaard Floer L-space knots in S^3 of a given genus. We examine this conjecture for twist families of knots $\{K_n\}$ obtained by twisting a knot K in S^3 along an unknot c, and establish the conjecture in the case where the linking number ω between K and c is not ± 1. In particular, if $\omega = 0$, we prove that $\{K_n\}$ contains at most three L-space knots. We address the case where $|\omega| = 1$ under an additional hypothesis about Seifert surgeries. To that end, we characterize a twisting circle c for which $\{(K_n, r_n)\}$ contains at least ten Seifert surgeries. We also pose a few questions about the nature of twist families of L-space knots, their expressions as closures of positive (or negative) braids, and their wrapping about the twisting circle. This is joint work with Ken Baker.

Jun Murakami (Department of Mathematics, Faculty of Science and Engineering, Waseda University)
Logarithmic invariant of knots and its applications

We introduce the logarithmic invariant of knots by two ways. One construction uses the colored Jones invariants and the other construction uses the colored Alexander invariants. The logarithmic invariant can be generalized to an invariant of knots in a three manifold. Its relation to the topological quantum field theory is also discussed.

Tomotada Ohtsuki (RIMS, Kyoto University)
On the asymptotic expansion of the Kashaev invariant of some hyperbolic knots

The asymptotic expansion of the Kashaev invariant of hyperbolic knots is a refinement of the volume conjecture, which is important from the viewpoint that it relates the quantum topology to the hyperbolic geometry. This asymptotic expansion for some hyperbolic knots can be calculated by using the Poisson summation formula and the saddle point method. By definition, the Kashaev invariant of a knot is presented by a certain sum. By using the Poisson summation formula, we can rewrite this sum by an integral. Further, by using the saddle point method, we can calculate the asymptotic expansion of this integral. In this talk, I will explain a survey on this asymptotic expansion.

Kyoji Saito (Kavli IPMU)
Dual Artin Monoid (joint work with T. Ishibe)

The submonoid of an Artin group (=the fundamental group of the complement of the discriminant loci of a finite reflection group) generated by simple generators (corresponding to simple reflections) is called the Artin monoid. Bessis introduced another submonoid of the Artin group using the generators corresponding to all reflections. We shall call it the dual Artin monoid. Surprisingly, both Artin and dual Artin monoids are lattices. That is, they admit least common multiples. Therefore, their skew growth functions are determined by the Möbius inversion formula. We show that the functions have exactly n (=rank of the group) number of real simple roots on the interval $(0, 1]$.

Mario Salvetti (Department of Mathematics, University of Pisa)
Local cohomology of Artin groups and applications

We are interested in the topology of the configuration spaces of Artin groups, in particular their local cohomology. Computation of such cohomology turns out to have interesting applications in several areas. Here we focus in particular, besides the case of classical braid groups, to the case of some affine Artin groups and some applications.

Dai Tamaki (Department of Mathematics, Shinshu University)
Discrete Morse theory and classifying spaces

Given a "good" discrete Morse function f on a finite regular cell complex K, we construct an acyclic category $C(f)$, called the flow category of f, whose objects are critical cells. Morphisms between critical cells are certain sequences of cells, called flow paths, which serve as an analogue of gradient flows in the case of smooth Morse theory. It turns out that the set of morphisms $C(f)(c, c')$ between two critical cells has a structure of poset, inducing a structure of 2-category on $C(f)$. We show that there is a zigzag of homotopy equivalences between the classifying space $BC(f)$ of the flow category as a 2-category and K. This result can be regarded as a discrete analogue of a refinement of smooth Morse theory announced by R. Cohen, J.D.S. Jones, and G.B. Segal in early 1990's. This is a joint work with Vidit Nanda and Kohei Tanaka.

Hiroaki Terao (Hokkaido University)
Weyl groups and their parabolic subgroups
—In the hyperplane arrangement setup—

The parabolic subgroups of a Weyl group are classified by the root systems determined by their fixed points. Each fixed point set is regarded as an element of the intersection lattice of the corresponding Weyl arrangement. In this talk we will study the restriction of the arrangement to a fixed point set. We will apply the recently proved height-free theorem by T. Abe, M. Barakat, M. Cuntz, T. Hoge and H. Terao (to appear in J. Euro. Math. Soc.) and use a new concept called the divisionally freeness due to T. Abe.

Alexander P. Veselov (Department of Mathematical Sciences, Loughborough University)
Logarithmic Frobenius structures and theory of hyperplane arrangements

The logarithmic Frobenius structures correspond to a special class of solutions of the famous Witten-Dijkgraaf-Verlinde-Verlinde equations. The examples include Coxeter configurations and their restrictions, but the general classification is still an open problem.

I will discuss some relations of this problem with the theory of hyperplane arrangements, including holonomy Lie algebras and logarithmic vector fields. The talk is based on recent joint work with M. Feigin.

Alexander Voronov (University of Minnesota)
Dijkgraaf-Witten theory using cohomology with coefficients in Picard groupoids

This is a new perspective on an old topic: Dijkgraaf-Witten theory, i.e., gauge theory with a finite gauge group. I will define cap product between homology and cohomology with coefficients in Picard groupoids and use it to construct Dijkgraaf-Witten theory in arbitrary dimension as a functor from the category of cobordisms to that of vector spaces. This is a joint work with my graduate student Amit Sharma.

Tadayuki Watanabe (Department of Mathematics, Shimane University)
Finite type invariants of nullhomologous knots in 3-manifolds by counting graphs

We study finite type invariants of nullhomologous knots in a closed 3-manifold M defined in terms of certain descending filtration $K_n(M)$ of the vector space $K(M)$ spanned by isotopy classes of nullhomologous knots in M. The filtration is defined by surgeries on special kinds of claspers in M. When M is fibered over S^1 and $H_1(M) = \mathbb{Z}$, we show that the natural surgery map from the space of $Q[t, t^{-1}]$-colored Jacobi diagrams on S^1 of degree n to the graded quotient $K_n(M)/K_{n+1}(M)$ is injective for $n = 1, 2$. In the proof, we construct a finite type invariant of nullhomologous knots in M up to degree 2 by counting certain graphs in M.