The notion of topological K-theory as an extraordinary cohomology theory of spaces was mainly forged in the end of 1950-ies due to the efforts of many mathematicians, first of all Michael Atiyah and Isadore Singer. Their ideas lead to the proof of the famous Index Theorem, one of the most renown Mathematical results in the second part of the XX-th century, and up to the modern days remain the source of inspiration for many results in various branches of Mathematics, such as Geometric Analysis, Riemannian Geometry, Topology of Manifolds, Algebra and other. The amazing power of this construction is due to the fact that it is situated at the crossroads of various mathematical disciplines, which allows one use rigorous analytical computations, geometric and topological intuition and subtle algebraic constructions at the same time to prove results, pertaining to either of the fields. Thus it is not surprising that in order to harness similar power to solve problems in other fields the construction of K-theory was further generalized in the most general algebraic and topological contexts, which lead to the development of Algebraic K-theory, K-theory of C*-algebras, noncommutative Geometry and other modern mathematical theories.

In the present series of lectures, I will try to give a concise introduction in the subject of K-theory within its traditional area of application; i.e. we shall not deal with the K-theory of algebras, neither topological, nor discrete, we shall not speak about the K-homology and its relation with elliptic operators (although it would be impossible and counterproductive to keep this subject totally behind the scene, so some traces of "elliptic science" will probably find their way into the course), we shall also avoid speaking about the equivariant theories and many other things. Unfortunately (or on the contrary, fortunately), even the principal classical constructions of the K-theory are quite variegated and cover a large scope of ideas and mathematical objects, hence it would be quite impossible and even ridiculous trying to fit everything in the short course. So we shall leave beyond the scope of this course (or give only a short mention of) such important ideas as the Bott's proof of the periodicity theorem, based on the Morse theory, Atiyah-Bott-Shapiro's famous construction, relating K-theory with the representation theory of Clifford algebras, we shall omit discussing the relation of K-theory to other extraordinary cohomology theories, and even the Adams' operations will have only a brief appearance in the course.

The outline of this course is as follows:

Lecture 1 We shall first address the general theory of fibred bundles; we shall begin with the definition of locally-trivial bundles, principal bundles and associated bundles and Čech cocycles, that determine them. We shall prove the theorem, identifying the set of all isomorphism classes of principal G-bundles over X with the elements of the first noncommutative Čech cohomology: $\check{H}^1(X, \mathscr{G})$, where $\mathscr{G}$ is the (pre)sheaf of G-valued local functions on X. As a corollary of this description we shall derive the homotopy invariance of the pull-back of a bundle. In addition, we shall obtain the first examples of characteristic classes, the Stiefel-Whitney classes w_1, w_2 and the Chern class c_1 of $\mathbb{C}^*$-bundles.

Lecture 2 Next we shall have a thorough discussion of vector bundles and their properties. We shall begin with the elementary algebraic operation on vector bundles, given by "continuous functors" on the category of vector spaces, the most important of which are the Whitney sum, and the tensor product. We shall prove the splitting property of the exact sequences of vecror bundles and use it to prove the existence of complementary bundles (i.e. such bundles, whose direct sum with the given one is trivial). This result will lead us to two important theorems: first, the *Serre-Swan theorem* that identifies the categories of vector bundles and the category of projective modules over functions; second, we shall use it to construct the *classifying space* of $U(n)$-bundles.

Lecture 3 Here we shall discuss the definitions, properties and main constructions of the K-groups of a cell space, $K^*(X)$ including the reduced and relative K-groups $\tilde{K}^*(X)$, $K^*(X,Y)$: we shall give both homotopy theoretic and geometric definitions of these objects. We shall construct the classifying space BU of K-theory and consider few examples of computations in K-theory.

Lecture 4 This lecture shall be dedicated to the discussion of one important property of K-theory, *Bott periodicity theorem.* We shall describe several different approaches leading to the proof of this result, including the Atiyah's proof, based on the notion of the index of a family of elliptic operators. In the end, if time permits, we shall give a very brief introduction to the theory of characteristic classes.

Lecture 5 After a brief introduction, concerning the definition and major properties of the Adams' cohomological operations in K-theory, we shall give the proof of the famous Adams' theorem about the dimensions of algebras with division over $\mathbb{R}$.

It is worth mentioning here that the course will consist not only of lectures but also of problem sessions; your results at these sessions will be taken into account when grading will be done.

Prerequisites for this course are more or less standard: elementary Algebra, Linear Algebra and Analysis. Some knowledge of point-set Topology and basics of algebraic Topology (including the homotopy groups of spaces and elements of cohomology theory) are also more than welcome.