Floer and Novikov homology, contact topology and related topics

Kavli IPMU Seminar Room B
April 21 – April 24, 2014

Program

Monday, April 21

10:30 – 12:00 **Urs Frauenfelder** (Seoul National University)
Spicy Hopf algebras and growth of Reeb chords

13:30 – 15:00 **Manabu Akaho** (Tokyo Metropolitan University)
On Morse homology of manifolds with boundary

15:30 – 17:00 **Mihai Damian** (Université de Strasbourg)
Lifted Floer homology and topology of monotone Lagrangian submanifolds

Tuesday, April 22

10:30 – 12:00 **Dan Burghelea** (Ohio State University)
A (computer friendly) alternative to Morse Novikov theory for angle valued maps.

13:30 – 15:00 **Fabiola Manjarrez-Gutierrez** (CIMAT, Mexico)
Additivity of Morse-Novikov number of a-small knots

15:30 – 16:30 **Tetsuya Ito** (RIMS, Kyoto University)
Overtwisted discs in planar open books

16:40 – 17:40 **Tadayuki Watanabe** (Shimane University)
Morse theory and Lescop's equivariant propagator

19:00 – Workshop Dinner

Wednesdasy, April 23

10:30 – 12:00 **Otto van Koert** (Seoul National University)
Fractional twists and invariant contact structures

13:30 – 15:00 **Sheila Sandon** (Université de Strasbourg)
On positive loops of contactomorphisms

15:30 – 17:00 **Kei Irie** (Kyoto University)
Hofer-Zehnder capacity, symplectic homology and loop product

Thursday, April 24

10:30 – 12:00 **Yasha Savelyev** (ICMAT, Madrid)
Global Fukaya category

13:30 – 15:00 **River Chiang** (National Cheng Kung University, Taiwan)
Examples of higher dimensional non-fillable contact manifolds

15:30 – 17:00 **Vincent Colin** (Université de Nantes)
Higher-dimensional Heegaard Floer homology

Titles and Abstracts

Manabu Akaho (Tokyo Metropolitan University)
title: On Morse homology of manifolds with boundary
abstract: In this talk we explain Morse homology of manifolds with boundary, motivated by Floer theory of Lagrangian submanifolds with concave end. First we observe Riemannian metrics and Morse functions on manifolds with boundary whose gradient vector fields are tangent to the boundary. Then we discuss their unstable manifolds and define our Morse homology, which is isomorphic to the absolute singular homology. Moreover we consider product on our Morse complexes, which satisfies the Leibnitz rule. Finally we mention some application to Floer theory of Lagrangian submanifolds.

Dan Burghelea (Ohio State University)
title: A (computer friendly) alternative to Morse Novikov theory for angle valued maps.
abstract: Morse-Novikov theory is a useful tool to analyze the dynamics of a large class of vector fields (which admit a Lyapunov closed one form) and relate rest points, trajectories between rest points and closed trajectories to the algebraic topology of the underlying manifold, at least in generic situation. A generic vector field which admits a closed one form as Lyapunov admits also an angle valued map as Lyapunov. We present a computer friendly alternative to the Morse-Novikov theory for an angle valued map and implicitly of Morse theory which works for a much larger class of spaces and maps (compact ANR and tame maps) which, from the point of view of algebraic topology, does as much as the smooth Morse-Novikov. It is based on invariants (in case of simplicial complexes computable by algorithms of the same complexity as of the ones which calculate Betti numbers). The invariants proposed provide refinements of the familiar topological invariants (Novikov and standard Betti numbers, monodromy) and reveal stability properties invisible in the smooth theory. The lecture is based on joint work with T. Dey and S. Haller and was influence by the work on Morse-Novikov theory and closed trajectories of Pajitnov, Hutchings-Lee and Burghelea-Haller.

River Chiang (National Cheng Kung University, Taiwan)
title: Examples of higher dimensional non-fillable contact manifolds
abstract: A bordered Legendrian open book, introduced by Massot, Niederkruger, and Wendl, is a higher dimensional analog of an overtwisted disk. Its existence in a contact manifold obstructs fillability. In this talk, we would discuss an extension of their construction of such objects using equivariant means. This is a joint work in progress with Y. Karshon.

Vincent Colin (Univ. de Nantes)
title: Higher-dimensional Heegaard Floer homology
abstract: In a work in progress with Ko Honda, we extend the definition of the hat version of Heegaard Floer homology to contact manifolds of arbitrary odd dimension using higher-dimensional open book decompositions and the theory of Weinstein domains. This also suggests a reformulation and an extension of Symplectic Khovanov homology to links in arbitrary 3-manifolds.

Mihai Damian (Université de Strasbourg)
title: Lifted Floer homology and topology of monotone Lagrangian submanifolds
abstract: We establish a new version of Floer homology for monotone Lagrangian submanifolds which is related to the homology of the universal cover and more generally to Novikov homology. We get some constraints on the topology of monotone Lagrangian submanifolds in $\mathbf{C}^n$ and in $\mathbf{C}P^n$. In particular we show that there are some Lagrangians in these manifolds which do not admit monotone embeddings.

