Amalgams: a machinery of the modern theory of finite groups

A.A. Ivanov

Abstract

Traditionally finite groups are studied either as automorphism groups of certain geometric configurations or as abstract groups in terms of generators and relations. The amalgam method is a *simbios* of these two approaches. I would like to report on the progress in the project to establish the existence and uniqueness of the Monster group adopting the Monster amalgam as the first principle. The Monster group M is the largest among the exceptional (sporadic) finite simple groups. The Monster is particularly famous due to its connection with modular forms and vertex operator algebras. The Monster amalgam $\mathcal{M}$ is formed by a triple of subgroups in M, one of which is the involution centralizer isomorphic to $2_+^{1+24}.Co_1$. Here Co_1 is the automorphism group of the Leech lattice Λ modulo $\{\pm 1\}$-scalar transformations and 2_+^{1+24} is an extension of $\Lambda/2\Lambda$ by a centre of order 2. The main stages of the project is to (1) axiomatise the Monster amalgam $\mathcal{M}$; (2) prove that subject to the axioms $\mathcal{M}$ exists and unique; (3) define M as the largest group which contains $\mathcal{M}$ and which is generated by the elements of $\mathcal{M}$; (4) through analysing of the coset geometry associated with the pair $(M, \mathcal{M})$ prove that M is the Monster as we know it that is a finite simple group of a very impressive order. The ultimate goal of the project is to produce a vertex-operator algebra free construction of the Moonshine module for the Monster.