INTRODUCTION TO K-STABILITY IN KÄHLER GEOMETRY

AKITO FUTAKI

GRADUATE SCHOOL OF MATHEMATICAL SCIENCES, THE UNIVERSITY OF TOKYO

K-stability is defined by using a weight for a group action on polarized varieties. This weight goes back to Kazdan and Warner who found an integrability condition for the 2-sphere to have a prescribed Gaussian curvature. The prescribing Gaussian curvature problem has the same type of non-linear elliptic PDE as the Kähler-Einstein problem in Kähler geometry, the complex Monge-Ampre equation, and indeed the integrability condition is extended to an obstruction to the existence of Kähler-Einstein metrics. This obstruction is an invariant which can be viewed as one of equivariant cohomology of the automorphism group of the complex manifold under consideration. This invariant is closely related to Chow weight in geometric invariant theory. In this lectures we will study its generalizations to other problems such as constant scalar curvature Kähler metrics, Kähler-Ricci solitons and Sasaki-Einstein metrics. The proof of sufficiency of K-stability for the existence of Kähler-Einstein metrics will be outlined.