INTRODUCTION TO LOCAL CLASS FIELD THEORY

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Local class field theory is a fundamental pillar of the number theory. Roughly speaking, its main theorem identifies the Galois group of a local field K (i.e. a complete discrete valuation field with finite residue field) with its multiplicative group $K^\times$ as a topological group. The basic aim of the lectures is to give a proof of the local class field theory. Prerequisites for the lectures are basic knowledge in algebra such as Galois theory and theory of simple algebras. A reference is J.-P. Serre's book 'Local fields'.

Lecture I: Valuations of fields

1.1 Valuations of fields: Basic definitions (valuation rings, residue fields, discrete valuations,..), basic examples ($\mathbb{Z}_{(p)}$, $F[[t]]$, ...).
1.2 Complete valuations: Completions of valuations, basic example $\mathbb{Z}_p$.
1.3 Hensel's lemma and its applications.

Lecture II: Unramified extensions of complete discrete valuation fields

2.1 Extensions of valuations in fields extensions: Existence and uniqueness of extensions of valuations in finite extensions of complete discrete valuation fields.
2.2 Unramifeid extensions: Definition of unramified extensions and basic properties.

Lecture III: Brauer groups of local fields

4.1 Review of simple algebras and Brauer groups.
4.2 Cyclic algebras and fundamental pairings.
4.3 A fundamental theorem on simple algebras over a local field.

Lecture IV: Local class field theory

Lecture V: Proof of local class field theory