Tokyo-Berkeley Summer School
“Geometry and Mathematical Physics” – 1st Week –

The purpose of the first week is to provide students systematic accounts for deformation of singularities, primitive integrals, infinite root systems, flat structures etc starting from basic materials. We will also focus on a relation to integrable systems and give an introduction to derived categories and stability conditions, which will give an introduction to some talks in the second week.

Program

Tuesday, July 21, 10:00-12:00, 13:00 - 15:00, Balcony A
Lecturer : Kyoji Saito (Kavli IPMU)

Wednesday, July 22, 10:00-12:00, 13:00 - 15:00, Seminar Room B
Lecturer : Todor Milanov (Kavli IPMU)

Thursday, July 23, 10:00-12:00, 13:00 - 15:00, Balcony A
Lecturer : Akishi Ikeda (Kavli IPMU)

Friday, July 24, 10:00-12:00, 13:00 - 15:00, Seminar Room B
Student Session

Tuesday, July 21, 10:00-12:00, 13:00 - 15:00, Balcony A

Lecturer : Kyoji Saito (Kavli IPMU)

An introduction to Period Integrals

Abstract : In the present lectures, I’ll give an introduction to an analytic theory of Period Integrals over open CY-manifolds, or, so called, B-side LG-model theory from mathematical view point. In the first half, I’ll start with the theory of elliptic integrals, and will show how such classical theory leads to the construction of the flat Frobenius structure and elliptic modular functions. As a toy model, we examine how the mirror symmetry appears in this

context. In the second part, I'll give a higher dimensional generalization of the elliptic integral theory by introducing 1) semi-infinite Hodge structure, 2) higher residue pairings on the Hodge structure, 3) the primitive elements (= the primitive forms) on the Hodge structure, and 4) the flat Frobenius structure on the parameter space. Finally, I'll discuss how these concepts lead to the construction of the (genus 0) pre-potential functions and its mirror symmetry. We shall also discuss some examples.

The references for Saito's lectures are given in the last 3 pages of the program.

Wednesday, July 22, 10:00-12:00, 13:00 - 15:00, Seminar Room B

Lecturer : Todor Milanov (Kavli IPMU)

If X is a compact complex orbifold with semi-simple quantum cohomology, then it was conjectured by Givental and proved by Teleman that the higher-genus Gromov-Witten (GW) invariants can be reconstructed from genus zero only. Moreover, Dubrovin and Zhang have constructed an integrable hierarchy (one for each target orbifold) that governs the GW invariants. I would like to give an introduction to another approach whose goal is to characterize the GW invariants and the Dubrovin-Zhang's hierarchies in terms of certain class of Vertex Operator Algebra (VOA) representations.

Lecture 1: Introduction to GW theory.

The main goal is to state Givental's higher-genus reconstruction and mirror symmetry. I am going to assume the most basic notions from the geometry of complex manifolds, such as: cohomology, vector bundles, connections and Chern classes (see Chapter 0 in Griffiths and Harris). Some familiarity with symplectic geometry: Hamiltonian vector fields and Poisson brackets (see e.g. the on-line available Ana Cannas da Silva's notes "Lectures on symplectic geometry").

Lecture 2: Twisted VOA representations.

I would like to present an explicit construction of a certain class of twisted vertex algebra representations, which are expected to govern GW invariants.

The precise mechanism to reconstruct the GW invariants from the VOA representation is known only for simple singularities and to some extend for Fano orbifold curves. The 2nd goal of this lecture is to explain the case of simple singularities.

References :

1. A. Givental. "A tutorial on quantum cohomology."
2. A. Givental. "Gromov – Witten invariants and quantization of quadratic hamiltonians."
3. A. Givental. "A_{n-1}-singularities and nKdV hierarchies."
4. V. Kac. "Vertex algebras for beginners."
5. B. Bakalov and T. Milanov. "W-constraints for the total descendant potential of a simple singularity."

Thursday, July 23, 10:00-12:00, 13:00 - 15:00, Balcony A

Lecturer : Akishi Ikeda (Kavli IPMU)

Introduction to derived categories and stability conditions

The purpose of this lecture is to be familiar with derived categories, triangulated categories and stability conditions. I plan the following contents.

Lecture 1: Derived categories

In the first part, after short review of the homological algebra, I would like to introduce the derived categories of abelian categories through the localization of categories. As important examples of derived categories, we deal with the derived categories of coherent sheaves on algebraic varieties and representations of algebras. I'll also give the construction of derived functors.

Lecture 2: Stability conditions

In the second part, we see axioms of triangulated categories. After that I would like to introduce bounded t-structures on triangulated categories and construct the standard t-structures on the derived categories. Finally I'll explain Bridgeland stability conditions on triangulated categories and state the basic theorem for the space of stability conditions.

References

- S. I. Gelfand and Yu. I. Manin, Methods of Homological Algebra, Springer-Verlag (1996).
- T. Bridgeland, Stability conditions on triangulated categories, Ann. of Math. 166:317-345, (2007), arXiv:0212237.

References for Saito's lectures

Reference for Part I.

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Reference for Part II

[F] Fan,H., Jarvis,T., and Ruan,Y.: The Witten equation, mirror symmetry, and quantum singularity theory, Ann. of Math. (2) 178 (2013), no. 1, 1-106.
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