

INTRODUCTION TO SYMPLECTIC GEOMETRY TOWARD MIRROR SYMMETRY

CHEOL-HYUN CHO

ABSTRACT. We give a gentle introduction to various subjects in and around symplectic geometry and mirror symmetry. Lectures will be given in omnibus style consisting of following topics: basic symplectic geometry, Morse homology theory, A_∞ -algebras, homological mirror symmetry.

Symplectic geometry is originated as a mathematical language to describe classical mechanics. On the other hand it is also one of the basic structures which exist in many important geometric spaces. The field has been one of central area of research partly due mirror symmetry conjecture. The mirror symmetry conjecture claims certain duality between symplectic and complex geometry. It also has revealed deep relations to other branches of mathematics as well. In this summer school topics are chosen in the hope of conveying some of the ideas in this exciting field. But inevitably due to time constraint, the lectures will only serve as an invitation to these topics.

Here is a tentative list of topics to be covered.

- Lecture 1. Basics of differentiable manifolds (Manifolds, tangent bundle, vector fields). Morse functions, and Morse-Smale-Witten complex.
- Lecture 2. More basics of differentiable manifolds (differential forms, calculus). Symplectic manifolds, and Hamiltonian vector fields. Lagrangian submanifolds.
- Lecture 3. More basics of symplectic geometry (compatible triple, moment map). Stasheff Polytope.
- Lecture 4. Introduction of A_∞ algebras and categories. Morse A_∞ category, Fukaya A_∞ category of a surface.
- Lecture 5. Matrix factorizations. Introduction to homological mirror symmetry.

Here are some references.

- Lectures on symplectic geometry, by Ana Cannas da Silva,
Book available at <https://people.math.ethz.ch/~acannas/>
- Lecture notes on Morse homology (with an eye towards Floer theory and pseudoholomorphic curves)
by M. Hutchings available at <https://math.berkeley.edu/~hutching/>
- Deformation theory, Homological Algebra, and Mirror symmetry, by K. Fukaya
available at <https://www.math.kyoto-u.ac.jp/~fukaya>

DEPARTMENT OF MATHEMATICAL SCIENCES, SEOUL NATIONAL UNIVERSITY, GWANAK-GU, SEOUL, SOUTH KOREA

E-mail address: chocheol@snu.ac.kr