Urs Frauenfelder (Seoul National University)
title: Spicy Hopf algebras and growth of Reeb chords
abstract: This is joint work with Felix Schlenk. We considerthe homology growth of based loop spaces of closed manifolds whose universal cover is not homotopy equivalent to a finite CW complex.The homology of the based loop space has the structure of a Hopf algebraand we take advantage of the action of the fundamental group on it.

Kei Irie (Kyoto University)
title: Hofer-Zehnder capacity, symplectic homology and loop product
abstract: Hofer-Zehnder(HZ) capacity is a quantitative invariant of symplectic manifolds, which reflects behavior of Hamiltonian flows on manifolds.

Symplectic homology is a version of Floer homology, which is defined for convex symplectic manifolds (e.g. cotangent bundles). We give estimates of HZ capacity using symplectic homology, in particular its product structure. An application to cotangent bundles which involves computations of the Chas-Sullivan loop product is also presented.

Tetsuya Ito (Kyoto University)
title: Overtwisted discs in planar open books
abstract: We show that overtwisted disc in a planar open book can be put in a topologically nice position so that each the intersection of disc and pages is essential. This provides a tightness criterion based on topological method which generalizes Bennequin's proof of the tightness of the standard contact structure of S^3.

Otto van Koert (Seoul National University)
title: Fractional twists and invariant contact structures
abstract: We define fractional twists, a generalization of Dehn twists, and discuss their role in constructing contact structures that are invariant under a circle action. We give some criteria to detect whether these fractional twists are symplectically isotopic to the identity, and then discuss the difference between right- and left-handed twists. We shall show that left-handed twists in an open book often give rise to so-called algebraically overtwisted contact manifolds. In particular, such manifolds are not symplectically fillable. This is joint work with River Chiang and Fan Ding.

Fabiola Manjarrez-Gutierrez (CIMAT, Mexico)
title: Additivity of Morse-Novikov number of a-small knots
abstract: A knot is a-small if its exterior does not contain closed incompressible surfaces disjoint from some incompressible Seifert surface for the knot.

The Morse-Novikov number of a knot is the minimal number of critical points of a Morse map of the knot exterior to the circle. This concept was introduced by Pajitnov, Rudolph and Weber, they also proved that the Morse-Novikov number is subadditive under connected sum of knots.

Given a Morse map of the knot exterior to the circle we can reorganize the critical points to obtain a decomposition of the knot exterior in such a way that the the preimages of regular values are Seifert surfaces which are alternatevely incompressible and weakly incompressible. This notion is

known as circular handle decomposition.

Using circular handle decomposition of knot exteriors we prove that Morse-Novikov number is additive for a-small knots.

Sheila Sandon (Université de Strasbourg)
title: On positive loops of contactomorphisms
abstract: A contact isotopy is said to be positive if it moves every point in a direction positively transverse to the contact distribution. In 2000 Eliashberg and Polterovich noticed that this notion induces for certain contact manifolds (which are called orderable) a partial order on the universal cover of the contactomorphism group. Orderability of a contact manifold turns out to be sensitive to the underlying topology (for example the standard real projective space is orderable while the standard sphere is not), and was later discovered by Eliashberg, Kim and Polterovich to be deeply related to a non-squeezing phenomenon in contact topology. In my talk I will review these topics and discuss the fact that orderability is equivalent to the non-degeneracy of a natural bi-invariant metric on the universal cover of the contactomorphism group (joint work with V. Colin). I will also present a recent result about small positive loops on overtwisted contact manifolds (joint with R. Casals and F. Presas).

Yasha Savelyev (ICMAT, Madrid)
title: Global Fukaya category
abstract: We introduce a kind of Fukaya category for a smooth Hamiltonian fibration over a general smooth manifold, and discuss an application to Hofer geometry. This story is intimately connected to Toen's derived Morita theory, and theory of quasi-categories after Lurie and Joyal. This gives a nice geometric context for some high powered abstraction in category theory, and raises a number of interesting questions and possibilities.

Tadayuki Watanabe (Shimane University)
title: Morse theory and Lescop's equivariant propagator
abstract: For a 3-manifold M with $b_1(M) = 1$ fibered over S^1 and the gradient ξ of a fiberwise Morse function on M, we introduce the notion of "amidakuji path" on M. An amidakuji path is a piecewise smooth path on M consisting of edges each of which is either a part of a critical locus of ξ or an integral curve of ξ. Counting closed amidakuji paths with signs gives the Lefschetz zeta function of M. The "moduli space" of amidakuji paths on M gives Lescop's equivariant propagator, which can be used to define $\mathbb{Z}$-equivariant version of Chern–Simons perturbation theory for M.