Tokyo-Berkeley Summer School
“Geometry and Mathematical Physics” – 2nd Week –

July 27 – July 31
Venue : Kavli IPMU Lecture Hall

Program

	Monday	Tuesday	Wednesday	Thursday	Friday
10:00-11:30	Kapranov I	Kapranov II	Reshetikhin I	Reshetikhin II	Toda II
13:00-14:30	Morrison I	Morrison II		Toda I	Yamazaki
15:00-15:30	Tea	Tea	Tea	Tea	Tea
15:30-17:00		Hori			

Mikhail Kapranov (Kavli IPMU)

I. Combinatorial approach to Fukaya categories of surfaces.

II. Perverse sheaves and their categorifications as coefficients for Fukaya categories.

I will probably not really need to assume a background on Fukaya categories (although it would be nice, and perhaps it is in the program anyway). I can recall all that is necessary for the particular example of surfaces.

The second lecture would be about “Fukaya categories with coefficients”, discussing first the general concepts and then switching to the particular case of surfaces.

David Morrison (UC Santa Barbara)

Mirror symmetry: theme and variations

Abstract: My theme is mirror symmetry and its origins in string theory. In non-perturbative string theory, mirror symmetry is an equivalence between the compactification of two of the string theories (type IIA and type IIB) on different Calabi-Yau threefolds. This equivalence implies that the open-string sectors of these theories (which only exist non-perturbatively) are also equivalent: i.e., there is an equivalence of D-brane states. Variation one (perturbative mirrror symmetry): in perturbative string theory, we find an equivalence between two-dimensional conformal field theories, and boundary conditions must also coincide. Variation two (GLSM): for a particular class of two-dimensional field theories ("abelian gauged linear sigma models") the equivalence is quite explicit and closely related to the combinatorial mirror symmetry discovered by Batyrev and Batyrev-Borisov. Variation three (SYZ): analyzing D-brane states leads to explicit predictions about a geometric connection between two mirror Calabi-Yau manifolds. Variation four (Gross-Seibert) analyzing degenerations of Calabi-Yau m anifolds near the boundary of moduli space allows one to recover the mirror family. Variation five (homological mirror symmetry): the data of D-branes determine categories, and there is a beautiful conjecture relating different types of categories on the different Calabi-Yau manifolds. Further variations take one beyond Calabi-Yau manifolds and I will give some indications of this along the way.

Kentaro Hori (Kavli IPMU)

The hemisphere and its applications

The lecture will be about the partition function on the hemisphere of 2d (2,2) supersymmetric gauged linear sigma models. It provides a general exact formula for the central charge of the D-brane placed at the boundary. It also coincides with the genus zero topological string with an insertion of the macroscopic loop associated with the D-brane. It takes the form of Mellin-Barnes integral and the question of its convergence leads to the grade restriction rule on D-branes in linear sigma models.

Nicolai Reshetikhin (UC Berkeley)

Introduction to BV quantization

Abstract: The BV (Batalin-Vilkovisky) quantization is a general method for constructing a perturbative path integral, i.e. Feynman diagram expansion, for classical field theories with degenerate action functional. When the degeneracy is given by a stabilizer free action of the gauge group it is equivalent to Faddeev-Popov and BRST quantization. In the BV framework fields are accompanied by ghosts, by anti-fields and by anti-ghosts. This extended space of BV field is an odd symplectic manifold. The integration over the space of fields is replaced the integration over a Lagrangian submanifold. The extended action functional is non-degenerate over this Lagrangian submanifold.

The goal of these lectures is to outline the method and to show it works for space-time manifolds with boundary.

References:

1. I. A. Batalin, G. A. Vilkovisky, Gauge algebra and quantization, Phys. Lett. B, 102 27 (1981).

2. Alberto S. Cattaneo, Pavel Mnev, Nicolai Reshetikhin, Perturbative quantum gauge theories on manifolds with boundary. arXiv:1507.01221.

3. A. S. Schwarz, The partition function of degenerate quadratic functionals and Ray?Singer invariants, Lett. Math. Phys. 2, 247?252 (1978).

4. A. S. Schwarz, " Geometry of Batalin-Vilkovisky quantization, " Commun. Math. Phys. 155 2, 249-260 (1993).

Yukinobu Toda (Kavli IPMU)

Lecture I: Moduli of Bridgeland semistable objects on 3-folds and Donaldson-Thomas invariants

The Donaldson-Thomas invariants count stable sheaves on Calabi-Yau 3-folds. It has been expected that the theory should be extended to count

Bridgeland semistable objects in the derived category. I will show the existence of such a theory on Calabi-Yau 3-folds satisfying the Bogomolov-Gieseker inequality conjecture proposed by Bayer, Macri and myself. This result is applied to A-type Calabi-Yau 3-folds, i.e. etale quotients of abelian 3-folds. This is a joint work with D. Piyaratne.

Lecture II: Non-commutative thickening of moduli spaces of stable sheaves

Non-commutative thickening of moduli spaces of stable sheaves Abstract: In the last year, I found a curious formula describing the dimensions of Donovan-Wemyss's non-commutative deformation algebras of floppable curves in 3-folds in terms of Donaldson-Thomas invariants. This phenomena suggests that there might be DT type invariants which capture non-commutative deformations of sheaves, having some relations to usual DT invariants. In order to study this, I need to show some foundational things: I will explain that the moduli spaces of stable sheaves admit non-commutative structures in the sense of Kapranov, which lead to the notion of non-commutative virtual structure sheaves.

Masahito Yamazaki (Kavli IPMU)

Dilogarithm identities and cluster algebras

In this lecture I will describe classical and quantum dilogarithm identities as constructed naturally from cluster algebras and quiver mutations. They have applications to a number of topics in mathematical physics, such as Donaldson-Thomas theory, geometry of 2/3-manifolds, integrable lattice models, and supersymmetric gauge theories in 2/3/4 dimensions. The lecture requires only minimal knowledge, and should be understandable to everyone.